Investigations of Parton Densities at Very High x

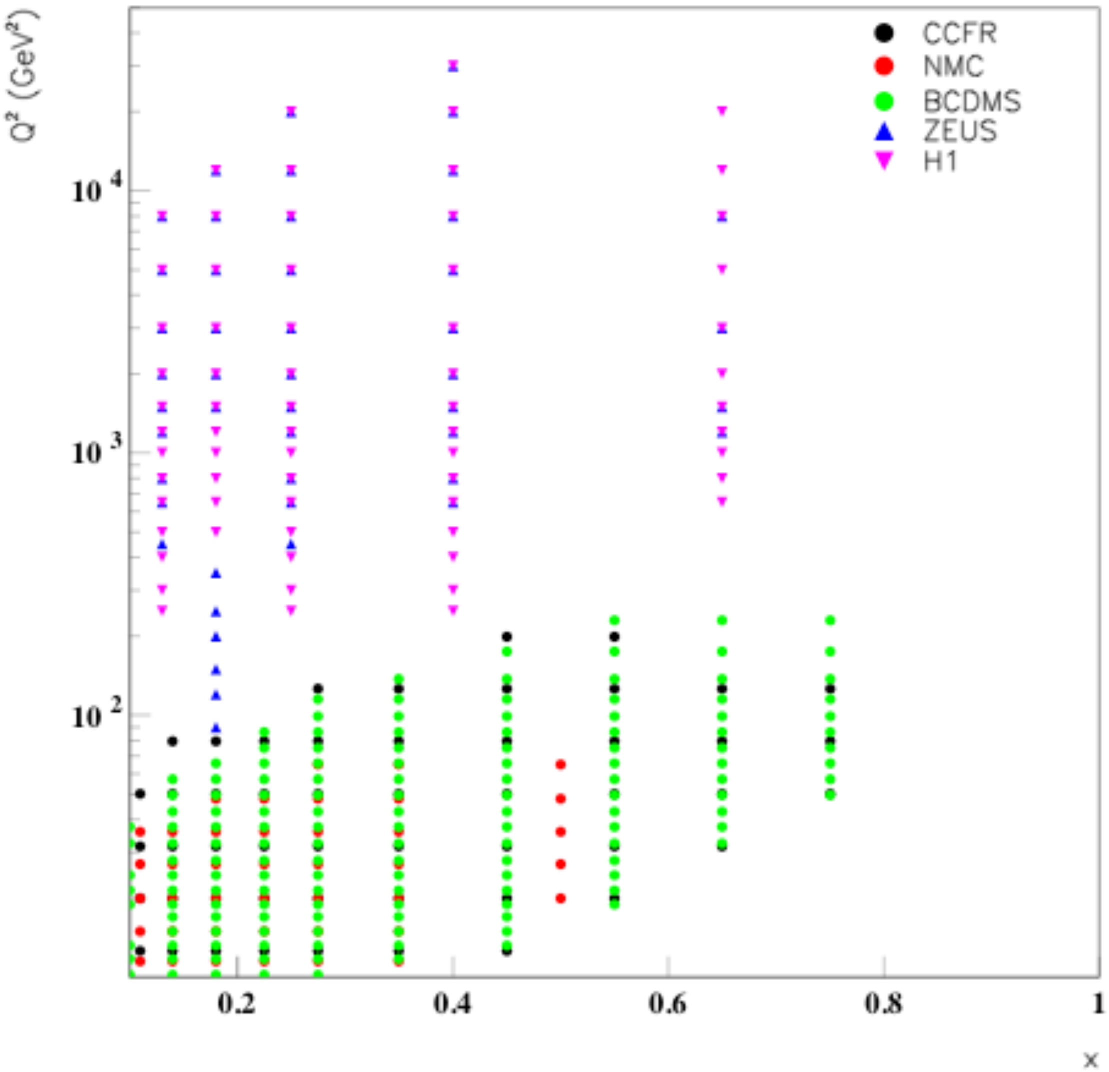
Allen Caldwell
Max Planck Institute for Physics
on behalf of the ZEUS Collaboration

