

# Combination of beauty and charm production cross section measurements in deep inelastic $ep$ scattering at HERA

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Measurements of open beauty and charm production cross sections in deep inelastic  $ep$  scattering at HERA from the H1 and ZEUS Collaborations are combined. Reduced cross sections for beauty and charm production are obtained in the kinematic range of photon virtuality  $2.5 \leq Q^2 \leq 2000 \text{ GeV}^2$  and Bjorken scaling variable  $3 \times 10^{-5} \leq x_{\text{Bj}} \leq 5 \times 10^{-2}$ . The combination method accounts for the correlations of the statistical and systematic uncertainties among the different data sets. The combined data are compared to perturbative QCD predictions and used together with the combined inclusive deep inelastic scattering cross sections from HERA to determine the charm and beauty quark masses.

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## 1. Introduction

Measurements of open charm and beauty production in deep inelastic electron<sup>1</sup>–proton scattering (DIS) at HERA provide important input for stringent tests of QCD. This analysis is an extension of the previous H1 and ZEUS combination [1] of charm measurements in DIS [2–9] with new charm and beauty data [2, 10–14]. The reduced charm,  $\sigma_{\text{red}}^{c\bar{c}}$ , and beauty,  $\sigma_{\text{red}}^{b\bar{b}}$ , cross sections are combined to create one consistent set of charm and beauty cross sections in the kinematic range of photon virtuality  $2.5 \leq Q^2 \leq 2000 \text{ GeV}^2$  and Bjorken scaling variable  $3 \times 10^{-5} \leq x_{\text{Bj}} \leq 5 \times 10^{-2}$ . The data are compared to theoretical predictions obtained in the fixed-flavour-number scheme (FFNS) at next-to-leading order (NLO) QCD and approximate next-to-next-to-leading order (NNLO) using different proton parton distribution functions (PDFs) and used together with the inclusive DIS cross sections from HERA [15] to extract the charm and beauty quark masses.

## 2. Input data and combination method

The input data samples [2–14] used in the combination are listed in Tab. 1. The quantities to be combined are the reduced charm and beauty cross sections, defined as:

$$\sigma_{\text{red}}^{Q\bar{Q}} = \frac{d^2\sigma^{Q\bar{Q}}}{dx_{\text{Bj}}dQ^2} \cdot \frac{x_{\text{Bj}}Q^4}{2\pi\alpha^2(1+(1-y)^2)}. \quad (2.1)$$

Here  $Q\bar{Q}$  stands for  $c\bar{c}$  or  $b\bar{b}$  quark-antiquark pairs, and  $y$  is the inelasticity. The combined cross sections are determined at common  $(x_{\text{Bj}}, Q^2)$  points.

Data set	Tagging	$Q^2$ range [GeV <sup>2</sup> ]	$\mathcal{L}$ [pb <sup>-1</sup> ]	$\sqrt{s}$ [GeV]	$N_c$	$N_b$
1 H1 VTX [2]	VTX	5 – 2000	245	318	29	12
2 H1 $D^{*+}$ HERA-I [3]	$D^{*+}$	2 – 100	47	318	17	
3 H1 $D^{*+}$ HERA-II (medium $Q^2$ ) [4]	$D^{*+}$	5 – 100	348	318	25	
4 H1 $D^{*+}$ HERA-II (high $Q^2$ ) [5]	$D^{*+}$	100 – 1000	351	318	6	
5 ZEUS $D^{*+}$ 96-97 [6]	$D^{*+}$	1 – 200	37	300	21	
6 ZEUS $D^{*+}$ 98-00 [7]	$D^{*+}$	1.5 – 1000	82	318	31	
7 ZEUS $D^0$ 2005 [8]	$D^0$	5 – 1000	134	318	9	
8 ZEUS $\mu$ 2005 [9]	$\mu$	20 – 10000	126	318	8	8
9 ZEUS $D^+$ HERA-II [10]	$D^+$	5 – 1000	354	318	14	
10 ZEUS $D^{*+}$ HERA-II [11]	$D^{*+}$	5 – 1000	363	318	31	
11 ZEUS VTX HERA-II [12]	VTX	5 – 1000	354	318	18	17
12 ZEUS $e$ HERA-II [13]	$e$	10 – 1000	363	318		9
13 ZEUS $\mu$ + jet HERA-I [14]	$\mu$	2 – 3000	114	318		11

**Table 1:** Data sets used in the combination. For each data set the  $Q^2$  range, integrated luminosity ( $\mathcal{L}$ ), centre-of-mass energy ( $\sqrt{s}$ ) and the numbers of charm ( $N_c$ ) and beauty ( $N_b$ ) measurements are given.

