

Forward Jet Production and BFKL Dynamics at HERA

August 30, 2007

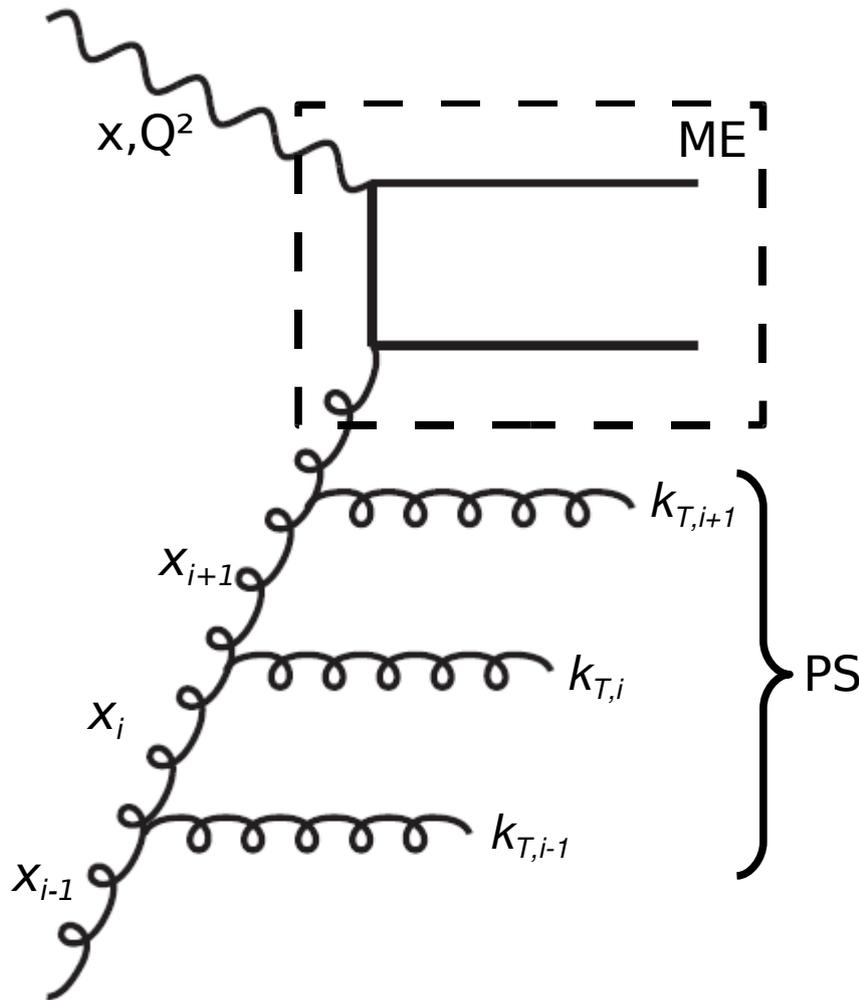
Workshop on low x physics, Helsinki

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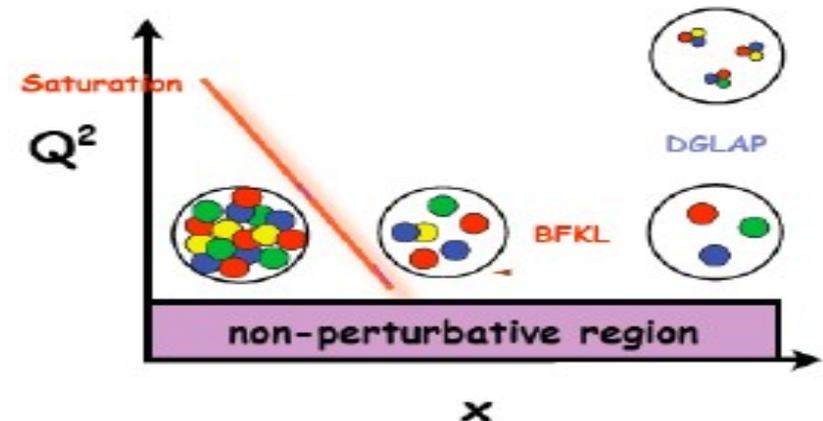
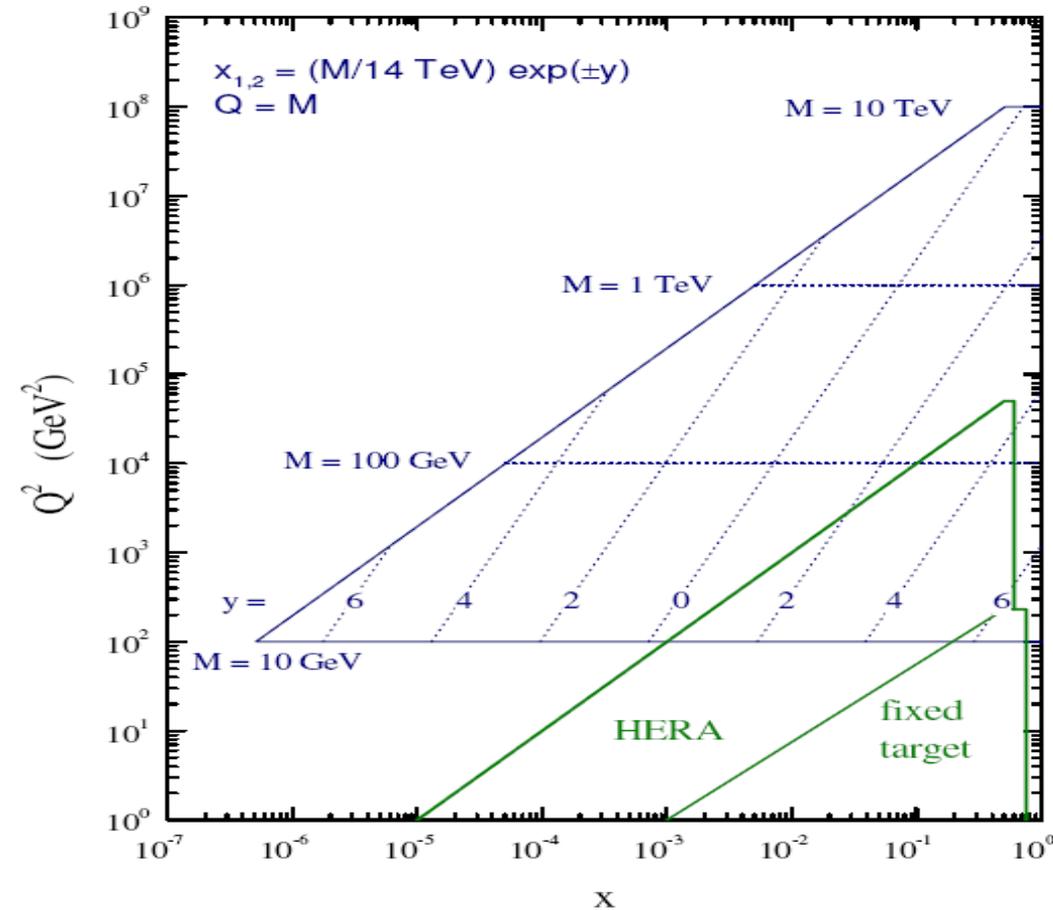
Matching of matrix element (ME) to parton showers (PS)



- ME can be $O(\alpha)$ (QPM), $O(\alpha\alpha_s)$ (BGF, QCD-Compton), $O(\alpha\alpha_s^2)$, ...
→ exact calculation for fixed orders
- higher orders are covered by PS which sum a subset of (leading) diagrams at each order
→ which diagrams are leading depends on kinematics (x, Q^2)
- different approaches for PS exist: DGLAP, BFKL, CCFM, ...:
 - resumming of different diagrams
 $\sim (\alpha_s \ln Q^2/Q_0^2)^n, (\alpha_s \ln 1/x)^n$
 - differences in ordering of $k_{T,i}, x_i$ of parton emissions

→ Final state jet studies can validate fixed order ME calculations and distinguish different PS approaches

