

Alexander von Humboldt
Stiftung / Foundation

Combined QCD and EW analysis of HERA data

DESY-16-039

[arXiv:1603.09628](https://arxiv.org/abs/1603.09628)

Accepted by PRD

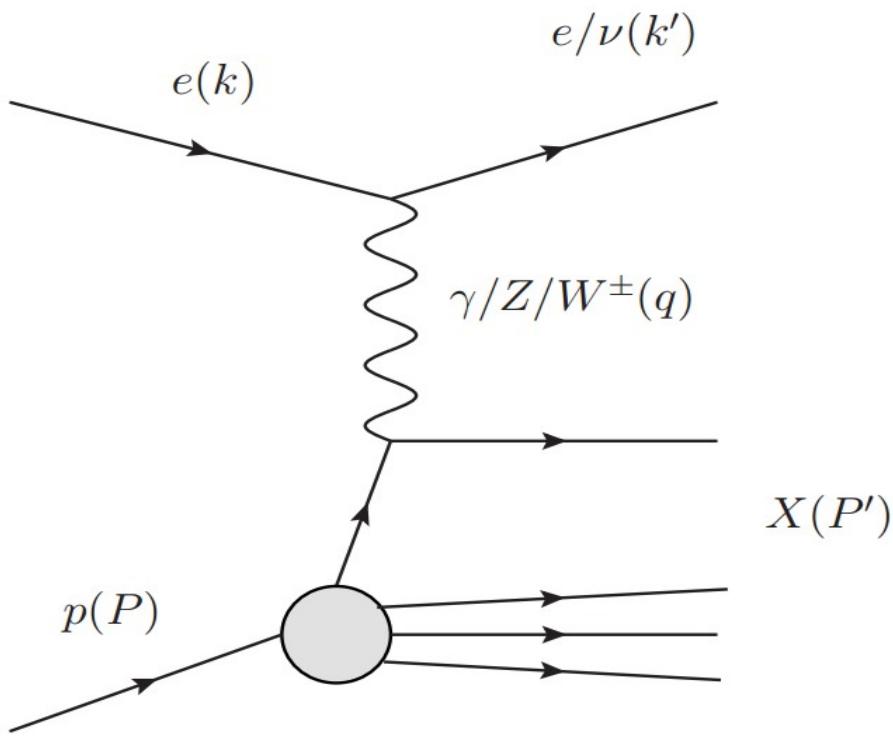
Volodymyr Myronenko
DESY

on behalf of
the ZEUS collaboration

Deep-Inelastic Scattering
Hamburg, Germany 2016



Deep Inelastic Scattering at HERA



$$E_P = 920(820, 460, 575) \text{ GeV}$$

$$E_e = 27.5 \text{ GeV}$$

$$\sqrt{s} = 318(300, 225, 252) \text{ GeV}$$

- ➊ Lepton beams were polarised at HERAII
 - Crucial for the EW measurements

$$Q^2 = -q^2 = -(k - k')^2$$

$$x_{Bj} = \frac{Q^2}{2pq}$$

$$s = (p + k)^2$$

$$y = \frac{pq}{pk}$$

$$Q^2 = xys$$

Experimental achievements (H1 & ZEUS):

~ 0.5 fb⁻¹ DIS data from each experiment

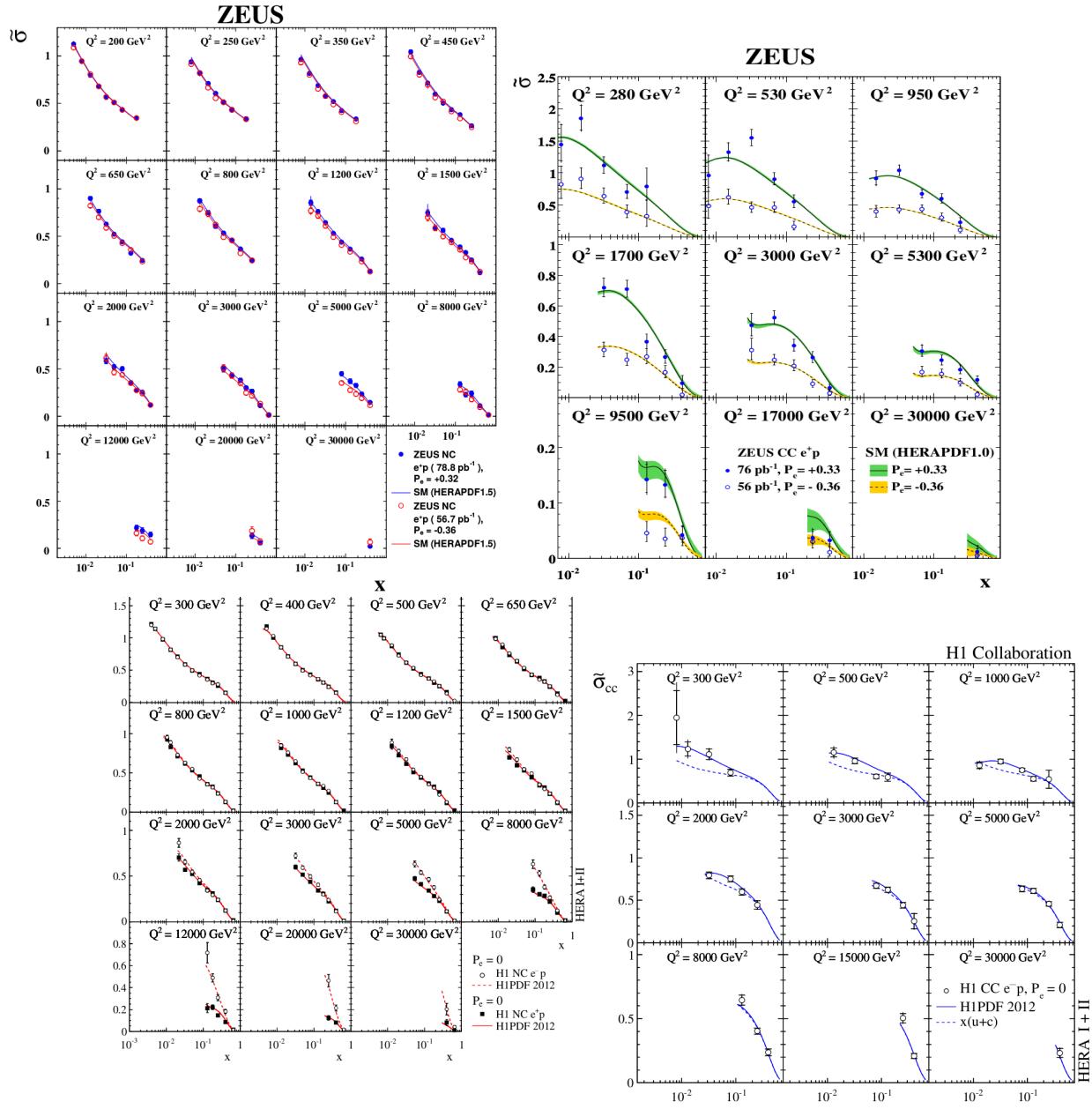
Data considered

- ◆ Data used in the analysis (uncombined **data sets**, correlations as in HERAPDF2.0):

- HERAI: H1 + ZEUS
 $P_e = 0$;

- Reduced E_p runs:
H1 + ZEUS, $P_e = 0$;

- HERAII:
 - H1 data with $P_e = 0$;
 - ZEUS data with $P_e \neq 0$;



Global QCD analysis



- ◆ $Q_{\min}^2 = 3.5 \text{ GeV}^2$.
- ◆ HF scheme: GM VFNS NLO (RT OPT).
- ◆ PDFs parametrised with **13p (HERAPDF2.0 - D \bar{U})** at $Q_0^2 = 1.9 \text{ GeV}^2$

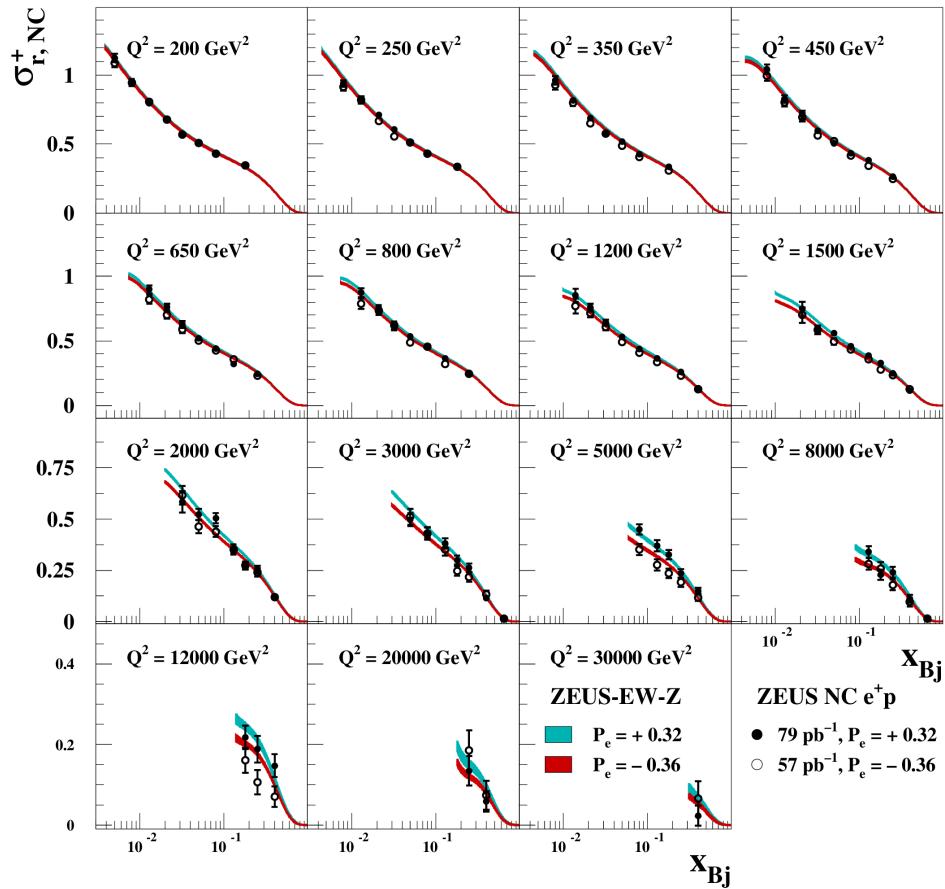
$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$
$$xg(x), xu_v(x), xd_v(x), x\bar{U}(x), x\bar{D}(x)$$

- ◆ Free parameters: PDF parameters + couplings of Z^0 to quarks (a_u, a_d, v_u, v_d), or M_W , or $\sin^2\theta_W$ (On-shell scheme).
- ◆ Model and parameterisation uncertainty estimation → HERAPDF2.0 strategy.
- ◆ Correction calculated using EPRC code: Δr . No ISR/FSR corrections.
desy.de/~hspiesb/eprc.html