The results of the H1 inclusive lifetime analysis (dataset 1) are directly taken from the original measurement in the form of  $\sigma_{\text{red}}^{c\bar{c}}$  and  $\sigma_{\text{red}}^{b\bar{b}}$  and transformed, when needed, to the common  $(x_{\text{Bj}}, Q^2)$

<sup>1</sup>In this note, ‘electron’ is used to denote both electron and positron if not stated otherwise. For  $D$  mesons, charge-conjugate modes are implied.

points using theoretical predictions. For all other measurements the reduced cross sections are obtained from the visible cross sections,  $\sigma_{\text{vis,bin}}$ , defined as the  $D$ -,  $\mu$ -,  $e$ - or jet-production cross sections in a particular kinematic range, using theoretical predictions. The combination of reduced cross sections is based on the procedure described elsewhere and used in previous HERA combinations [1, 15–18], accounting for all correlations in the uncertainties. For datasets 1, 8 and 11 statistical correlations between charm and beauty cross sections are accounted for.

### 3. Theoretical predictions

The cross-section predictions are obtained using the HVQDIS program [19] and the xFITTER (formerly HERAFITTER) framework (version 1.2.0) [20] which provide NLO QCD ( $O(\alpha_s^2)$ ) calculations in the 3-flavour FFNS for charm and beauty production in DIS. The predictions obtained with HVQDIS, which allows fully differential cross sections to be calculated, are used for phase-space corrections, while for the comparison to the combined data the predictions obtained with the xFITTER framework are used, which provides reduced cross sections only, but has the advantage of using the running heavy-quark mass definition as implemented in the OPENQCDRAD program [21]. The following parameters are used in the calculations:

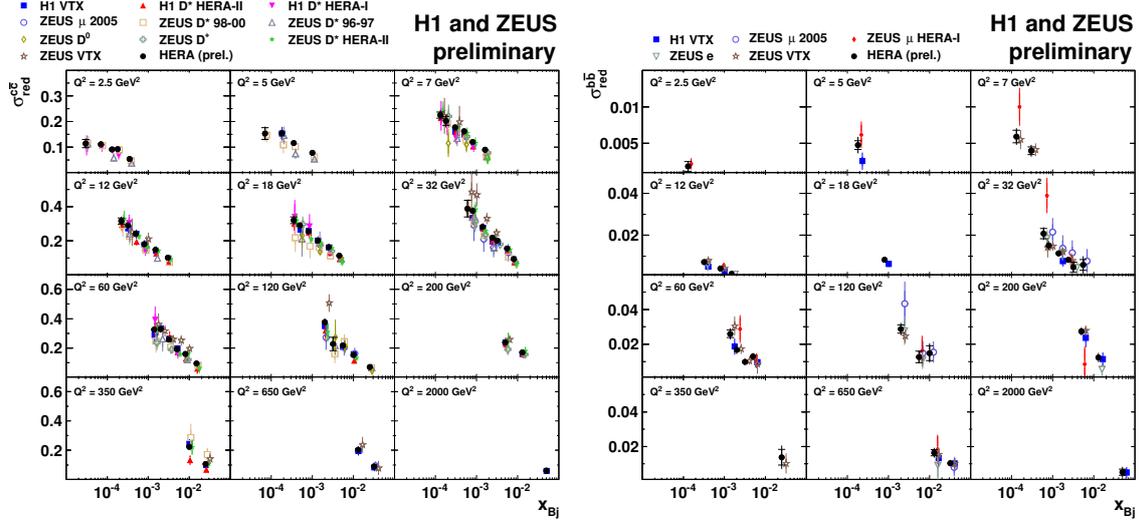
- The **pole masses of the  $c$  and  $b$  quarks** are set to  $m_c = 1.50 \pm 0.15$  GeV and  $m_b = 4.50 \pm 0.25$  GeV, respectively; the **running  $c$  and  $b$  quark masses** are set to the PDG values  $m_c(m_c) = 1.27 \pm 0.03$  GeV and  $m_b(m_b) = 4.18 \pm 0.03$  GeV [22].
- The **renormalisation and factorisation scales** are taken as  $\mu_r = \mu_f = \sqrt{Q^2 + 4m_Q^2}$ , where  $m_Q$  is  $m_c$  or  $m_b$ . The scales are varied simultaneously up or down by a factor of two.
- For the **strong coupling constant** the value  $\alpha_s^{n_f=3}(M_Z) = 0.105 \pm 0.002$  is chosen which corresponds to  $\alpha_s^{n_f=5}(M_Z) = 0.116 \pm 0.002$ .
- The **proton PDFs** are described by a series of FFNS variants of the HERAPDF1.0 set [18].

The NLO calculations performed with the HVQDIS program are extended with fragmentation models to provide hadron level cross sections. The fragmentation model for  $c$  quarks is described in detail in [1]. The fragmentation model for  $b$  quarks uses the Peterson et al. [23] parametrisation with  $\varepsilon_b = 0.0035 \pm 0.0020$  [24]. The fragmentation fractions of  $c$  quarks into specific charmed hadrons are taken from [25]. The branching fractions of semi-leptonic decays of heavy-quarks to a muon or electron are taken from [22] with the decay spectra of leptons modelled according to [26].