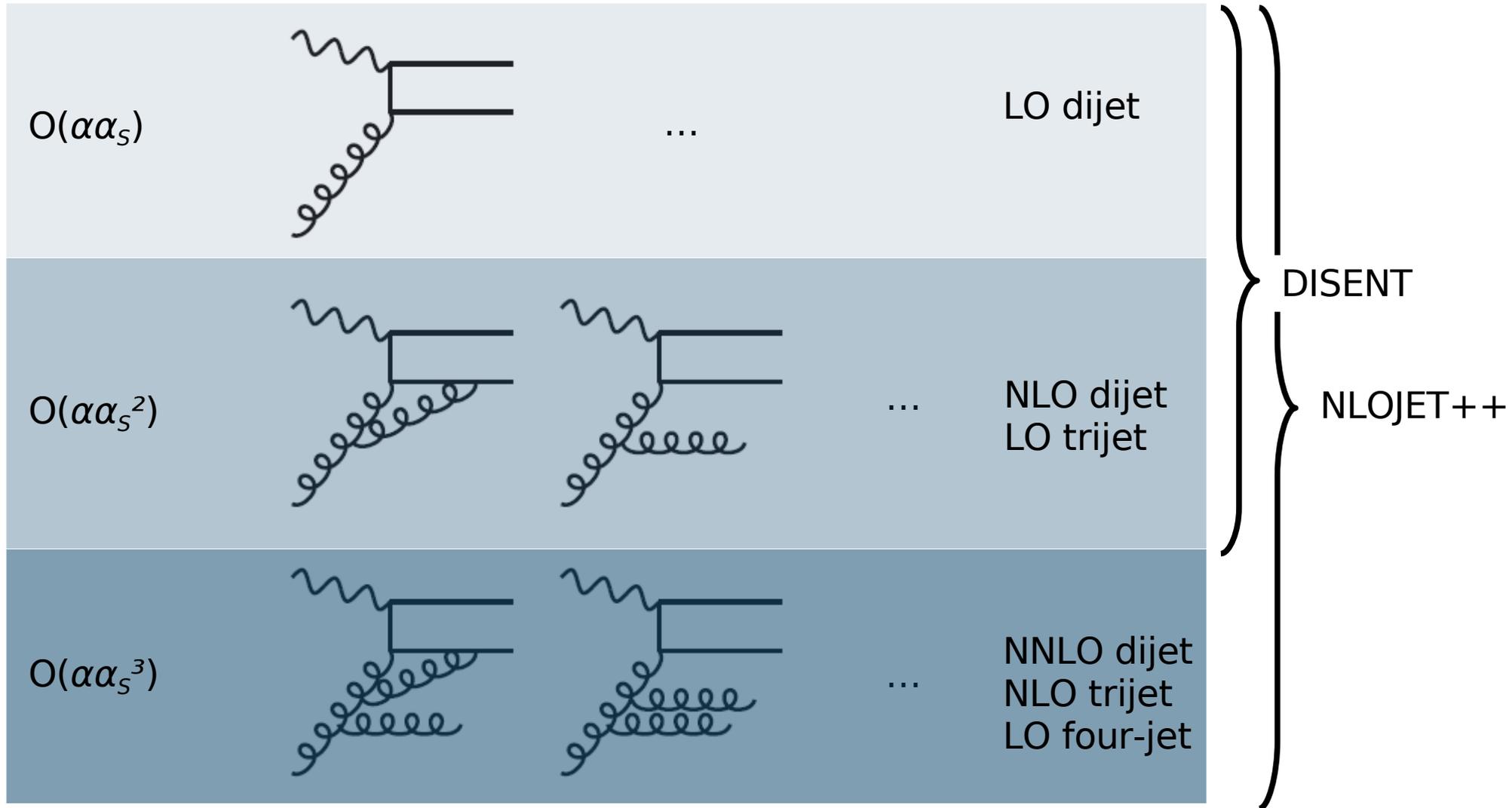
- the inclusive DIS cross section (F_2) is very well described by DGLAP, but also allows for BFKL terms
 - DGLAP fails to describe forward jet production at small x
 - novel QCD effects at low x , where the gluon density becomes very large, are expected (saturation, colour glass condensate, ...)
- Consequences for the LHC?
- many SM and BSM processes at the LHC involve the collision of partons with small x
 - standard prescription: take pdfs from HERA and evolve them to higher Q^2 according to DGLAP
- Forward and multijet studies at HERA are needed to predict low- x effects at the LHC