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Motivation

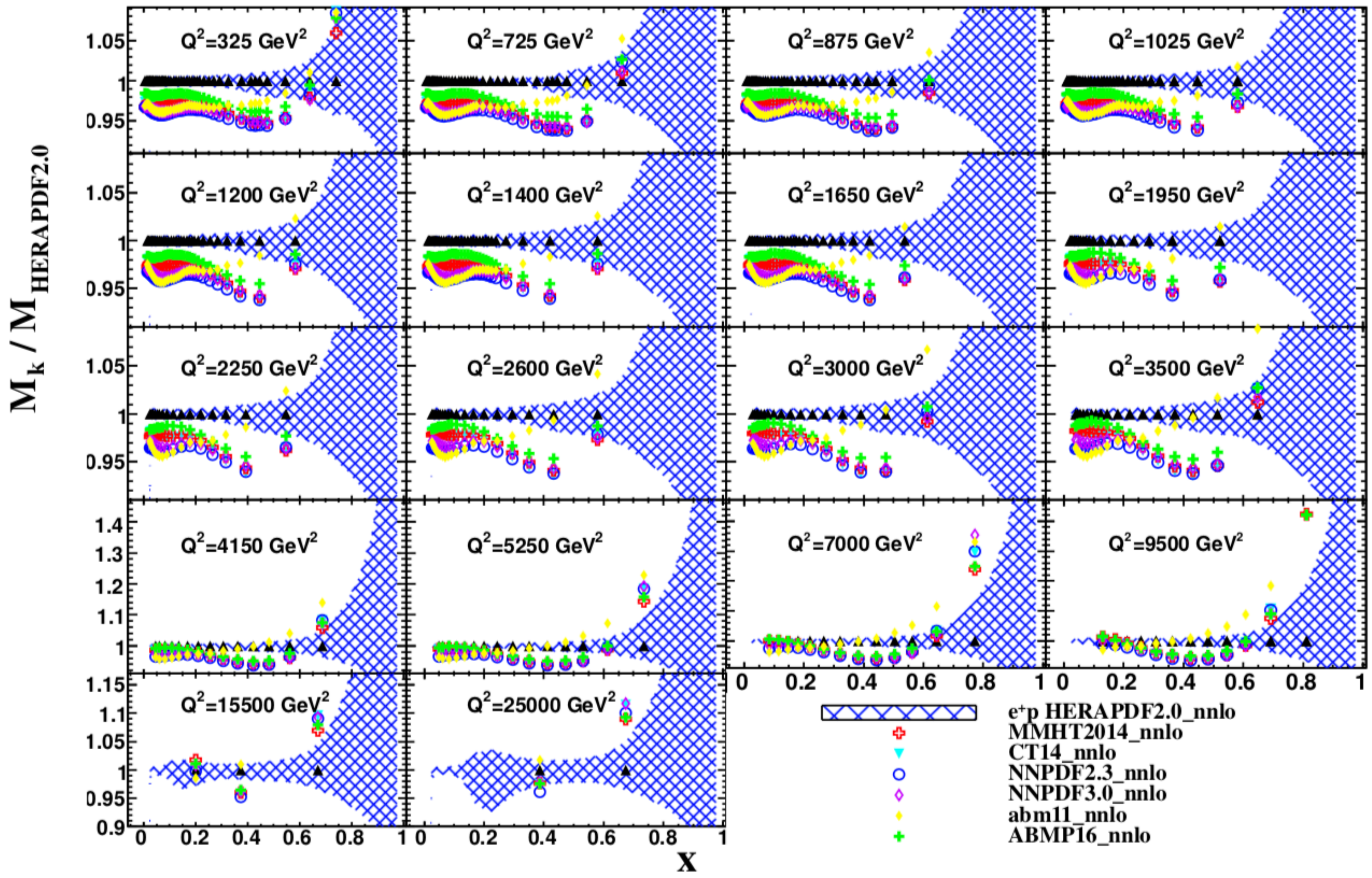
Our information on the very high x behavior of the parton densities is primarily theoretical.