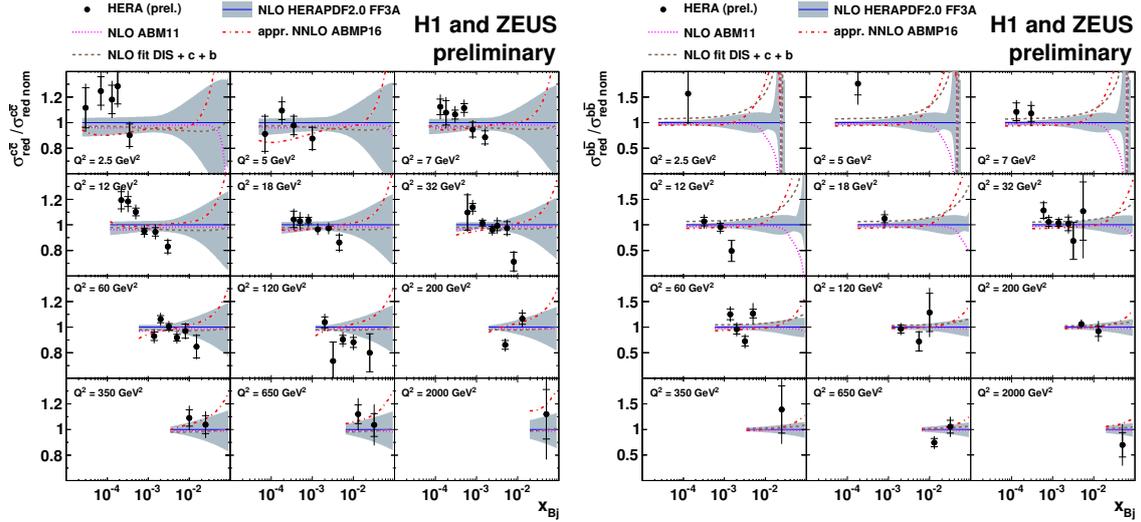
### 4. Combined data and QCD analysis

In total, 209 charm and 57 beauty data points are combined simultaneously to 52 reduced charm and 27 beauty cross-section measurements, respectively. A total  $\chi^2$  of 149 for 187 degrees of freedom (dof) is obtained in the combination indicating consistency of input data and conservative estimates of the uncertainties. The individual datasets as well as the results of the combination are shown in Fig. 1, while Fig. 2 present a comparison of the NLO QCD predictions in the FFNS to the combined data. The theoretical uncertainties on these plots present the mass, scale and PDF<sup>2</sup> variations added in quadrature. The predictions describe the data reasonably well within the uncertainties in the whole kinematic range.

<sup>2</sup>Only experimental uncertainties ('EIG') of HERAPDF2.0 are considered.



**Figure 1:** Combined reduced charm (left) and beauty (right) cross sections (full circles) as a function of  $x_{Bj}$  for different values of  $Q^2$ . The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in  $x_{Bj}$ .



**Figure 2:** Combined reduced charm (left) and beauty (right) cross sections, compared to the NLO and approximate NNLO QCD predictions obtained using various PDFs, normalised to HERAPDF2.0 FF3A.

The combined beauty and charm data are included in a QCD analysis at NLO, performed using xFITTER [20], together with the combined HERA inclusive DIS data [15]. The number of active flavours is set to  $n_f = 3$  at all scales. The heavy-quark masses are left free in the fit. For the light-flavour contributions to the inclusive DIS cross sections, the pQCD scales are set to  $\mu_r = \mu_f = Q$ . The massless contribution to the longitudinal structure function  $F_L$  is calculated to  $O(\alpha_s)$ . The strong coupling is set to  $\alpha_s^{n_f=3}(M_Z) = 0.106$ . The  $Q^2$  range of the inclusive HERA data is restricted to  $Q^2 > Q_{\min}^2 = 3.5 \text{ GeV}^2$ . No such cut is applied to the charm and beauty data since  $Q^2 + 4m_Q^2$  is always above  $3.5 \text{ GeV}^2$ .