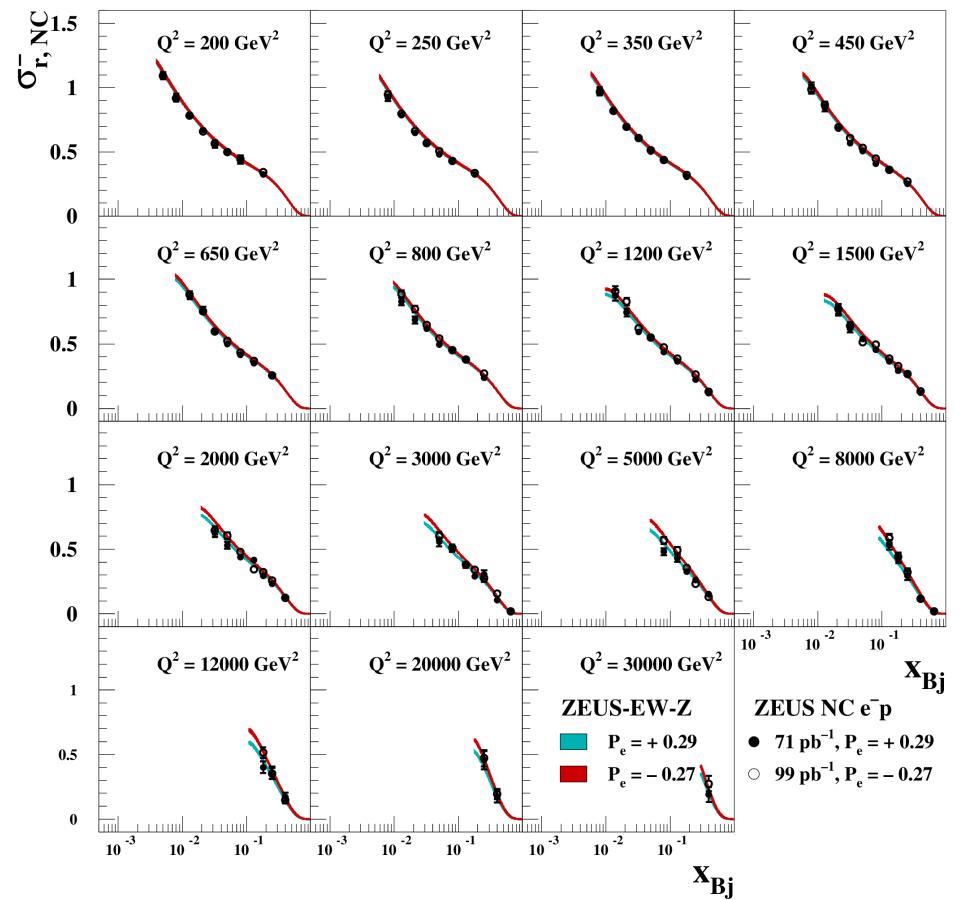
Data description (ZEUS-EW-Z)

◆ Fitted predictions describe data well.

ZEUS

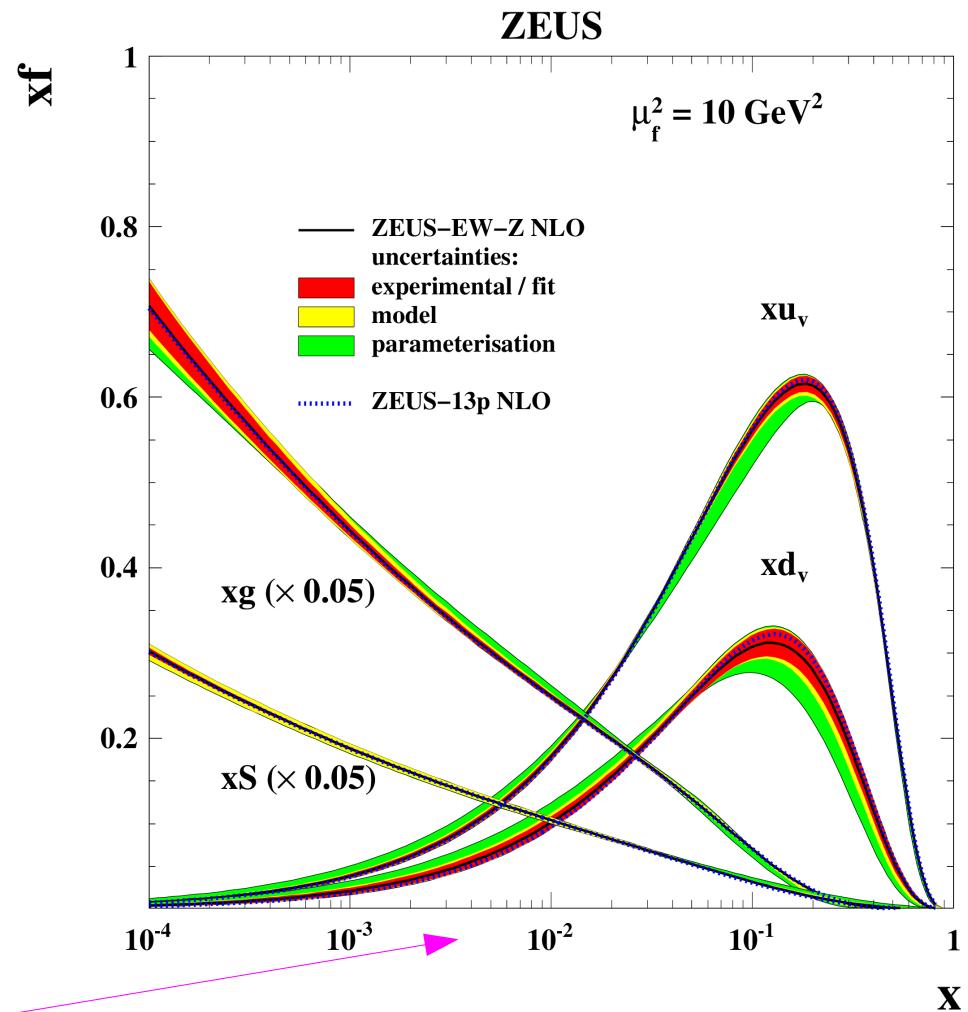
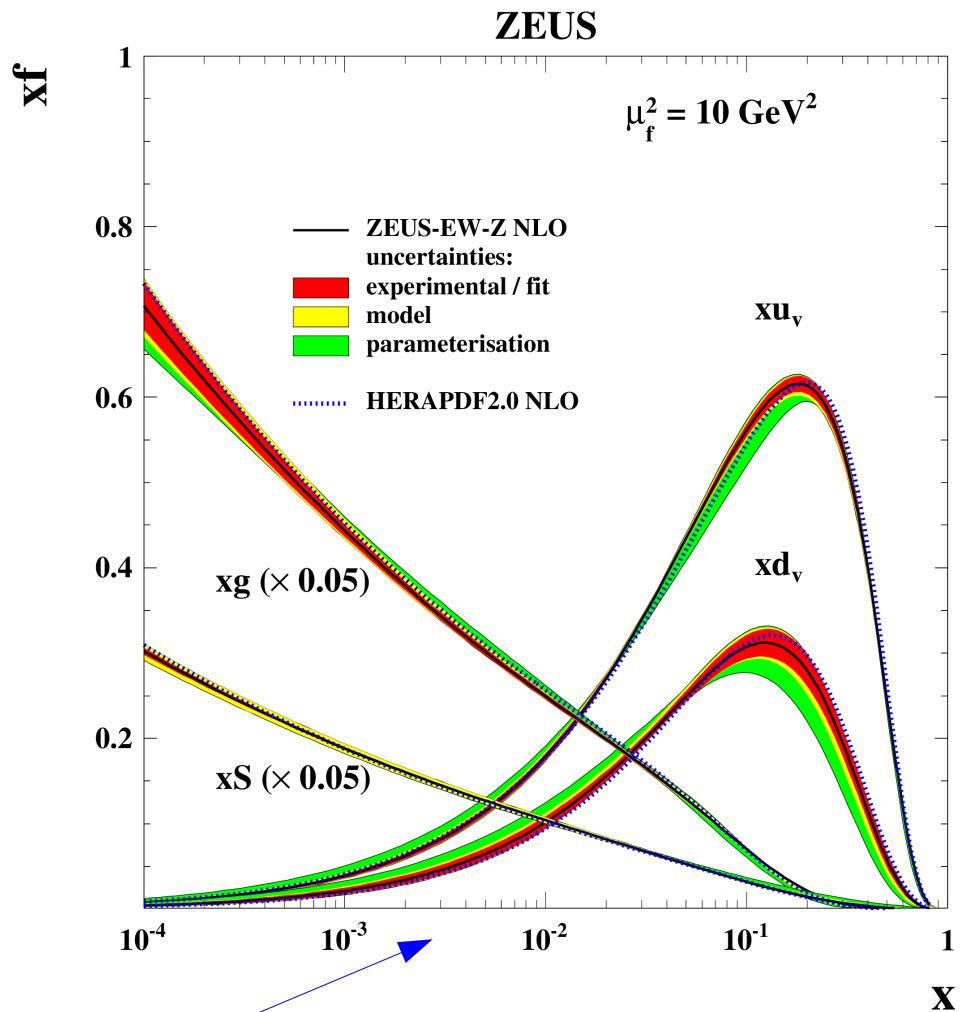


ZEUS



◆ $\chi^2 = 3270 / 2925 = 1.12$

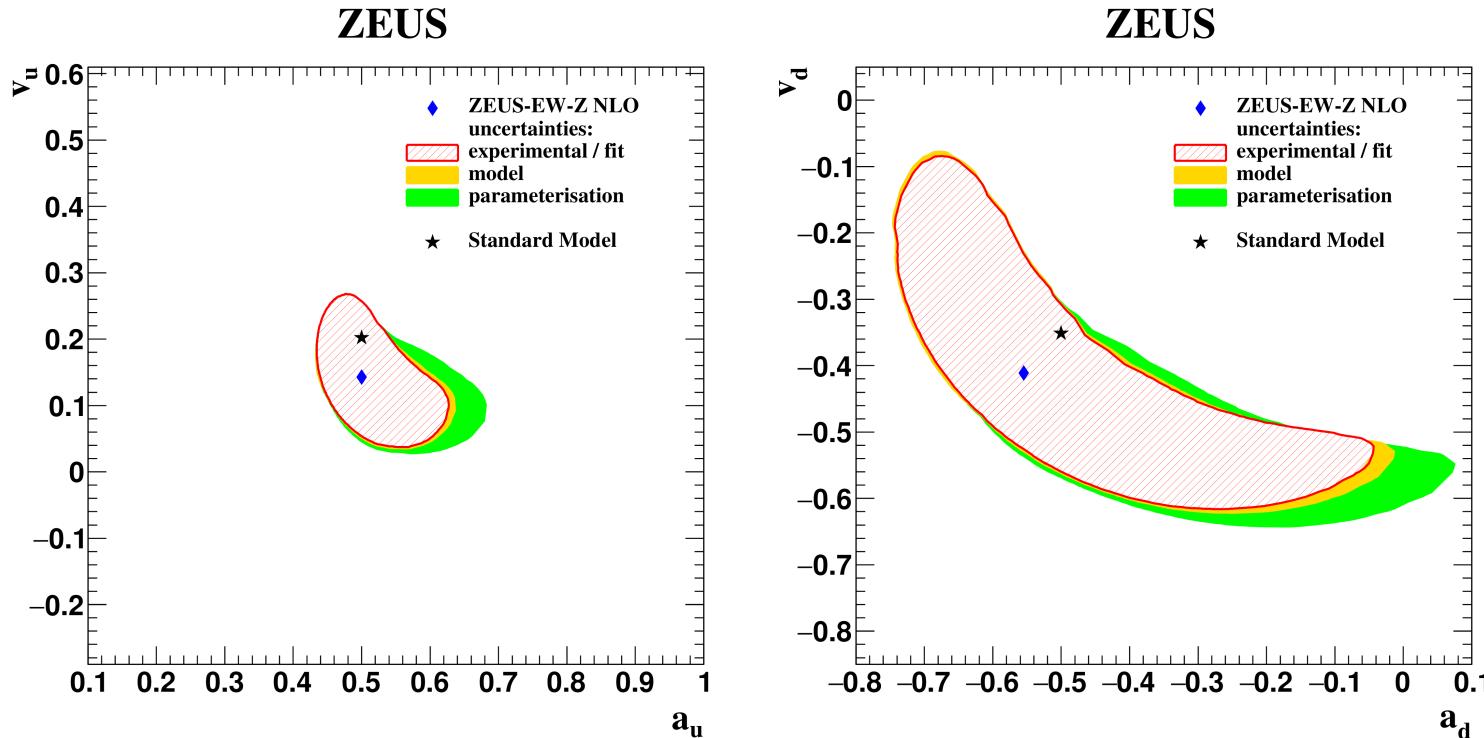
Effect of coupling determination on PDFs