BCDMS has measured F_2 up to x=0.75

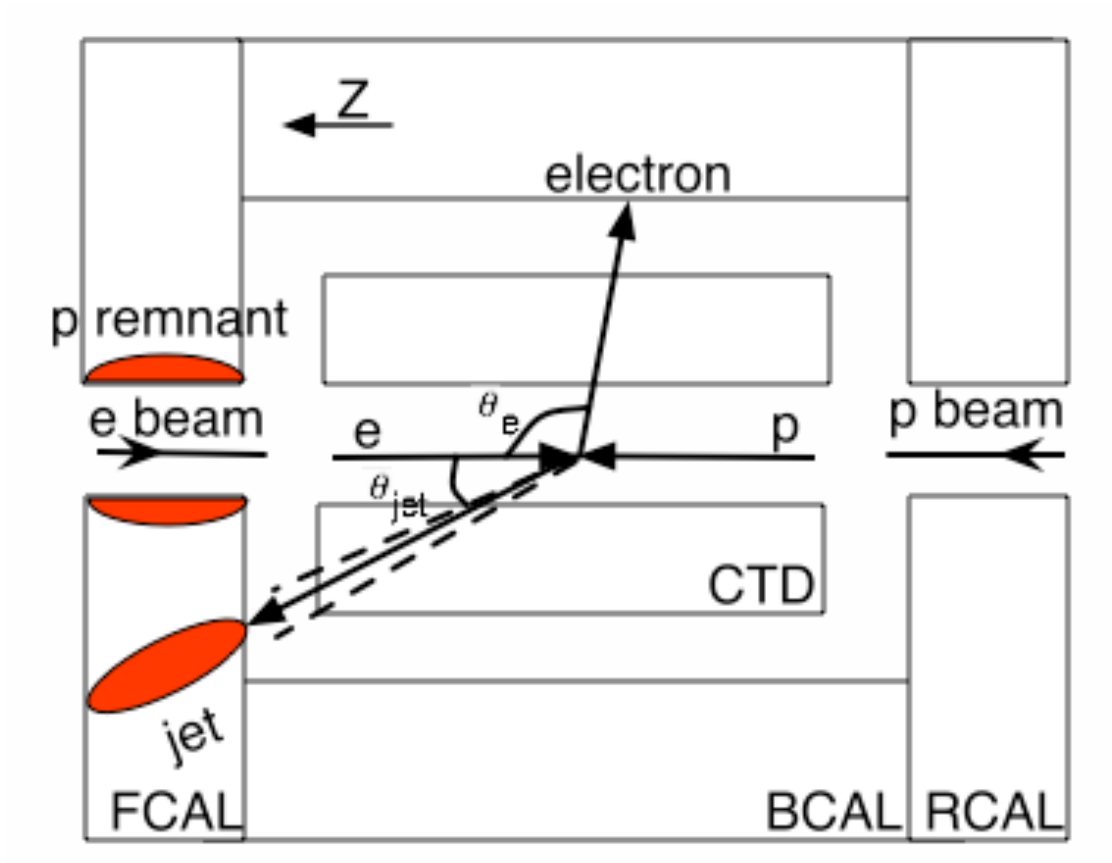
H1, ZEUS have measured F_2 up to x=0.65

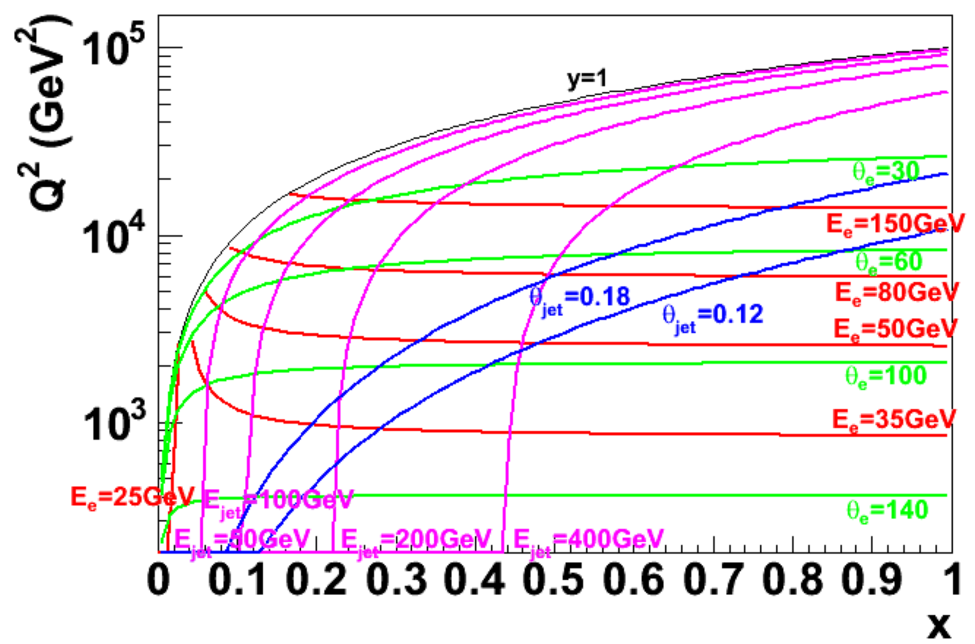
ZEUS has measured up to x=1, but these data are not (yet) included in PDF fits.



The PDF's are poorly determined at high-x. Sizable differences in expectations (much bigger than quoted uncertainties) despite the fact that fits typically use similar parametrization $xq \propto (1-x)^{\eta}$. Is it possible to improve this situation ?