QCD models and calculations

- Fixed order matrix element calculations

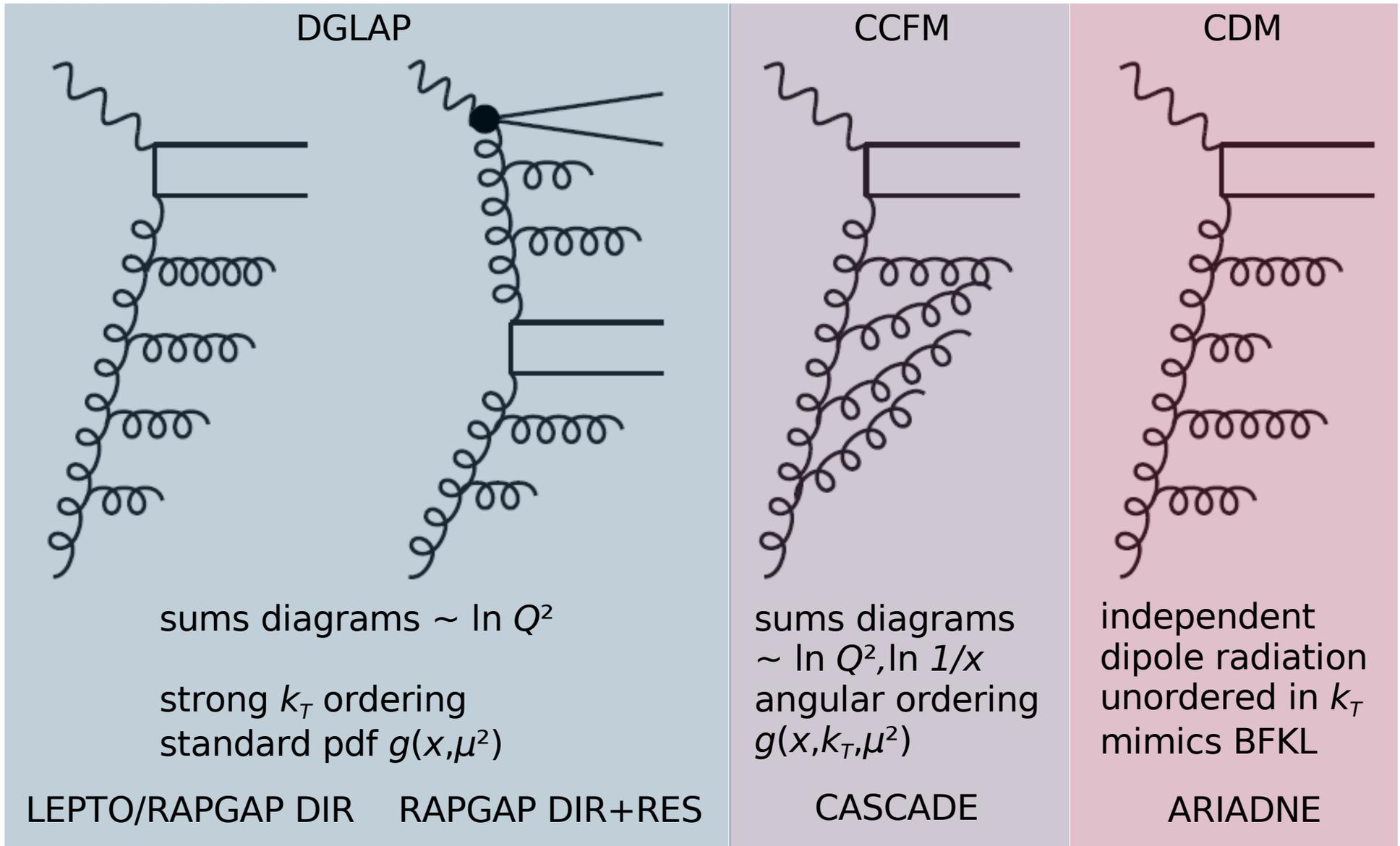


fixed order ME calculations yield parton level predictions
→ hadronisation corrections must be applied to these calculations



QCD models and calculations

- Parton shower models



Full Monte Carlo models use LO matrix elements + parton showers + hadronisation \rightarrow yield hadron level predictions

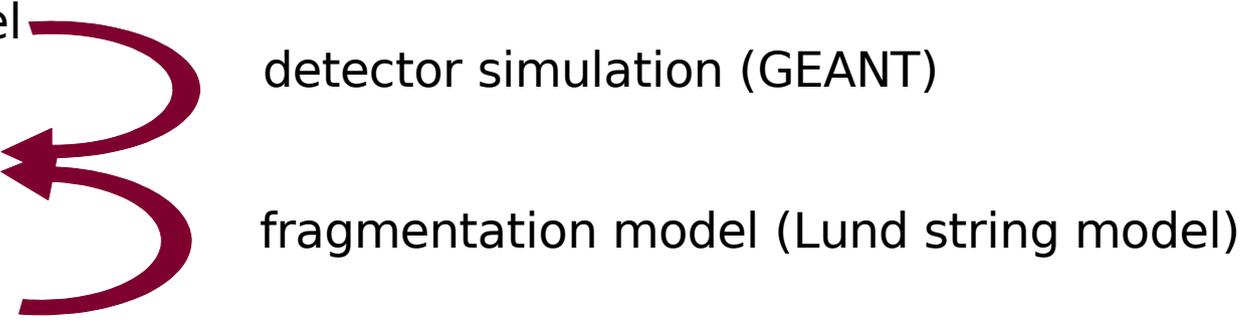


Experimental results

- Measurements covered in this talk:
 - Multijet production at low x_{Bj} in deep inelastic scattering at HERA
ZEUS Collaboration, DESY-07-062 (May 2007), submitted to Nucl. Phys. B
arXiv:0705.1931v1 [hep-ex]
 - Forward-jet production in deep inelastic ep scattering at HERA
ZEUS Collaboration, DESY-07-100 (July 2007), submitted to Eur. Phys. J. C
arXiv:0707.3093v2 [hep-ex]
 - Three- and four-jet production in deep-inelastic scattering at HERA and low- x parton dynamics
H1 Collaboration, to be published
- Previous results:
 - Forward jet production in deep inelastic scattering at HERA
H1 Collaboration, Eur. Phys. J. C46 (2006) 27-42
arXiv:hep-ex/0508055
 - Forward jet production in deep inelastic ep scattering and low- x parton dynamics at HERA
ZEUS Collaboration, Phys.Lett. B632 (2006) 13-26
arXiv:hep-ex/0502029v1



Experimental analysis

- DIS selection...
- Jet finding algorithm
 - inclusive k_T algorithm in γ^*p centre of mass or Breit frame
- Jet pseudorapidity range: determined by calorimeter coverage
 - ZEUS CAL: $-1.5 < \eta^{jet} < 2$ (multijet analysis)
 - ZEUS CAL + FPC: $2 < \eta^{jet} < 4.3$ (forward jet analysis)
 - H1 LAr calorimeter: $-1 < \eta^{jet} < 2.5$ (3- and 4-jet analysis)
- Data and model corrections
 - detector level
 - hadron level
 - parton level

detector simulation (GEANT)

fragmentation model (Lund string model)