- HERAPDF2.0 and ZEUS-13p PDFs with couplings set to SM agree with ZEUS-EW-Z PDFs.
- Releasing couplings has little effect on PDFs.

Couplings of quarks to Z boson

◆ Couplings were determined simultaneously with PDFs ([ZEUS-EW-Z](#))



$$a_u = 0.50^{+0.09}_{-0.05(\text{exp/fit})} \, {}^{+0.04}_{-0.02(\text{mod})} \, {}^{+0.08}_{-0.01(\text{par})} = \mathbf{0.50^{+0.12}_{-0.05(\text{tot})}} \quad \mathbf{0.5}$$

$$a_d = -0.56^{+0.34}_{-0.14(\text{exp/fit})} \, {}^{+0.11}_{-0.05(\text{mod})} \, {}^{+0.20}_{-0.00(\text{par})} = \mathbf{-0.56^{+0.41}_{-0.15(\text{tot})}} \quad \mathbf{-0.5}$$

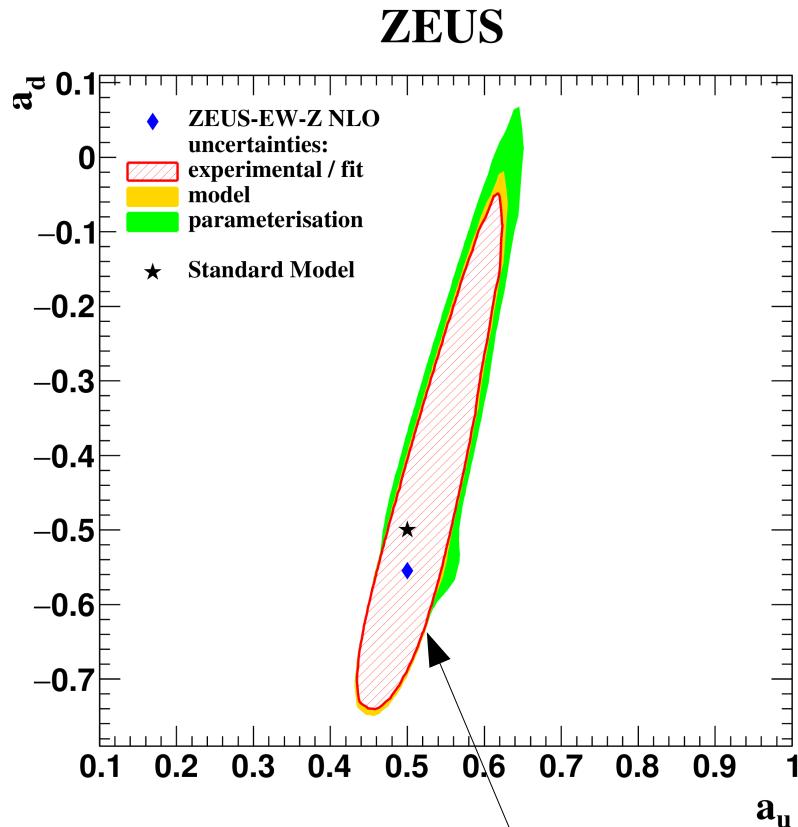
$$v_u = 0.14^{+0.08}_{-0.08(\text{exp/fit})} \, {}^{+0.01}_{-0.00(\text{mod})} \, {}^{+0.03}_{-0.01(\text{par})} = \mathbf{0.14^{+0.09}_{-0.09(\text{tot})}} \quad \mathbf{0.202}$$

$$v_d = -0.41^{+0.24}_{-0.16(\text{exp/fit})} \, {}^{+0.04}_{-0.07(\text{mod})} \, {}^{+0.00}_{-0.08(\text{par})} = \mathbf{-0.41^{+0.25}_{-0.20(\text{tot})}} \quad \mathbf{-0.351}$$

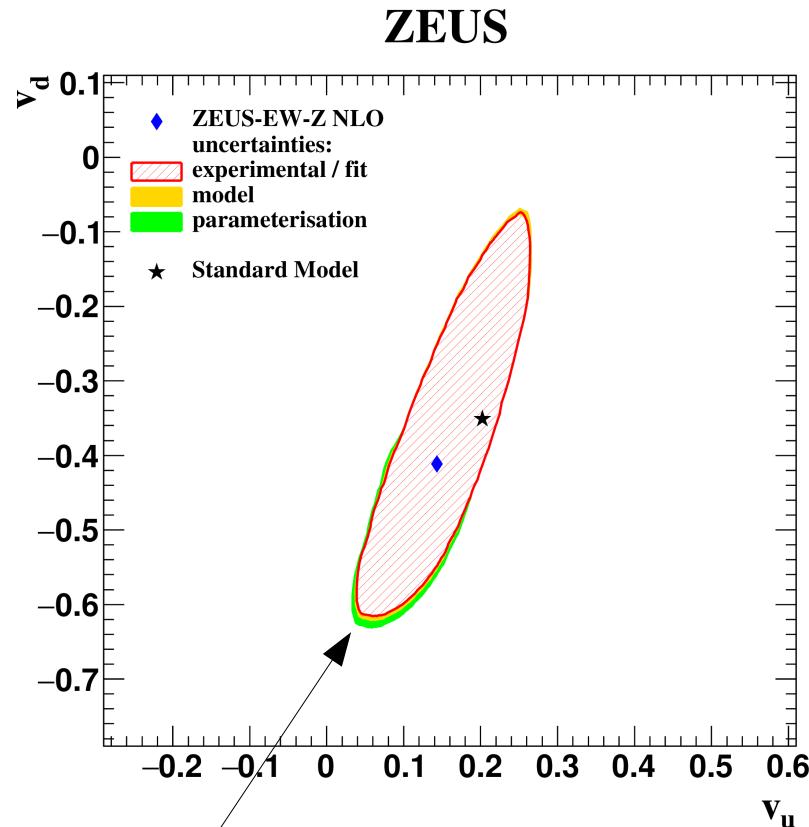
Standard Model

Couplings of quarks to Z boson

Vector and axial-vector couplings in the fit show high correlation



	a_u	a_d	v_u	v_d
a_u	1.000	0.861	-0.555	-0.729
a_d	0.861	1.000	-0.636	-0.880
v_u	-0.555	-0.636	1.000	0.851
v_d	-0.729	-0.880	0.851	1.000