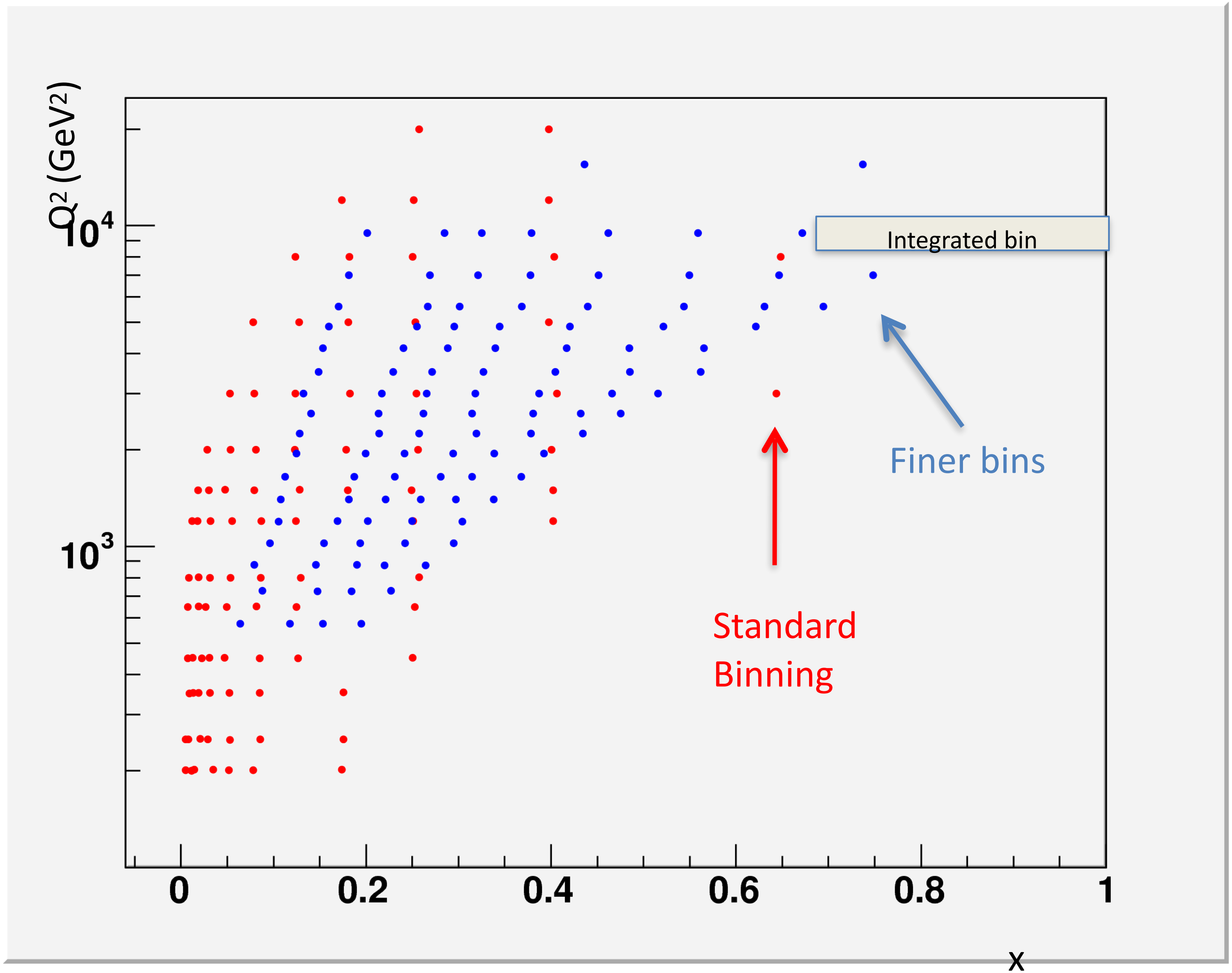
ZEUS high-x analysis

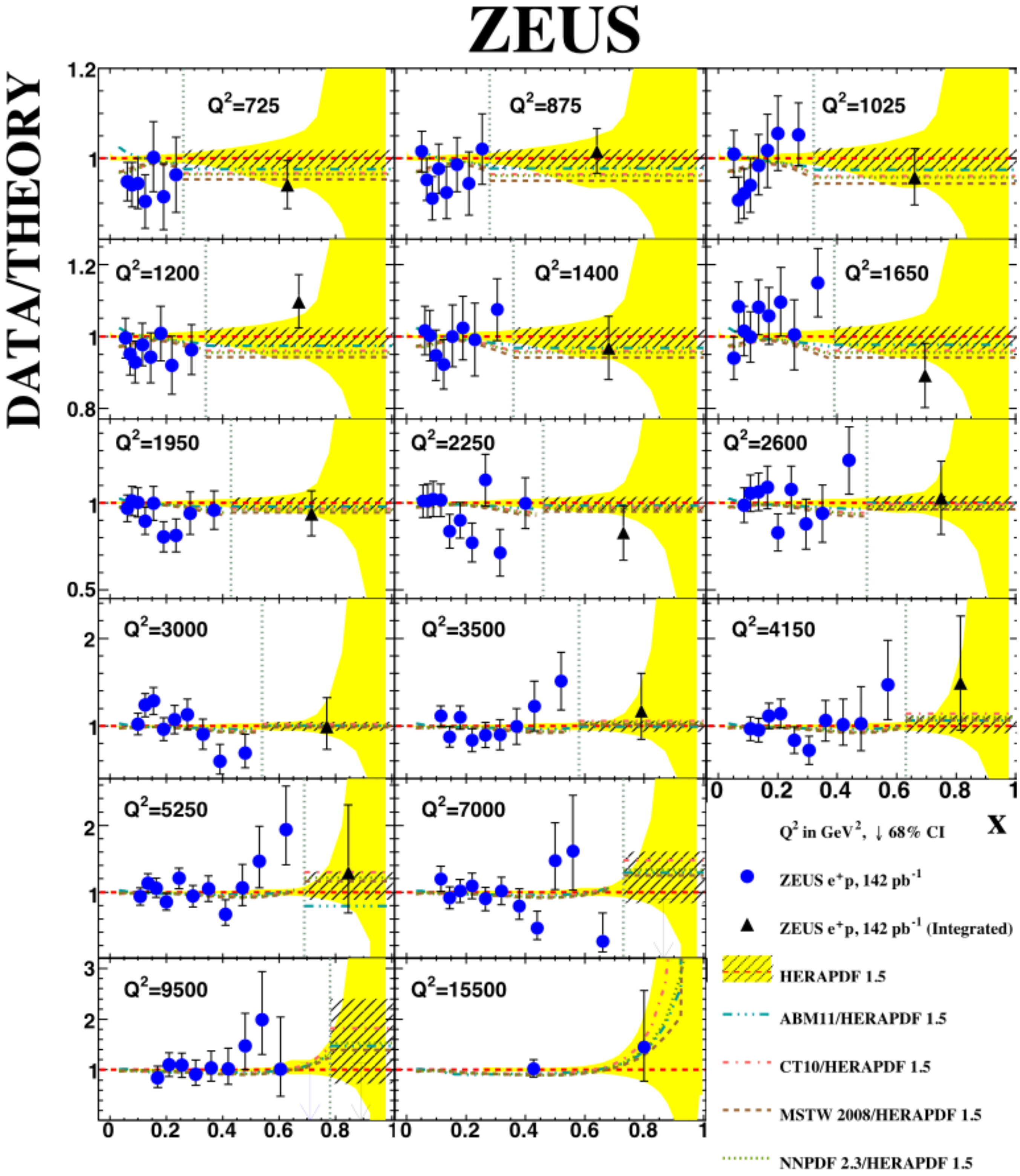




- At high Q^2, scattered electron seen with ≈100% acceptance
- For not too high x, measure x from jet: $$\frac{d^2\sigma}{dx dQ^2}$$
- For $x>x_{Edge}$, measure $$\int_{x_{Edge}}^{1} \frac{d^2\sigma}{dx dQ^2} dx$$

Fine-grained cross section measurements





Error bars indicate the range of probable values for the underlying cross section given the measured data. How to use this information in a fit ?

Use the observed number of events & calculate the probability to see this number given the model expectation.