Multijet production at low x

- **Aim:** check ME calculations for 2- and 3-jet production at NLO
- **Method:**
 - measure 2- and 3-jet single differential cross section and cross section ratios
 - use energy-momentum balance to search for gluon radiation beyond NLO calculations



$$\begin{aligned}\Delta E_T^{jet1,2*} &\approx 0 \\ |\Sigma \mathbf{p}_T^{jet1,2*}| &\approx 0 \\ |\Delta \mathbf{p}_T^{jet1,2*}| / 2E_T^{jet1*} &\approx 1 \\ |\Delta \phi^{jet1,2*}| &\approx \pi\end{aligned}$$

$$\begin{aligned}\Delta E_T^{jet1,2*} &> 0 \\ |\Sigma \mathbf{p}_T^{jet1,2*}| &> 0 \\ |\Delta \mathbf{p}_T^{jet1,2*}| / 2E_T^{jet1*} &< 1 \\ |\Delta \phi^{jet1,2*}| &< \pi\end{aligned}$$

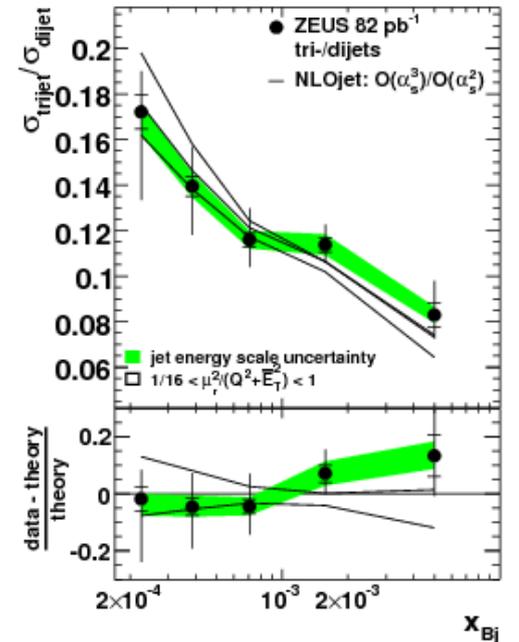
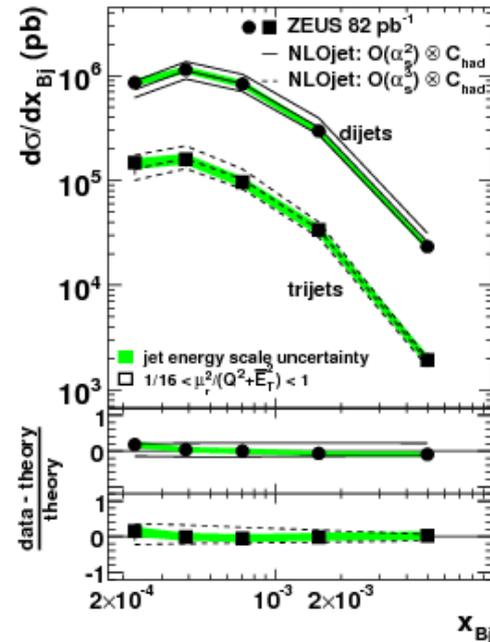
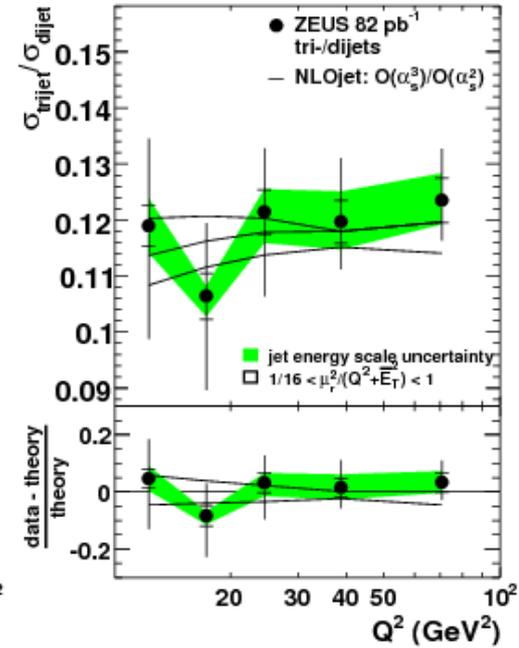
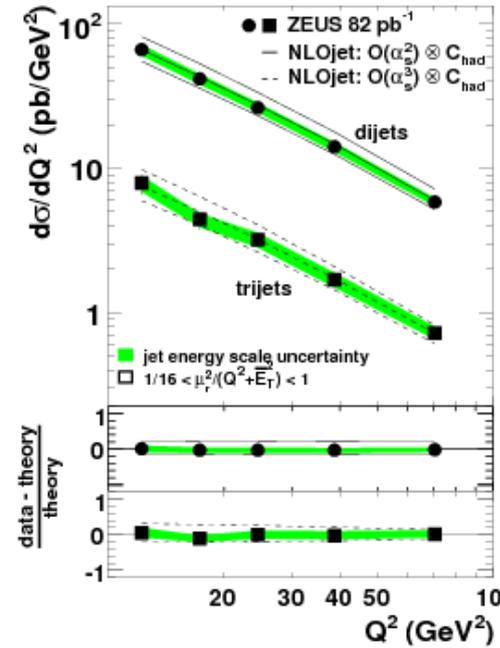
- **Kinematic selection:**
 - $10 < Q^2 < 100 \text{ GeV}^2$, $10^{-4} < x < 10^{-2}$, $0.1 < y < 0.6$
 - $-1.0 < \eta^{jet} < 2.5$, $E_T^{jet1*} > 7 \text{ GeV}$, $E_T^{jet2,3*} > 5 \text{ GeV}$