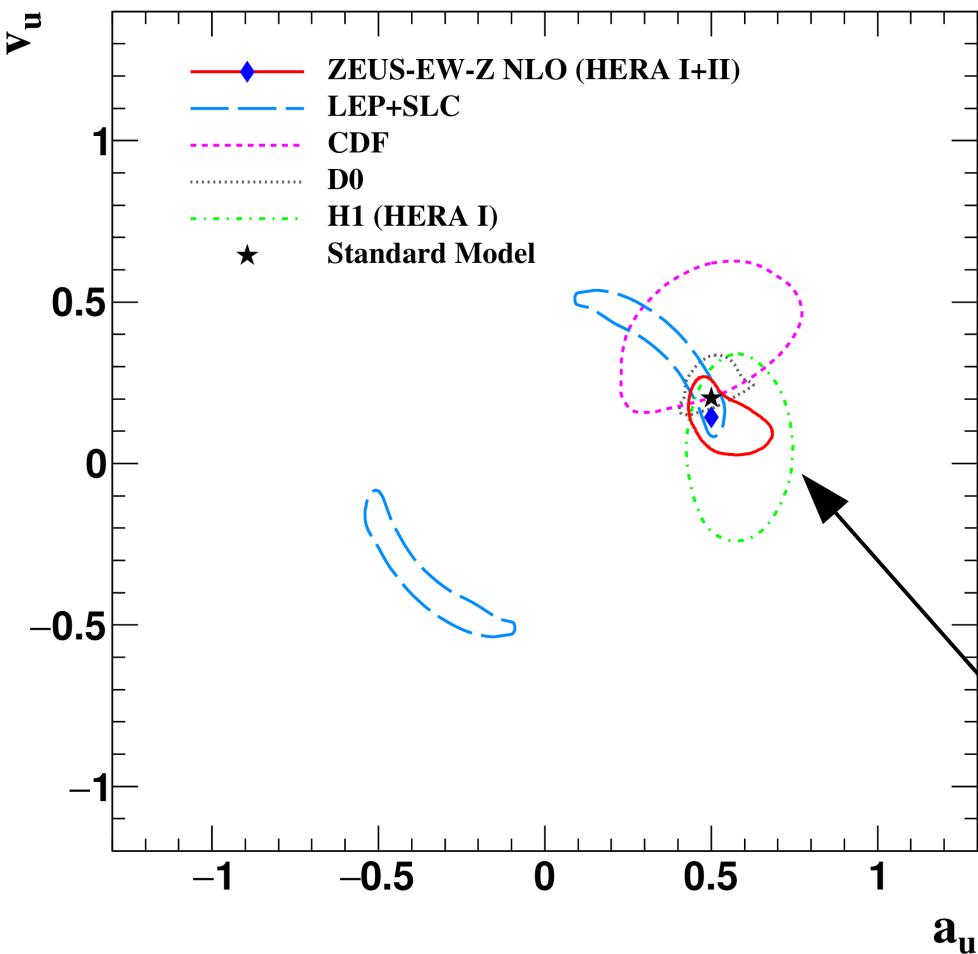


Insignificant correlations of couplings to PDF parameters

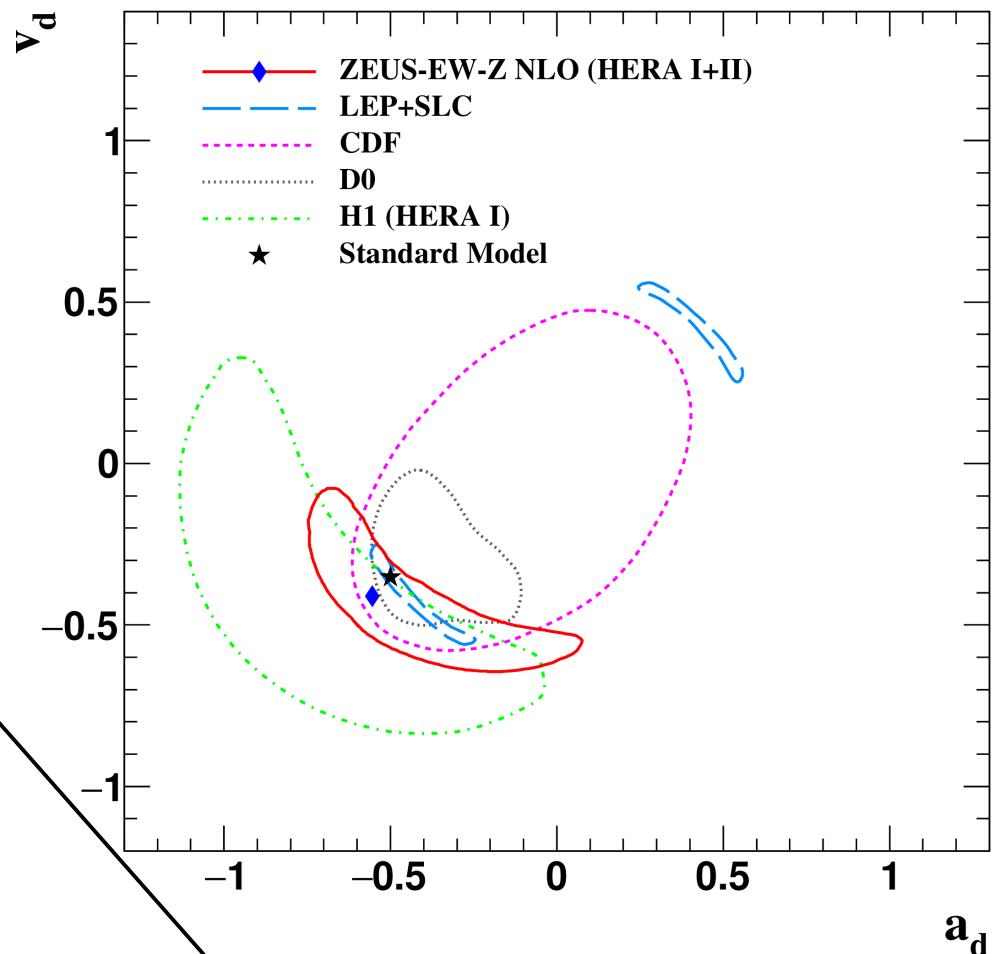
Couplings of quarks to Z boson

◆ ZEUS-EW-Z results are compatible with previous measurements

ZEUS



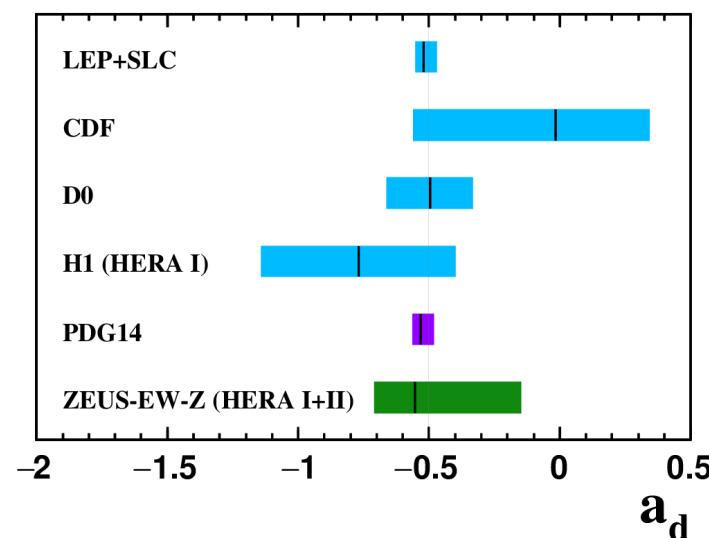
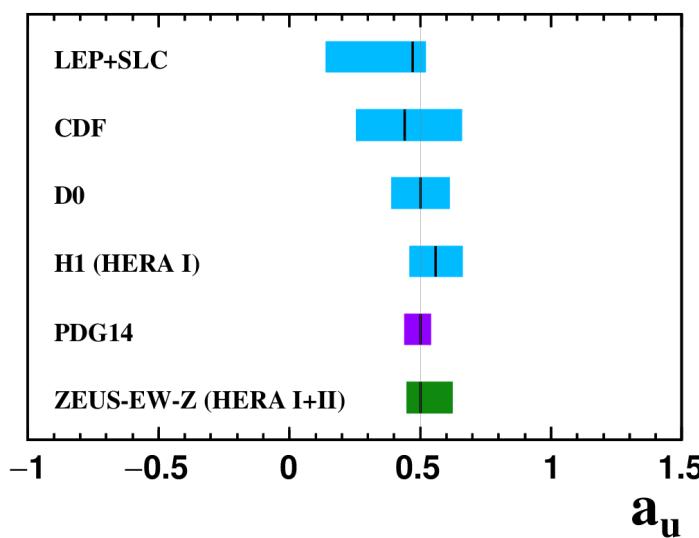
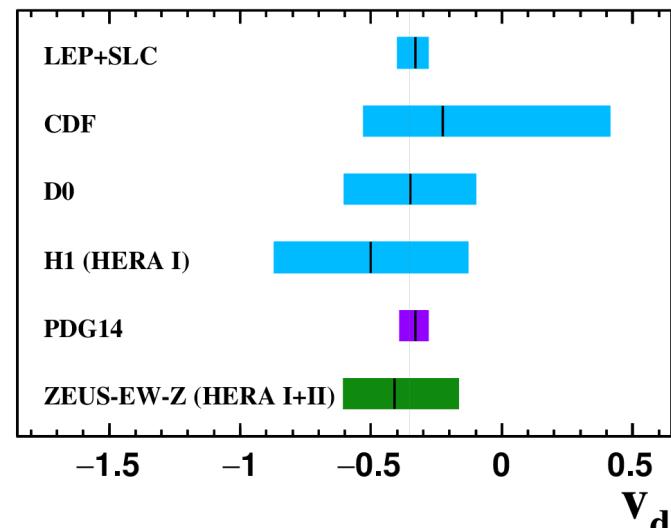
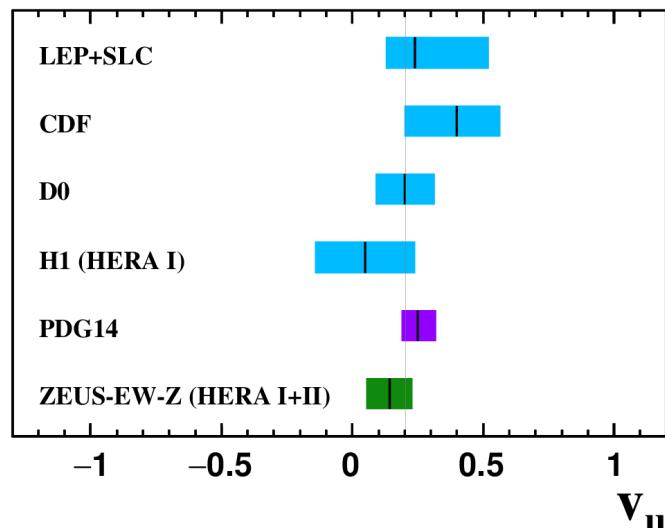
ZEUS



◆ HERA data show remarkable sensitivity to the **u-type** quark couplings.

Couplings of quarks to Z boson

ZEUS



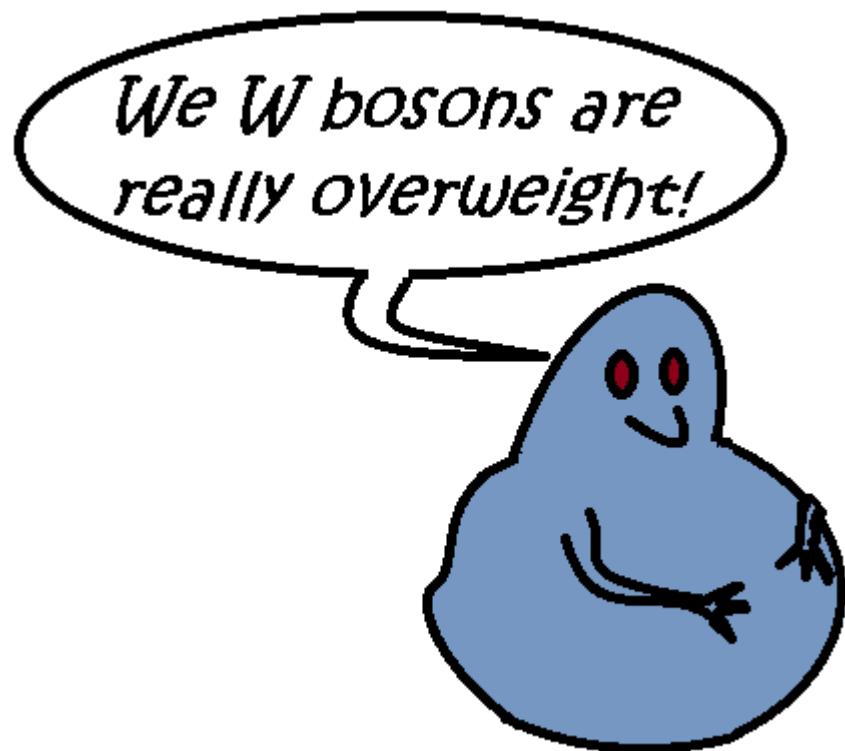
◆ PDG average values do not yet include current ZEUS-EW-Z results.