5250	0.62	5	$1.76e-04$	$^{+55.2}_{-35.2}$	$^{+11.1}_{-10.5}$	$^{+9.2}_{-9.1}$	$^{-3.2}_{+4.6}$	$^{+0.1}_{+0.1}$	$^{+3.7}_{-3.7}$	$^{+0.6}_{-0.6}$
7000	0.12	93	$1.61e-02$	$^{+10.4}_{-10.4}$	$^{+4.0}_{-5.1}$	$^{+3.3}_{-3.6}$	$^{-1.1}_{+0.8}$	$^{+0.8}_{-0.5}$	$^{-0.7}_{+0.7}$	$^{+0.0}_{-0.0}$
7000	0.14	89	$1.25e-02$	$^{+10.6}_{-10.6}$	$^{+3.7}_{-5.2}$	$^{+3.4}_{-3.5}$	$^{-1.3}_{+1.2}$	$^{-0.5}_{+0.2}$	$^{-0.9}_{+0.9}$	$^{+0.0}_{-0.0}$
7000	0.18	68	$7.02e-03$	$^{+12.1}_{-12.1}$	$^{+3.9}_{-3.6}$	$^{+3.4}_{-3.4}$	$^{-0.6}_{+0.6}$	$^{-0.6}_{+0.4}$	$^{-0.4}_{+0.4}$	$^{+0.0}_{-0.0}$
7000	0.22	56	$5.60e-03$	$^{+13.4}_{-13.4}$	$^{+4.2}_{-4.2}$	$^{+3.9}_{-3.9}$	$^{-1.4}_{+1.1}$	$^{-0.4}_{+1.0}$	$^{-0.4}_{+0.4}$	$^{+0.0}_{-0.0}$
7000	0.26	49	$3.79e-03$	$^{+14.3}_{-14.3}$	$^{+4.6}_{-4.8}$	$^{+3.9}_{-4.0}$	$^{-0.2}_{+2.1}$	$^{+0.2}_{-0.2}$	$^{-0.5}_{+0.5}$	$^{+0.0}_{-0.0}$
7000	0.32	41	$2.70e-03$	$^{+15.6}_{-15.6}$	$^{+5.1}_{-4.7}$	$^{+5.3}_{-4.5}$	$^{-1.4}_{+0.8}$	$^{-0.4}_{+0.4}$	$^{-0.2}_{+0.2}$	$^{+0.0}_{-0.0}$
7000	0.38	23	$1.52e-03$	$^{+20.9}_{-20.9}$	$^{+6.4}_{-6.2}$	$^{+5.5}_{-5.5}$	$^{-1.8}_{+1.7}$	$^{-0.7}_{+0.4}$	$^{+2.0}_{-2.0}$	$^{+0.0}_{-0.0}$
7000	0.44	17	$1.15e-03$	$^{+27.2}_{-21.3}$	$^{+8.4}_{-7.9}$	$^{+7.1}_{-7.1}$	$^{-2.7}_{+2.7}$	$^{-0.0}_{+0.2}$	$^{-2.4}_{+2.4}$	$^{+0.0}_{-0.0}$
7000	0.50	8	$5.38e-04$	$^{+41.8}_{-29.4}$	$^{+9.7}_{-10.3}$	$^{+9.5}_{-9.5}$	$^{-1.6}_{+1.4}$	$^{-0.3}_{+0.4}$	$^{+2.4}_{-2.4}$	$^{+0.1}_{-0.1}$
7000	0.56	4	$2.37e-04$	$^{+63.2}_{-38.2}$	$^{+12.3}_{-11.8}$	$^{+11.3}_{-11.3}$	$^{-3.4}_{+3.4}$	$^{+0.1}_{-0.0}$	$^{+1.2}_{-1.2}$	$^{+0.2}_{-0.2}$
7000	0.66	10	$2.30e-04$	$^{+36.7}_{-26.8}$	$^{+12.6}_{-13.6}$	$^{+12.1}_{-12.3}$	$^{-4.3}_{+2.5}$	$^{-0.3}_{-0.0}$	$^{+2.0}_{-2.0}$	$^{+0.9}_{-0.9}$
9500	0.17	76	$6.77e-03$	$^{+11.5}_{-11.5}$	$^{+5.6}_{-7.7}$	$^{+4.9}_{-4.9}$	$^{-2.0}_{+2.3}$	$^{+0.2}_{-0.2}$	$^{-0.6}_{+0.6}$	$^{+0.0}_{-0.0}$
9500	0.21	53	$3.87e-03$	$^{+13.7}_{-13.7}$	$^{+5.8}_{-5.1}$	$^{+4.3}_{-4.5}$	$^{-1.1}_{+1.8}$	$^{-0.7}_{+0.4}$	$^{-1.1}_{+1.1}$	$^{+0.0}_{-0.0}$
9500	0.25	40	$2.27e-03$	$^{+15.8}_{-15.8}$	$^{+4.8}_{-4.9}$	$^{+4.5}_{-4.5}$	$^{-2.0}_{+1.5}$	$^{+0.2}_{+0.4}$	$^{+0.1}_{-0.1}$	$^{+0.0}_{-0.0}$
9500	0.31	27	$1.50e-03$	$^{+19.2}_{-19.2}$	$^{+5.7}_{-8.1}$	$^{+5.2}_{-5.3}$	$^{-2.6}_{+1.5}$	$^{-0.5}_{+0.2}$	$^{+1.5}_{-1.5}$	$^{+0.0}_{-0.0}$
9500	0.36	19	$8.89e-04$	$^{+25.5}_{-20.3}$	$^{+6.6}_{-6.1}$	$^{+5.9}_{-5.9}$	$^{-1.0}_{+1.9}$	$^{-0.4}_{+0.2}$	$^{-0.3}_{+0.3}$	$^{+0.0}_{-0.0}$
9500	0.42	12	$5.64e-04$	$^{+33.1}_{-24.8}$	$^{+11.3}_{-7.5}$	$^{+13.4}_{-7.3}$	$^{-1.0}_{+2.4}$	$^{-0.8}_{+0.5}$	$^{-0.8}_{+0.8}$	$^{+0.0}_{-0.0}$
9500	0.48	8	$3.63e-04$	$^{+41.7}_{-29.4}$	$^{+10.5}_{-10.4}$	$^{+9.2}_{-9.2}$	$^{-2.6}_{+2.4}$	$^{-0.4}_{+0.6}$	$^{-3.4}_{+3.4}$	$^{+0.0}_{-0.0}$
9500	0.54	5	$2.31e-04$	$^{+55.2}_{-35.2}$	$^{+14.3}_{-13.7}$	$^{+12.5}_{-12.2}$	$^{-1.7}_{+3.6}$	$^{-0.2}_{+0.6}$	$^{+5.7}_{-5.7}$	$^{+0.1}_{-0.1}$
9500	0.61	4	$1.39e-04$	$^{+63.3}_{-38.3}$	$^{+15.5}_{-15.4}$	$^{+14.6}_{-14.8}$	$^{-4.2}_{+4.2}$	$^{+0.0}_{-0.1}$	$^{+0.4}_{-0.4}$	$^{+0.4}_{-0.4}$
9500	0.71	1	$1.50e-05$	$^{+158.0}_{-58.0}$	$^{+21.1}_{-19.8}$	$^{+18.9}_{-18.9}$	$^{-3.3}_{+4.5}$	$^{-0.4}_{+0.3}$	$^{+4.8}_{-4.8}$	$^{+1.3}_{-1.3}$

Not many events at high x

This uncertainty refers to how well we know the underlying cross section assuming that our only knowledge is the observed number of events. **Not the uncertainty that belongs in a fit.**

Statistical Procedure

Need to use Poisson statistics since event counts are small.