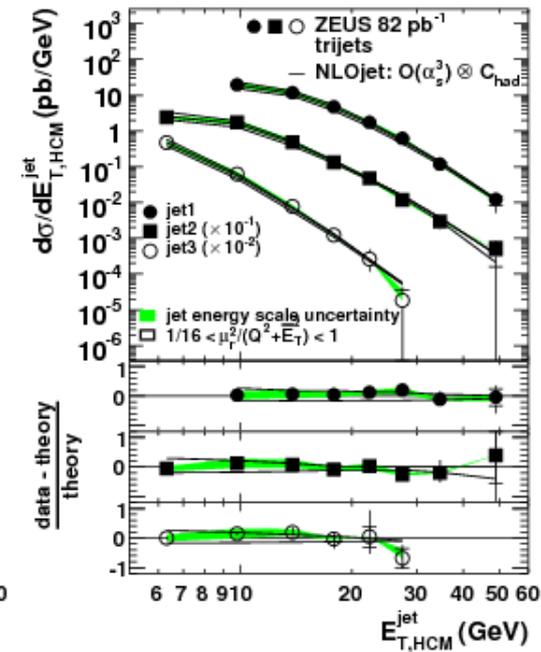
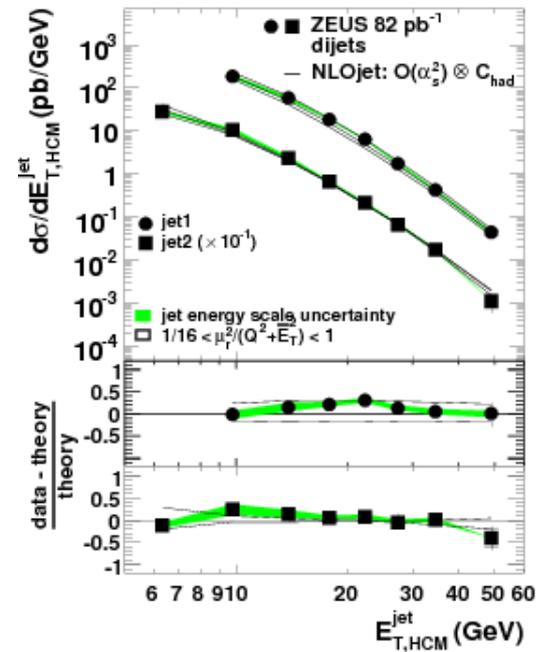
Inclusive 2-jet and 3-jet cross sections



- Q^2 and x dependence
 - trijet to dijet ratio is Q^2 independent but increases steeply towards small x
 - cross sections and cross section ratios are well described by NLO $O(\alpha_s^2)$, $O(\alpha_s^3)$ calculations