◆ Results presented here have a potential to decrease uncertainties of average values (u-quark in particular)

Mass of W boson

- ◆ Mass of W boson was determined simultaneously with PDFs ([ZEUS-EW-W](#))

$$M_W = 80.68 \pm 0.28_{(\text{exp/fit})}^{+0.12}_{-0.01(\text{mod})} {}^{+0.23}_{-0.01(\text{par})} \text{ GeV} = \mathbf{80.68^{+0.38}_{-0.28(\text{tot})} \text{ GeV}}$$



$$M_W^{\text{PDG } 14} = 80.385 \pm 0.015 \text{ GeV}$$

G_F in CC was re-expressed with:

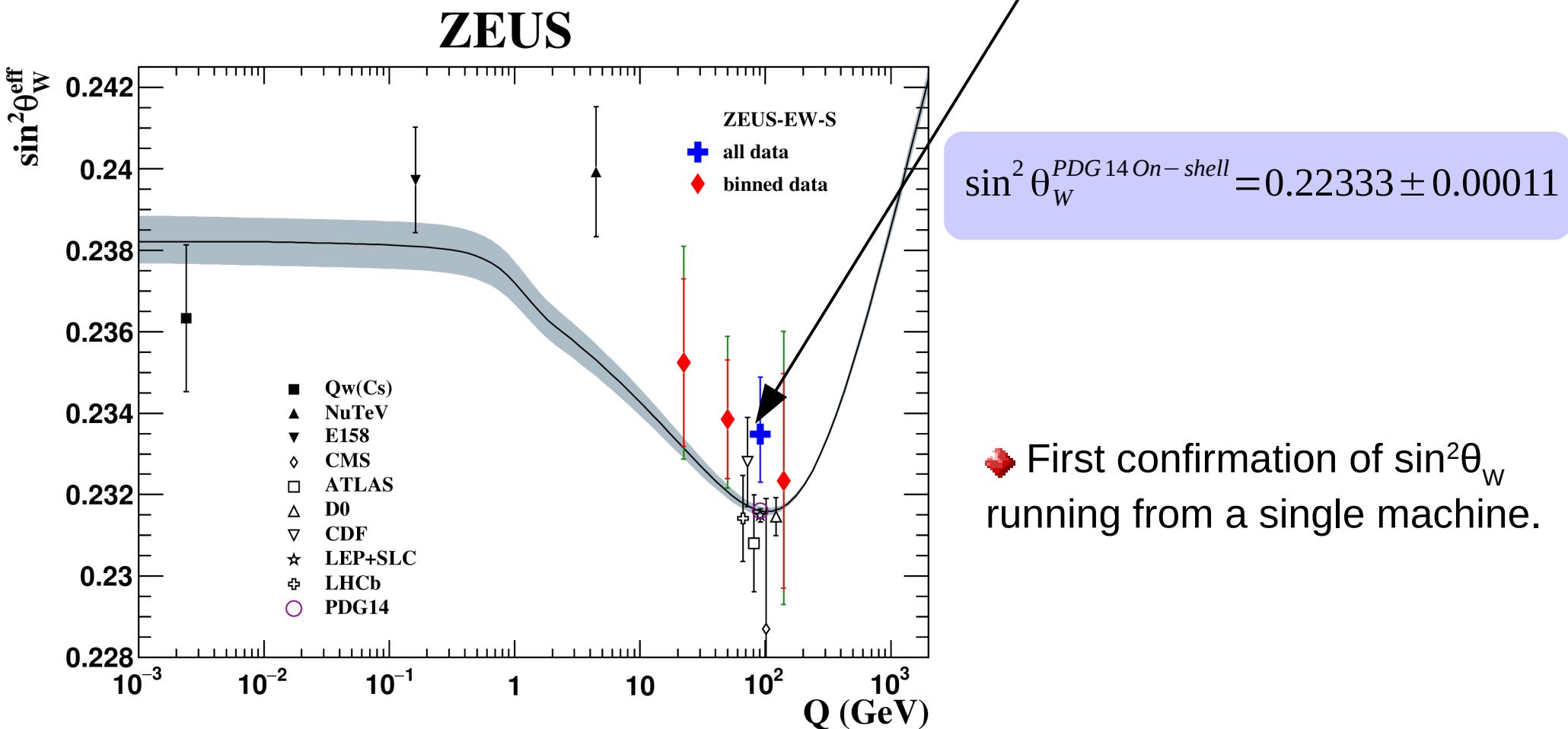
$$G_F = \frac{\pi\alpha}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

- ◆ M_W from [ZEUS-EW-W](#) is consistent with current world average.

$\sin^2\theta_w$ from HERA data

- sin $^2\theta_w$ was determined simultaneously with PDFs (ZEUS-EW-S)

$$\sin^2\theta_w = 0.2252 \pm 0.0011_{(\text{exp/fit})} {}^{+0.0003}_{-0.0001} {}^{+0.0007}_{-0.0001} {}^{(+0.0013)}_{(-0.0011)} = \mathbf{0.2252^{+0.0013}_{-0.0011} (\text{tot})}$$

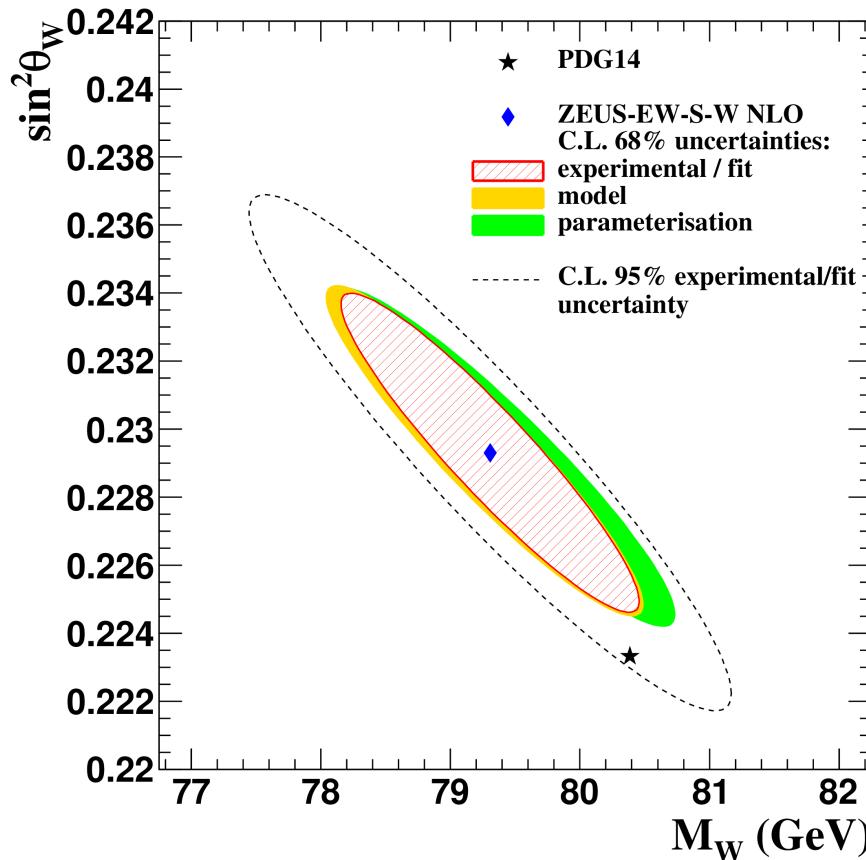


- On-shell measurements were translated to $\sin^2\theta_w^{\text{eff}}$.

$\sin^2\theta_W$ and mass of W boson

- sin $^2\theta_W$ and M $_W$ were determined simultaneously with PDFs ([ZEUS-EW-S-W](#))

ZEUS



$M_W = 79.30 \pm 0.76_{(\text{exp/fit})}^{+0.38}_{-0.08(\text{mod})}^{+0.48}_{-0.10(\text{par})} \text{ GeV} = 79.30_{-0.77(\text{tot})}^{+0.98} \text{ GeV}$

$\sin^2\theta_W = 0.2293 \pm 0.0031_{(\text{exp/fit})}^{+0.0005}_{-0.0001(\text{mod})}^{+0.0003}_{-0.0001(\text{par})} = 0.2293_{-0.0031(\text{tot})}^{+0.0032}$

$$\text{corr}(M_W, \sin^2\theta_W) = -0.930$$

$$M_W^{\text{PDG 14}} = 80.385 \pm 0.015 \text{ GeV}$$

$$\sin^2\theta_W^{\text{PDG 14 On-shell}} = 0.22333 \pm 0.00011$$

- All extracted quantities agree with World average values.

Summary

◆ The simultaneous QCD and EW analysis of HERA data was performed.

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◆ Couplings of u- and d-type quarks to Z boson were determined:

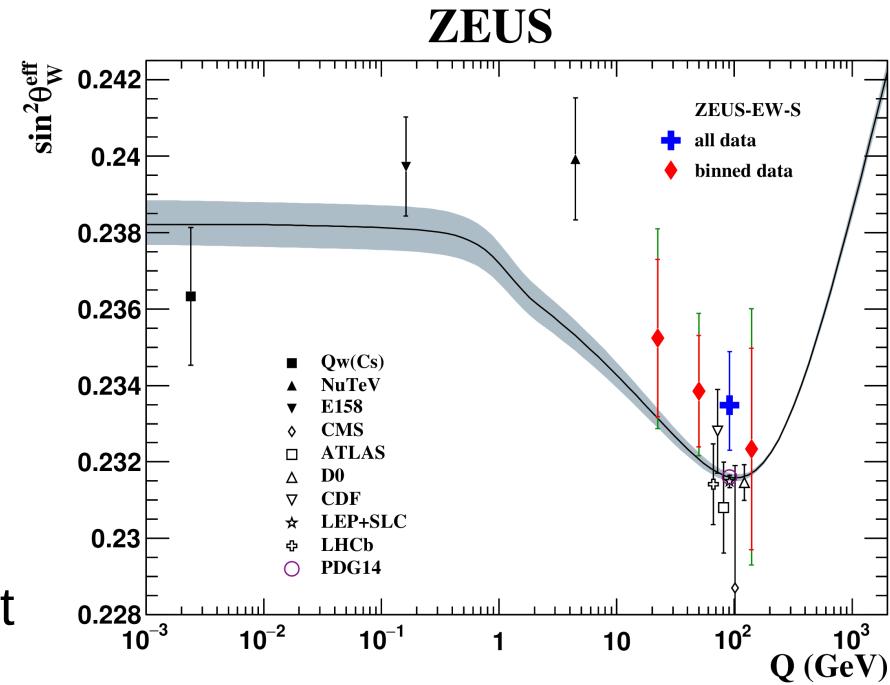
- Fitted couplings are consistent with SM predictions;
- Results are compatible with those from other measurements;
- Couplings of u-quarks are constrained significantly better than those of d-quarks.

◆ $\sin^2\theta_W$ at on-shell scheme was determined:

- Fitted value is competitive with measurements from other experiments;
- Result is consistent with current world average.

◆ Mass of W boson was determined:

- Fitted value of M_W is consistent with current world average.



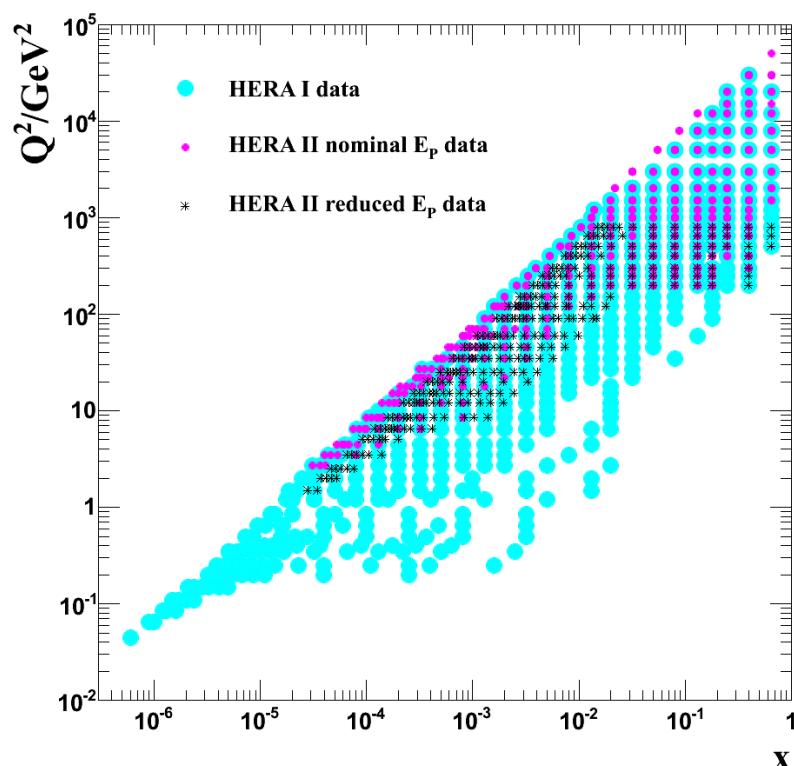
Backup not necessarily useful...