$$P(D\,|\,M_k) = \prod_j \frac{e^{-\nu_{j,k}}\nu_{j,k}^{n_j}}{n_j!}$$

Probability of the data (likelihood)

$$\nu_{j,k} = \mathscr{L}\int_{(\Delta x,\Delta Q^2)_j}\left[\int T(x_{\mathrm{rec}},Q^2_{\mathrm{rec}}\,|\,x,Q^2)\frac{d^2\sigma(x,Q^2\,|\,M_k)}{dxdQ^2}dxdQ^2\right]dx_{\mathrm{rec}}dQ^2_{\mathrm{rec}}$$

expectation

$$\nu_{j,k} \approx \sum_i t_{ij}\nu_{i,k}$$

transfer matrix realization

$$t_{ij} = K_{il}a_{lj}$$

separate radiative & detectors/analysis effects

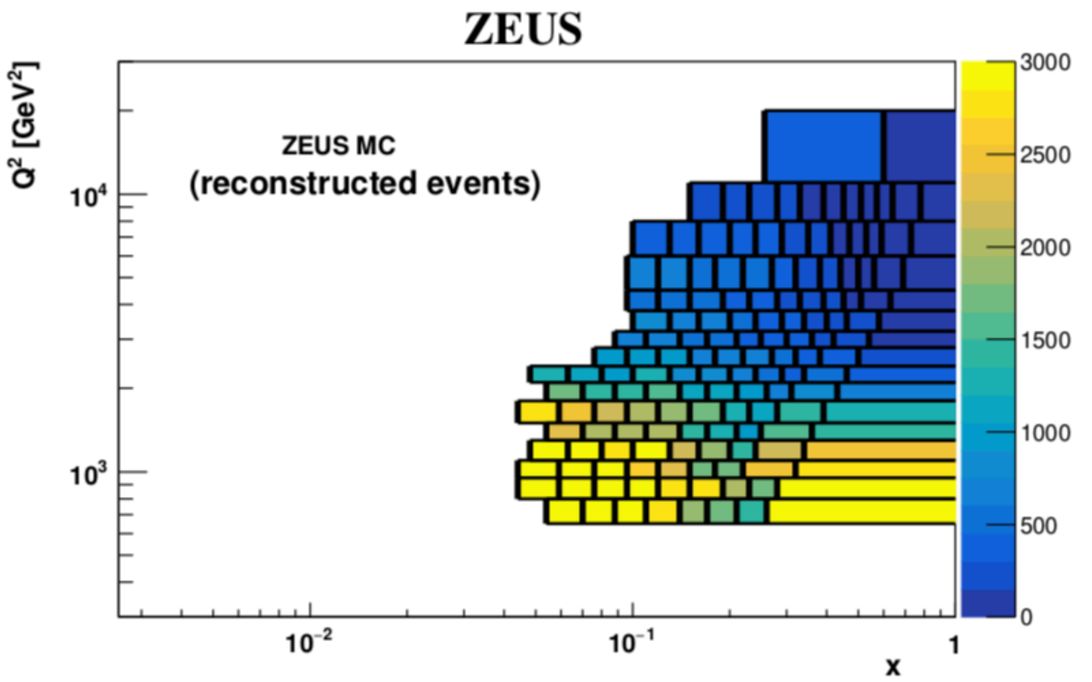
Radiative matrix from HERACLES - M generated number of events

$$K_{ii} = \frac{M_i}{\mathscr{L}^{\mathrm{MC}}\sigma_{i,o}}$$

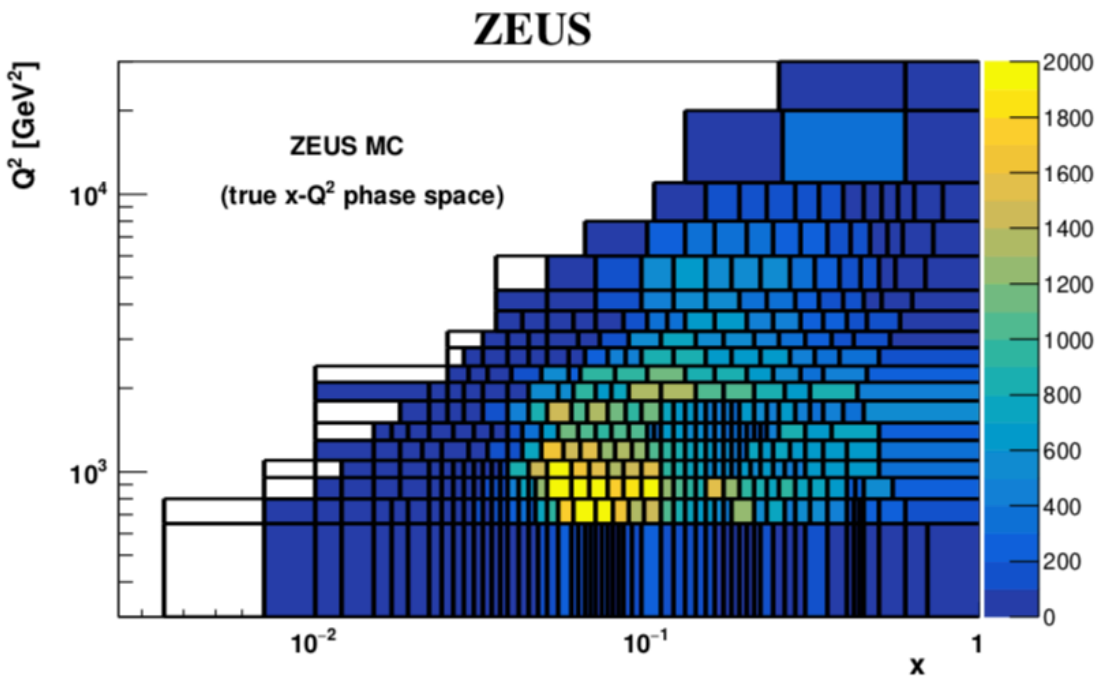
$$a_{ij} = \frac{\sum_{m=1}^{M_i}\omega_m I(m\in j)}{\sum_{m=1}^{M_i}\omega_m^{MC}}$$

detector/analysis matrix from ZEUS simulation

Detector/Analysis transfer matrix



Reconstructed variables/bins



generated variables/bins. Much finer binning used.