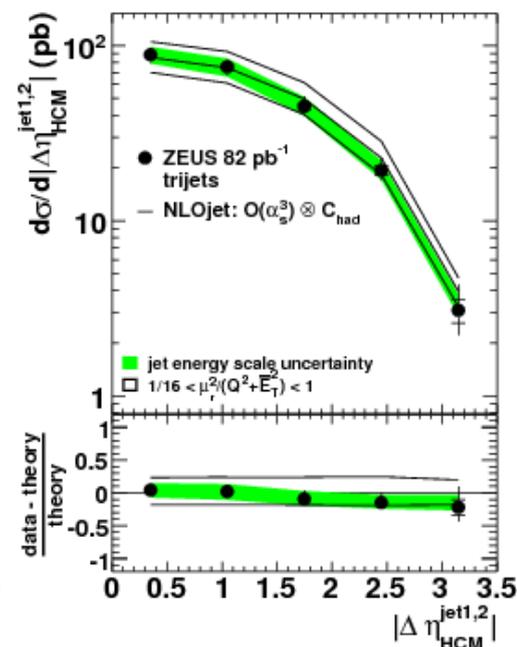
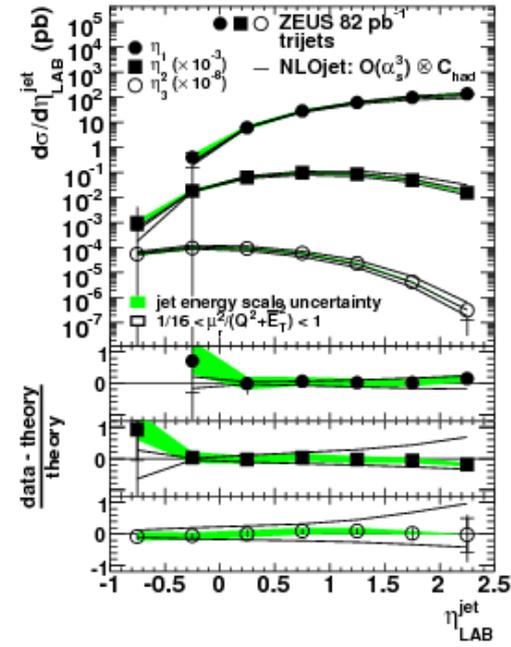
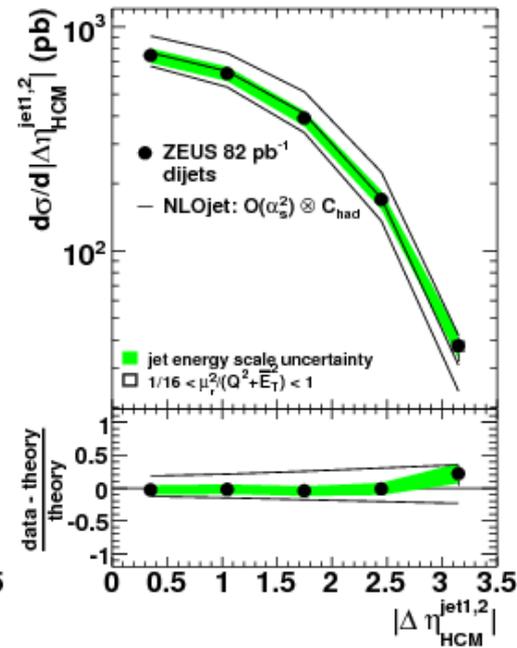
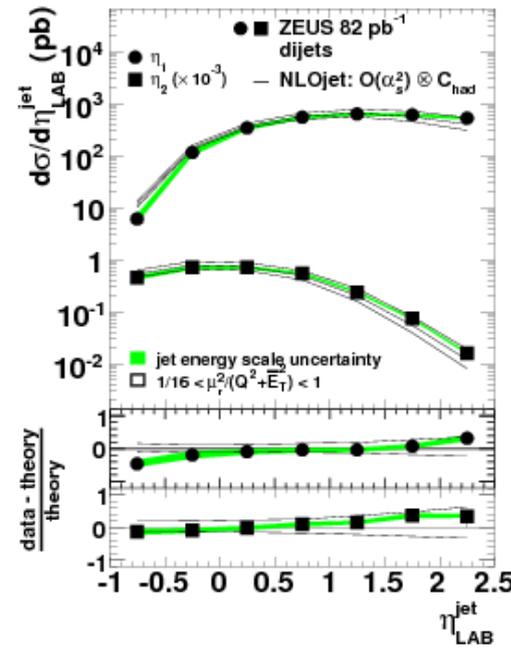
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- $E_T^{jet^*}$ and η^{jet} dependence
 - cross sections well described by NLO $O(\alpha_s^2)$, $O(\alpha_s^3)$ calculations over whole $E_T^{jet^*}$ range



Inclusive dijet and trijet cross sections



- Q^2 and x dependence
 - trijet to dijet ratio is Q^2 independent but increases steeply towards small x
 - cross sections and cross section ratios are well described by NLO $O(\alpha_s^2)$, $O(\alpha_s^3)$ calculations
- $E_T^{jet^*}$ and η^{jet} dependence
 - cross sections well described by NLO $O(\alpha_s^2)$, $O(\alpha_s^3)$ calculations over whole $E_T^{jet^*}$ range
 - η^{jet} and $\Delta(\eta^{jet1,2^*})$ distribution well described by NLO $O(\alpha_s^2)$, $O(\alpha_s^3)$ calculations