Full HERA data collection

HERAPDF1.0

HERAPDF1.5

HERAPDF2.0



Data Set	x_{Bj} Grid from	x_{Bj} Grid to	$Q^2[\text{GeV}^2]$ Grid from	$Q^2[\text{GeV}^2]$ Grid to	\mathcal{L} pb^{-1}	e^+/e^-	\sqrt{s} GeV
HERA I $E_p = 820 \text{ GeV}$ and $E_p = 920 \text{ GeV}$ data sets							
H1 svx-mb	95-00	0.000005	0.02	0.2	12	2.1	e^+p 301, 319
H1 low Q^2	96-00	0.0002	0.1	12	150	22	e^+p 301, 319
H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p 301
H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p 301
H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p 319
H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p 319
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p 319
H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p 319
H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p 319
ZEUS BPC							
ZEUS BPT	95	0.000002	0.00006	0.11	0.65	1.65	e^+p 300
ZEUS SVX	97	0.000006	0.001	0.045	0.65	3.9	e^+p 300
ZEUS NC	95	0.000012	0.0019	0.6	17	0.2	e^+p 300
ZEUS NC	96-97	0.00006	0.65	2.7	30000	30.0	e^+p 300
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p 300
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p 318
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p 318
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p 318
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p 318
HERA II $E_p = 920 \text{ GeV}$ data sets							
H1 NC $^{1.5}p$	03-07	0.0008	0.65	60	30000	182	e^+p 319
H1 CC $^{1.5}p$	03-07	0.008	0.40	300	15000	182	e^+p 319
H1 NC $^{1.5}p$	03-07	0.0008	0.65	60	50000	151.7	e^-p 319
H1 CC $^{1.5}p$	03-07	0.008	0.40	300	30000	151.7	e^-p 319
H1 NC med $Q^2 \cdot {}^{*y.5}$	03-07	0.0000986	0.005	8.5	90	97.6	e^+p 319
H1 NC low $Q^2 \cdot {}^{*y.5}$	03-07	0.000029	0.00032	2.5	12	5.9	e^+p 319
ZEUS NC	06-07	0.005	0.65	200	30000	135.5	e^+p 318
ZEUS CC $^{1.5}p$	06-07	0.0078	0.42	280	30000	132	e^+p 318
ZEUS NC $^{1.5}$	05-06	0.005	0.65	200	30000	169.9	e^-p 318
ZEUS CC $^{1.5}$	04-06	0.015	0.65	280	30000	175	e^-p 318
ZEUS NC nominal *y	06-07	0.000092	0.008343	7	110	44.5	e^+p 318
ZEUS NC satellite *y	06-07	0.000071	0.008343	5	110	44.5	e^+p 318
HERA II $E_p = 575 \text{ GeV}$ data sets							
H1 NC high Q^2	07	0.00065	0.65	35	800	5.4	e^+p 252
H1 NC low Q^2	07	0.0000279	0.0148	1.5	90	5.9	e^+p 252
ZEUS NC nominal	07	0.000147	0.013349	7	110	7.1	e^+p 251
ZEUS NC satellite	07	0.000125	0.013349	5	110	7.1	e^+p 251
HERA II $E_p = 460 \text{ GeV}$ data sets							
H1 NC high Q^2	07	0.00081	0.65	35	800	11.8	e^+p 225
H1 NC low Q^2	07	0.0000348	0.0148	1.5	90	12.2	e^+p 225
ZEUS NC nominal	07	0.000184	0.016686	7	110	13.9	e^+p 225
ZEUS NC satellite	07	0.000143	0.016686	5	110	13.9	e^+p 225



All inclusive DIS results are final and published!

Full HERA I data

HERA II data HER

HERA II data LER

Correlation matrix for the fit parameters

NO.	Bg	Cg	Aprig	Bprig	Buv	Cuv	Euv	Bdv	Cdv	CUbar	ADbar	BDbar	CDbar	auEW	adEW	vuEW	vdEW
Bg	1.000-0.014-0.449 0.824-0.216 0.172 0.250-0.084-0.085-0.098-0.107-0.136 0.046 0.025 0.003 0.015 0.018																
Cg	-0.014 1.000 0.831 0.457 0.341-0.373-0.550 0.010 0.296-0.018-0.082-0.103 -0.434 0.105 0.095 -0.098 -0.111																
Aprig	-0.449 0.831 1.000 0.120 0.548-0.404-0.629 0.233 0.274 0.159 0.081 0.072 -0.148 -0.052 0.000 -0.043 -0.054																
Bprig	0.824 0.457 0.120 1.000 0.106-0.037-0.082 0.075 0.047 0.043 0.011-0.014 0.012 -0.029 -0.011 -0.001 -0.002																
Buv	-0.216 0.341 0.548 0.106 1.000-0.409-0.774 0.465-0.086 0.690 0.476 0.395 0.439 -0.360 -0.178 0.079 0.070																
Cuv	0.172-0.373-0.404-0.037-0.409 1.000 0.828-0.297-0.235-0.188-0.095-0.069 -0.040 0.110 0.029 0.040 0.028																
Euv	0.250-0.550-0.629-0.082-0.774 0.828 1.000-0.296-0.066-0.363-0.170-0.117 -0.092 0.192 0.087 -0.023 -0.017																
Bdv	-0.084 0.010 0.233 0.075 0.465-0.297-0.296 1.000 0.518 0.405 0.350 0.291 0.673 -0.335 -0.134 0.038 0.021																
Cdv	-0.085 0.296 0.274 0.047-0.086-0.235-0.066 0.518 1.000-0.137-0.186-0.193 -0.139 0.110 0.128 -0.101 -0.128																
CUbar	-0.098-0.018 0.159 0.043 0.690-0.188-0.363 0.405-0.137 1.000 0.673 0.635 0.329 -0.320 -0.137 0.055 0.052																
ADbar	-0.107-0.082 0.081 0.011 0.476-0.095-0.170 0.350-0.186 0.673 1.000 0.959 0.477 -0.272 -0.137 0.056 0.059																
BDbar	-0.136-0.103 0.072-0.014 0.395-0.069-0.117 0.291-0.193 0.635 0.959 1.000 0.415 -0.239 -0.120 0.047 0.053																
CDbar	0.046-0.434-0.148 0.012 0.439-0.040-0.092 0.673-0.139 0.329 0.477 0.415 1.000 -0.449 -0.271 0.148 0.153																
auEW	0.025 0.105-0.052-0.029-0.360 0.110 0.192-0.335 0.110-0.320-0.272-0.239 -0.449 1.000 0.861 -0.555 -0.729																
adEW	0.003 0.095 0.000-0.011-0.178 0.029 0.087-0.134 0.128-0.137-0.137-0.120 -0.271 0.861 1.000 -0.636 -0.880																
vuEW	0.015-0.098-0.043-0.001 0.079 0.040-0.023 0.038-0.101 0.055 0.056 0.047 0.148 -0.555 -0.636 1.000 0.851																
vdEW	0.018-0.111-0.054-0.002 0.070 0.028-0.017 0.021-0.128 0.052 0.059 0.053 0.153 -0.729 -0.880 0.851 1.000																

World results (full uncertainties)

	a_u	a_b	v_u	v_d
LEP	$0.47^{+0.05}_{-0.33}$	$-0.52^{+0.05}_{-0.03}$	$0.24^{+0.28}_{-0.11}$	$-0.33^{+0.05}_{-0.07}$
D0	0.50 ± 0.11	-0.50 ± 0.17	0.20 ± 0.11	0.35 ± 0.25
CDF	$0.44^{+0.22}_{-0.19}$	$-0.02^{+0.36}_{-0.54}$	$0.40^{+0.17}_{-0.20}$	$-0.23^{+0.64}_{-0.30}$
H1: HERA1 (publ.)	0.56 ± 0.10	-0.77 ± 0.37	0.05 ± 0.19	-0.50 ± 0.37
ZEUS: HERA1+2 (prel.)	0.51 ± 0.20	-0.54 ± 0.37	0.05 ± 0.10	-0.64 ± 0.24
ZEUS-EW-Z	$0.500^{+0.122}_{-0.050}$	$-0.555^{+0.407}_{-0.152}$	$0.143^{+0.085}_{-0.088}$	$-0.411^{+0.246}_{-0.195}$
PDG14	$0.50^{+0.04}_{-0.06}$	$-0.523^{+0.050}_{-0.029}$	$0.25^{+0.07}_{-0.06}$	$-0.33^{+0.05}_{-0.06}$
SM	0.5	-0.5	0.202	-0.351

Effect of PDFs determination on couplings

- ◆ Couplings, fitted at fixed PDFs are well compatible with those from [ZEUS-EW-Z fit](#).

	a_u	exp	tot	a_d	exp	tot	v_u	exp	tot	v_d	exp	tot
EW-Z	+.500	+.086 -.047	+.122 -.050	-.555	+.337 -.144	+.407 -.152	+.143	+.084 -.081	+.085 -.088	-.411	+.243 -.164	+.246 -.195
13p	+.485	+.073 -.038		-.567	+.295 -.130		+.145	+.079 -.076		-.402	+.216 -.171	
HPDF1*	+.474	+.059 -.033		-.619	+.233 -.107		+.156	+.076 -.076		-.353	+.215 -.190	
HPDF2*	+.486	+.061 -.034		-.634	+.239 -.110		+.149	+.078 -.078		-.357	+.220 -.194	
SM	+.500			-.500			+.202			-.351		

- ◆ Differences in the experimental uncertainties can give a **rough estimate of PDF uncertainties** in the measurement.

* HERAPDF2.0 used $\sin^2\theta_w$ @ $\overline{\text{MS}}$ - HPDF2, this analysis uses $\sin^2\theta_w$ @ On-schell - HPDF1.
The influence of $\sin^2\theta_w$ for PDF extraction only is minimal (checked).