The  $\chi^2$  definition follows that of Eq. (32) in Ref. [15]. At the initial QCD evolution scale  $\mu_{f0}^2 = 1.9 \text{ GeV}^2$ , the gluon distribution  $xg(x)$ , the valence quark distributions  $xu_v(x)$  and  $xd_v(x)$ , and the  $u$ - and  $d$ -type antiquark distributions  $x\bar{U}(x)$  and  $x\bar{D}(x)$  are parametrised as:

$$\begin{aligned} xg(x) &= A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}, \\ xu_v(x) &= A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2), \\ xd_v(x) &= A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}, \\ x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x), \\ x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}, \end{aligned} \quad (4.1)$$

assuming  $x\bar{U}(x) = x\bar{u}(x)$  and  $x\bar{D}(x) = x\bar{d}(x) + x\bar{s}(x)$ . Here,  $x\bar{u}(x)$ ,  $x\bar{d}(x)$ , and  $x\bar{s}(x)$  are the up, down, and strange antiquark distributions, respectively. The parameters  $A_{u_v}$ ,  $A_{d_v}$ , and  $A_g$  are determined by the sum rules. The parameter  $C'_g$  is fixed to 25 [27]. Additional constraints  $B_{\bar{U}} = B_{\bar{D}}$  and  $A_{\bar{U}} = A_{\bar{D}}(1 - f_s)$  are imposed. The strangeness fraction  $f_s = xs/(xd + xs)$  is fixed to  $f_s = 0.4$  [15].

The PDF uncertainties are estimated as in the general approach of HERAPDF2.0 [15] in which the fit, model, and parametrisation uncertainties are taken into account. Fit uncertainties are determined using the tolerance criterion of  $\Delta\chi^2 = 1$ . Model uncertainties arise from the simultaneous variation of the factorisation and renormalisation scales up and down by a factor of two, the variations of  $\alpha_s^{n_f-3}(M_Z) = 0.106 \pm 0.0015$ ,  $0.3 \leq f_s \leq 0.5$ , and  $2.5 \leq Q_{\min}^2 \leq 5.0 \text{ GeV}^2$ . The parametrisation uncertainty is estimated by extending the functional form in Eq. (4.1) with additional parameters  $D$  and  $E$  added one at a time. An additional parametrisation uncertainty is considered by using the functional form with  $E_{u_v} = 0$ , as the  $\chi^2$  in this variant of the fit is only 5 units worse than with the released  $E_{u_v}$  parameter. Furthermore,  $\mu_{f0}^2$  is varied within  $1.6 \text{ GeV}^2 < \mu_{f0}^2 < 2.2 \text{ GeV}^2$ . The parametrisation uncertainty is constructed as an envelope at each  $x$  value, built from the maximal differences between the central PDFs and all parametrisation variations. The total PDF uncertainty is obtained by adding the fit, model, and parametrisation uncertainties in quadrature.

The results for the fitted heavy-quark masses extracted are:

$$\begin{aligned} m_c(m_c) &= 1290_{-41}^{+46}(\text{fit})_{-14}^{+62}(\text{mod})_{-31}^{+7}(\text{par}) \text{ MeV}, \\ m_b(m_b) &= 4049_{-109}^{+104}(\text{fit})_{-32}^{+90}(\text{mod})_{-31}^{+1}(\text{par}) \text{ MeV}. \end{aligned} \quad (4.2)$$

The model uncertainties are dominated by theoretical uncertainties arising from the scale variations. The fit yields  $\chi^2/\text{dof} = 1435/1208$ . The resulting theoretical predictions are shown in Fig. 2.

The results obtained in the fit using the inclusive data only are:  $m_c(m_c) = 1798_{-134}^{+144}(\text{fit}) \text{ MeV}$ ,  $m_b(m_b) = 8450_{-1812}^{+2282}(\text{fit}) \text{ MeV}$  (only the fit uncertainties are quoted). In the variant of the fit using the inclusive data only and the parametrisation with  $E_{u_v} = 0$  the central fitted values for the heavy-quark masses are:  $m_c(m_c) = 1450 \text{ MeV}$ ,  $m_b(m_b) = 3995 \text{ MeV}$ . A cross check is performed using the Monte Carlo method [28, 29]. The obtained heavy-quark masses and their fit uncertainties are in agreement with those quoted in Eq. (4.2). We conclude that the inclusive data alone can not reliably constrain the quark masses, and that the systematics from including the inclusive data in the global mass fit are covered by the parametrisation uncertainties applied.

The predictions for the combined data are also calculated using the ABM11 PDFs [30] at NLO, and ABMP16 PDFs [31] at approximate NNLO as implemented in the OPENQCDRAD program

interfaced in XFITTER. They are compared to the combined data in Fig. 2. Both calculations yield very similar description of the data to the one obtained using HERAPDF2.0 FF3A.

## 5. Summary

Measurements of beauty and charm production cross sections in deep inelastic  $ep$  scattering by the H1 and ZEUS experiments were combined at the level of reduced cross sections, accounting for their statistical and systematic correlations. The beauty cross sections have been combined for the first time. The data sets were found to be consistent and the combined data have significantly reduced uncertainties. The combined data were compared to NLO QCD predictions, which are found to describe the data reasonably well. The running charm and beauty masses were extracted in the QCD analysis using the inclusive HERA DIS and new combined charm and beauty data.

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