Comparison to Data

PDF	e^-p	e^+p
$HERAPDF2.0$	0.05	0.5
$CT14$	0.002	0.8
$MMHT2014$	0.002	0.8
$NNPDF2.3$	0.00007	0.6
$NNPDF3.0$	0.0002	0.7
$ABMP16$	0.01	0.8
$ABM11$	0.001	0.6

p-value for e-p and e+p data sets are shown on comparison to different PDFs
(includes only statistical fluctuation from Poisson probabilities).

Significant differences in agreement between different pdf sets. Also significant differences between electron & positron data.

In both cases, differences in $\Delta\chi^2_{k,l} = -2\ln\frac{P(D|M_k)}{P(D|M_l)}$ of >20 units.

Comparison to Data

	e^-p		e^+p	
PDF	$x < 0.6$	$x \geq 0.6$	$x < 0.6$	$x \geq 0.6$
HERAPDF2.0	0.06	0.2	0.6	0.1
CT14	0.0008	0.2	0.7	0.6
MMHT2014	0.00003	0.1	0.6	0.6
NNPDF2.3	0.00007	0.2	0.6	0.6
NNPDF3.0	0.00003	0.2	0.6	0.6
ABMP16	0.01	0.2	0.8	0.5
ABM11	0.03	0.3	0.7	0.4

p-value for e-p and e+p data sets are shown on comparison to different PDFs for two different x ranges.

biggest discrepancy: lower x electron data. But p-values are not the whole story …

Systematic Uncertainties

Categories investigated:

A. Uncertainties on expectation at generator level
   1. **normalization due to luminosity uncertainty**
   2. knowledge of radiative effects
   3. pdf set uncertainties

B. Uncertainties on transfer matrix
   4. binning effects (too crude ?)
   5. finite Monte Carlo statistics
   6. imperfect events & detector simulation

By far the dominant effect in terms of the probability of the data given a pdf set is the normalization.

Nomalization Error : Vary generated events by 1.8 % up and down and calculate new p-value

+1.8 %				
	e^-p		e^+p	
PDF	$x < 0.6$	$x \geq 0.6$	$x < 0.6$	$x \geq 0.6$
HERAPDF2.0	0.02	0.1	0.2	0.3
CT14	0.02	0.3	0.8	0.5
MMHT2014	0.008	0.2	0.8	0.5
NNPDF2.3	0.009	0.3	0.8	0.4
NNPDF3.0	0.008	0.3	0.8	0.4
ABMP16	0.04	0.3	0.6	0.4
ABM11	0.03	0.3	0.4	0.2
−1.8 %				
	e^-p		e^+p	
PDF	$x < 0.6$	$x \geq 0.6$	$x < 0.6$	$x \geq 0.6$
HERAPDF2.0	0.03	0.3	0.8	0.2
CT14	0.0	0.08	0.4	0.6
MMHT2014	0.0	0.04	0.2	0.6
NNPDF2.3	0.0	0.08	0.2	0.6
NNPDF3.0	0.0	0.08	0.2	0.6
ABMP16	0.0003	0.1	0.7	0.6
ABM11	0.004	0.2	0.7	0.5

(Scale M by 1.8% up)

	e^-p		e^+p	
PDF	$x < 0.6$	$x \geq 0.6$	$x < 0.6$	$x \geq 0.6$
HERAPDF2.0	0.06	0.2	0.6	0.1
CT14	0.0008	0.2	0.7	0.6
MMHT2014	0.00003	0.1	0.6	0.6
NNPDF2.3	0.00007	0.2	0.6	0.6
NNPDF3.0	0.00003	0.2	0.6	0.6
ABMP16	0.01	0.2	0.8	0.5
ABM11	0.03	0.3	0.7	0.4

(Scale M by 1.8% down)

Dominant systematics : due to error in normalization of data quoted as 1.8 %

Main difference - normalization. Cannot be moved both up/down -> not all the pdf sets can represent the data well.

Summary

- There is very little data as x-> 1 and high virtuality
- ZEUS high-x data unique, but not used in PDF fits
- uncertainties in the PDFs appear to be underestimated, judged by comparing their predictions to each other (**my personal conclusion**)
- a transfer matrix formulation makes it possible to compare PDF set predictions to the ZEUS high-x data and calculate probabilities
- systematic uncertainties can be included in the analysis
- effect on PDF extractions to be evaluated: may require redefinition of PDF uncertainties (**my personal conclusion**).