On $\sin^2\theta_W(+X)$ fits to DIS data

◆ DIS inclusive cross sections depend on $\sin^2\theta_W$ through:

- Z propagator in NC cross sections;
- Vector couplings of Z to quarks;

$$\tilde{F}_2^\pm = F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z$$

$$x \tilde{F}_3^\pm = -(a_e \pm P_e v_e) \chi_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \chi_Z^2 x F_3^Z$$

- W propagator (G_F);

$$\frac{d^2\sigma_{CC}(e^+ p)}{dx_{Bj} dQ^2} = (1 + P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(\bar{u} + \bar{c}) + (1 - y)^2 (d + s + b)]$$

$$\frac{d^2\sigma_{CC}(e^- p)}{dx_{Bj} dQ^2} = (1 - P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(u + c) + (1 - y)^2 (\bar{d} + \bar{s} + \bar{b})]$$

ΔR is an EW correction.

$$\chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2} \frac{1}{1 - \Delta R}$$

$$G_F = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

[arXiv:hep-ph/9902277](https://arxiv.org/abs/hep-ph/9902277)

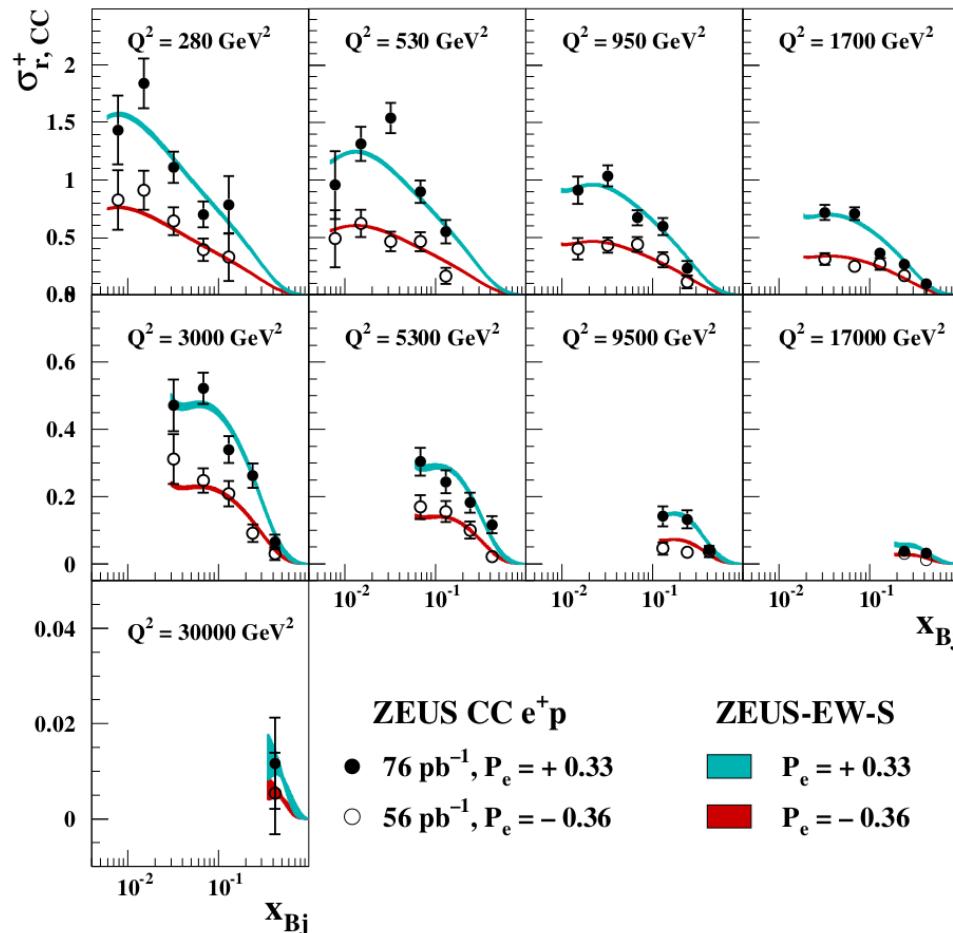
◆ Re-expressing G_F through $\sin^2\theta_W$ and M_W allows to use both CC and NC for $\sin^2\theta_W$ determination.

- Current analysis exploits all three dependences for $\sin^2\theta_W$ extraction.
- $\sin^2\theta_W$ values extracted in current analysis correspond to **On-shell scheme**.

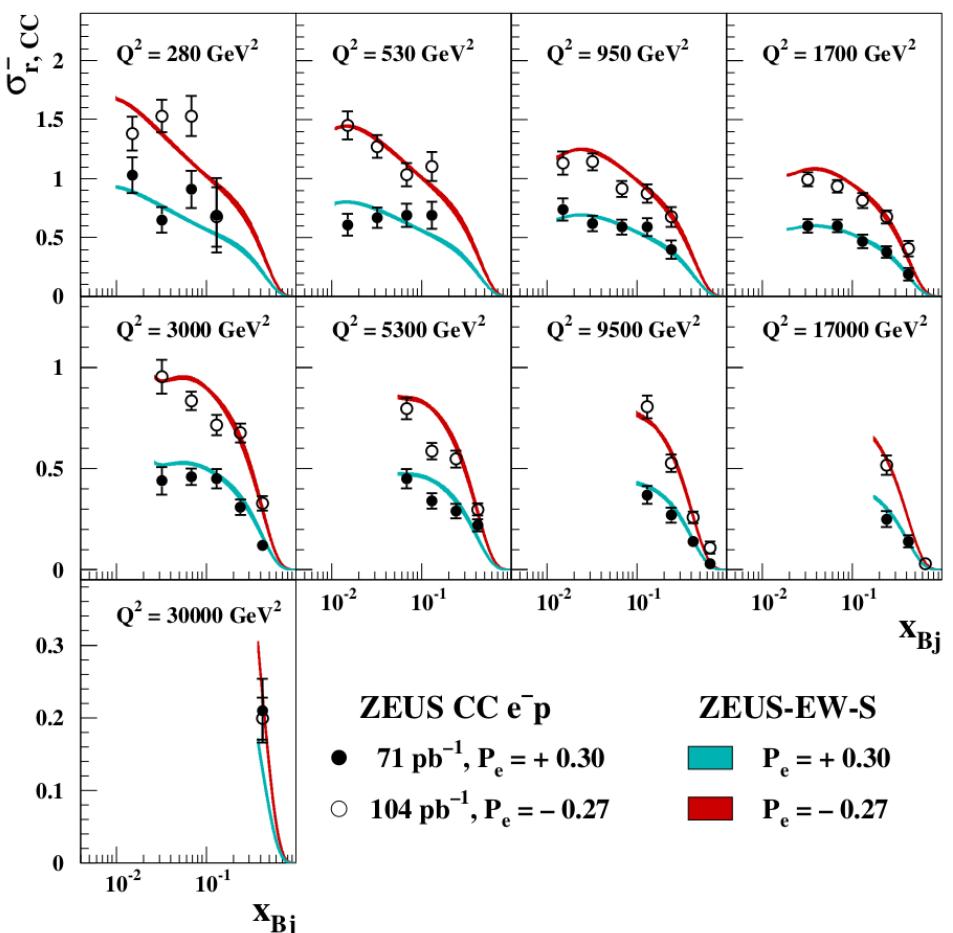
Data description (ZEUS-EW-S)

◆ Fitted predictions describe data reasonably well.

ZEUS

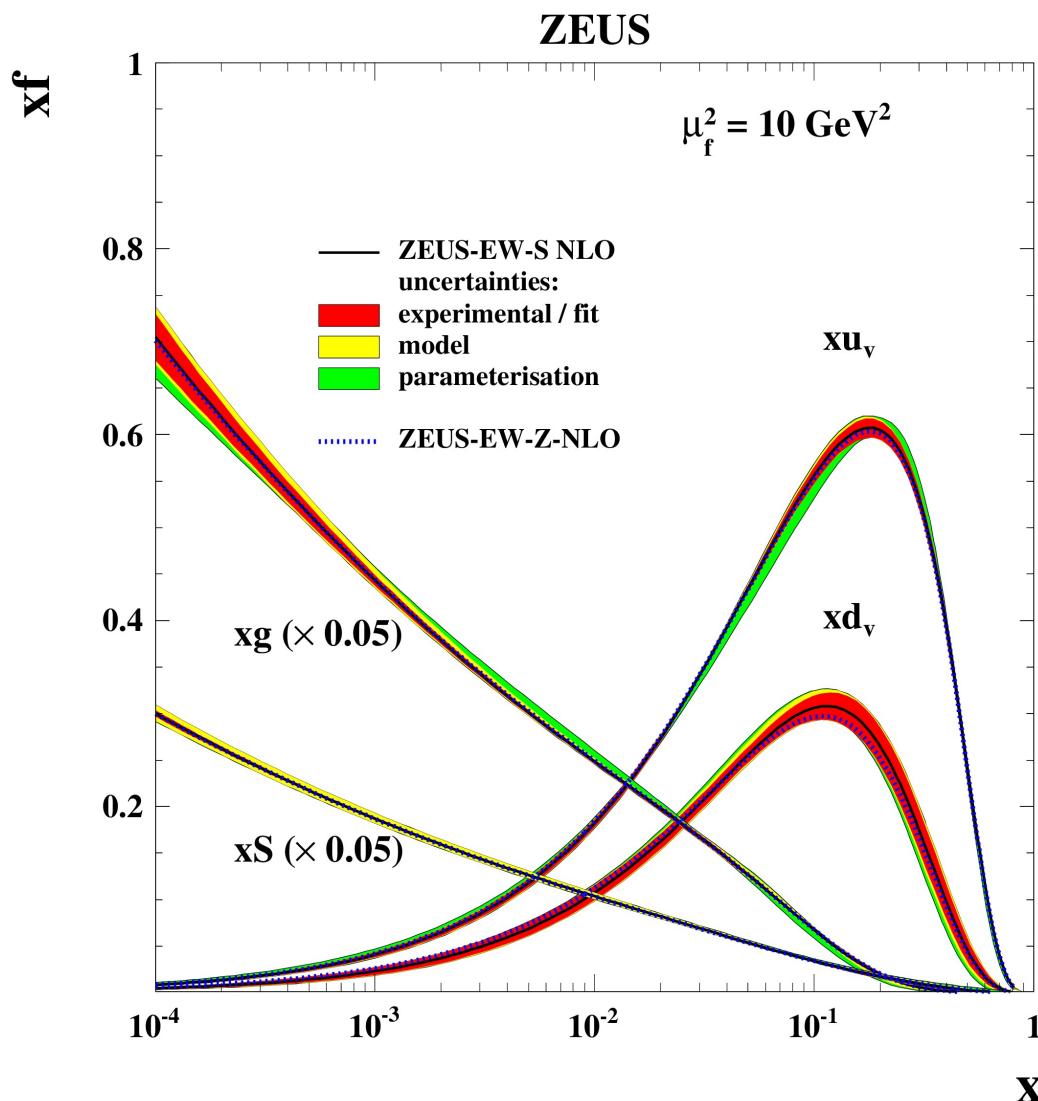


ZEUS



◆ $\chi^2 = 3270 / 2928 = 1.118$

ZEUS-EW-Z vs ZEUS-EW-S

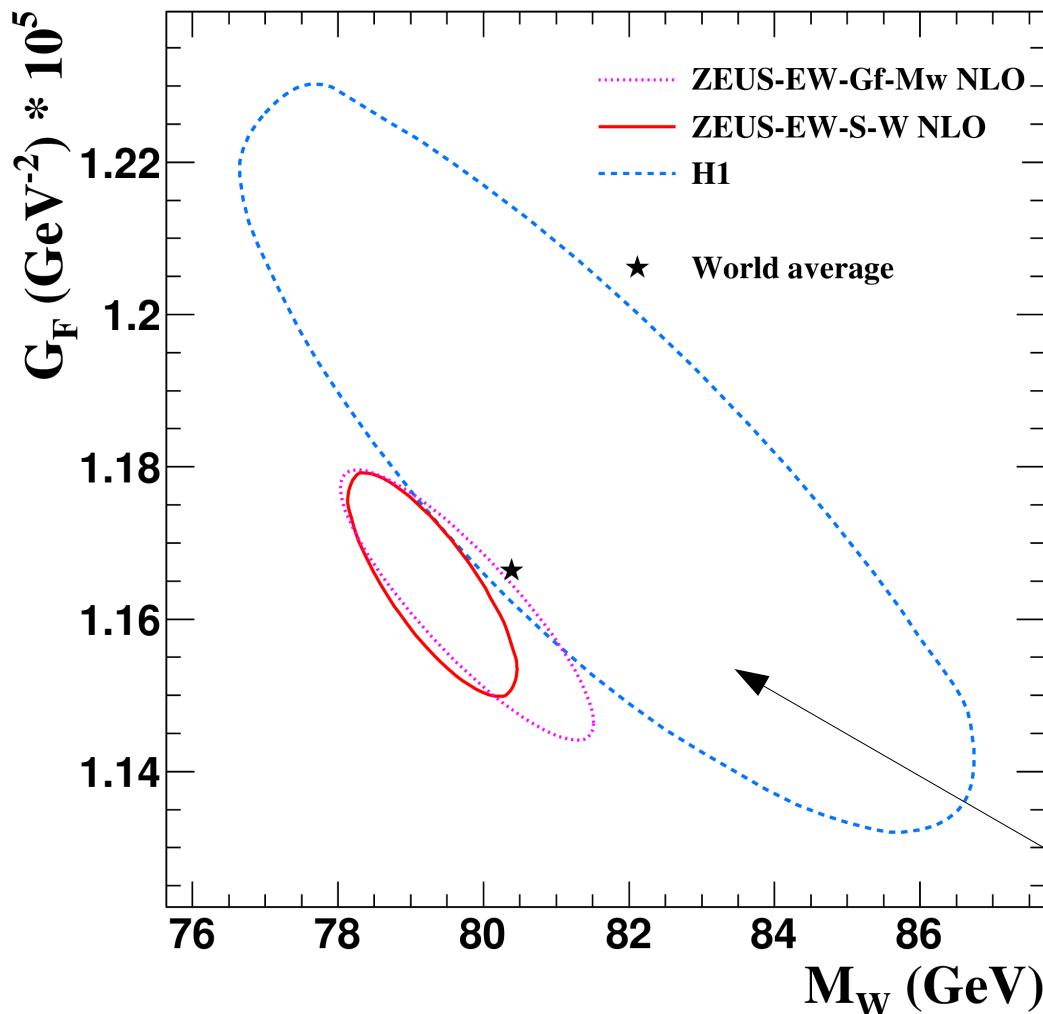


PDFs from ZEUS-EW-Z and ZEUS-EW-S agree very well.

G_F and mass of W boson

- ◆ G_F and M_W were also determined simultaneously with PDFs as a consistency check.

ZEUS



$$M_W = 79.77 \pm 1.15 (\text{exp}) \text{ GeV}$$

$$G_F = (1.1618 \pm 0.0117) * 10^{-5} (\text{exp}) \text{ GeV}^{-2}$$

$$\text{corr}(M_W, G_F) = -0.87$$

$$M_W^{\text{World average}} = 80.385 \pm 0.015 \text{ GeV}$$

$$G_F^{\text{World average}} = 1.1663787 * 10^{-5} \pm 6 * 10^{-12} \text{ GeV}^{-2}$$

Experimental/fit uncertainties only!!

- ◆ Fitter G_F and M_W are consistent with current world average values.

Quark couplings to Z

Now consider fits to electroweak NC couplings as well as PDF parameters

The total cross-section : $\sigma = \sigma^0 + P \sigma^P$

The unpolarised cross-section is given by $\sigma^0 = Y_+ F_2^0 + Y_- xF_3^0$

$$F_2^0 = \sum_i A_i^0(Q^2) [xq_i(x, Q^2) + x\bar{q}_i(\bar{x}, Q^2)]$$

$$xF_3^0 = \sum_i B_i^0(Q^2) [xq_i(x, Q^2) - x\bar{q}_i(\bar{x}, Q^2)]$$

$$A_i^0(Q^2) = e_i^2 - 2 e_i v_i v_e P_Z + (v_e^2 + a_e^2)(v_i^2 + a_i^2) P_Z^2$$

$$B_i^0(Q^2) = -2 e_i a_i a_e P_Z + 4 a_i a_e v_i v_e P_Z^2$$

$$P_Z = \frac{1}{\sin^2 2\theta} \frac{Q^2}{(M_Z^2 + Q^2)}$$

The polarised cross-section is given by $\sigma^P = Y_+ F_2^P + Y_- xF_3^P$

$$F_2^P = \sum_i A_i^P(Q^2) [xq_i(x, Q^2) + x\bar{q}_i(\bar{x}, Q^2)]$$

$$xF_3^P = \sum_i B_i^P(Q^2) [xq_i(x, Q^2) - x\bar{q}_i(\bar{x}, Q^2)]$$

$$A_i^P(Q^2) = 2 e_i v_i a_e P_Z - 2 v_e a_e (v_i^2 + a_i^2) P_Z^2$$

$$B_i^P(Q^2) = 2 e_i a_i v_e P_Z - 2 a_i v_i (v_e^2 + a_e^2) P_Z^2$$

$P_Z \gg P_Z^2$ (γZ interference is dominant)

v_e is very small (~ 0.04).

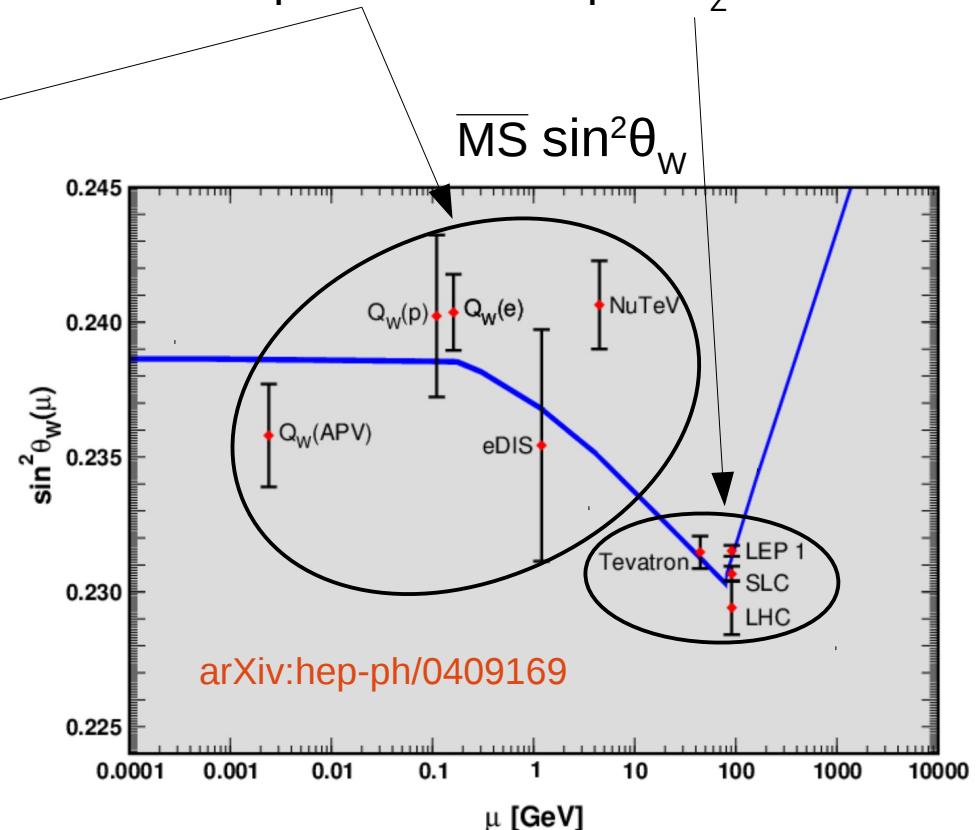
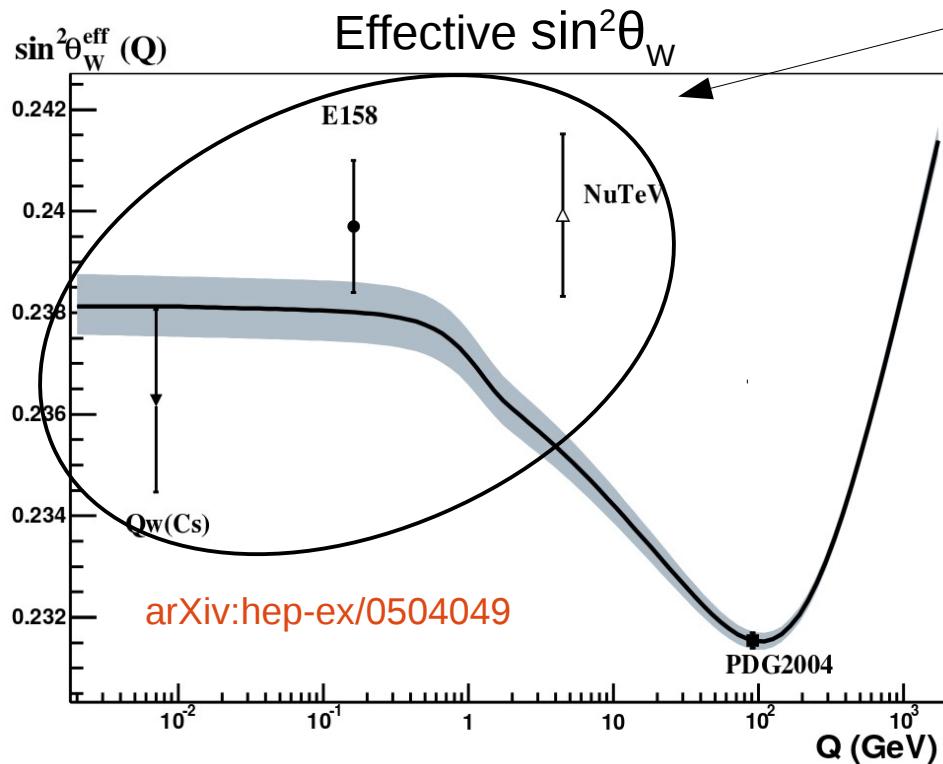


unpolarized $xF_3 \rightarrow a_i$,
polarized $F_2 \rightarrow v_i$

From slides by Amanda Cooper-Sarkar

On $\sin^2\theta_w$ running with a scale

- All the measurements were so far done either at the scale $\mu \leq 1 \text{ GeV}$ or $\mu = M_Z$.



Both of the variants more-or-less follow the same approach:

$$1 - 4 \kappa(Q^2) \sin^2 \theta_w(M_Z) = 1 - 4(Q^2) \sin^2 \theta_w(Q^2)$$

$$\kappa = \kappa_f(Q^2, \alpha, T_{3f}, Q_f, m_f, M_Z) + \kappa_b(Q^2, \alpha, M_W) \quad \text{Fermion and boson loop.}$$