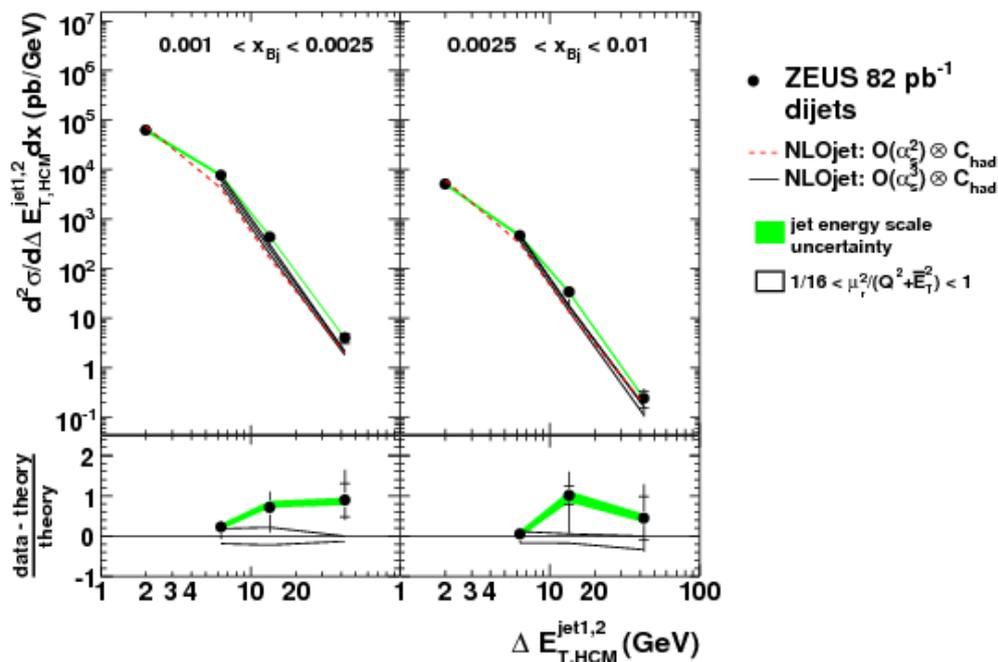
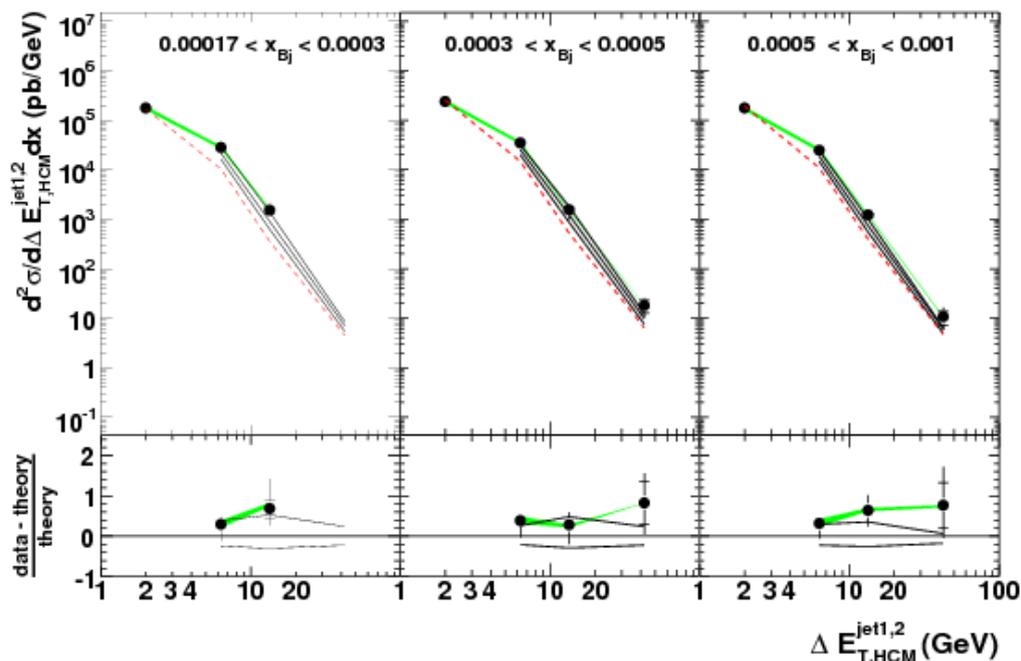
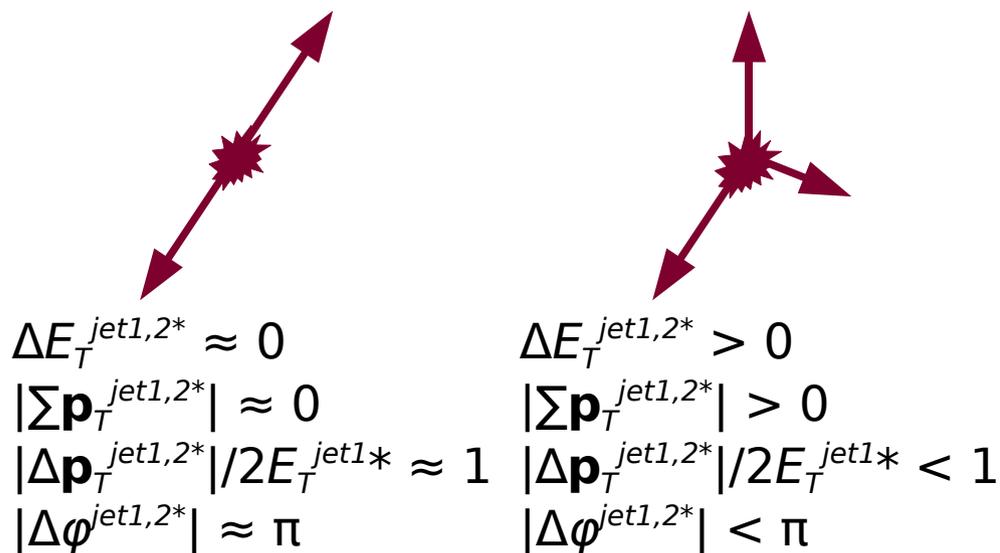




Jet energy-momentum balance



- Transverse energy correlations:
 - high- $\Delta E_T^{jet1,2*}$ tail not well described by $O(\alpha_s^2)$ calculations for 2-jet production at low x
 - $O(\alpha_s^3)$ calculations fine for 2-jet and 3-jet production

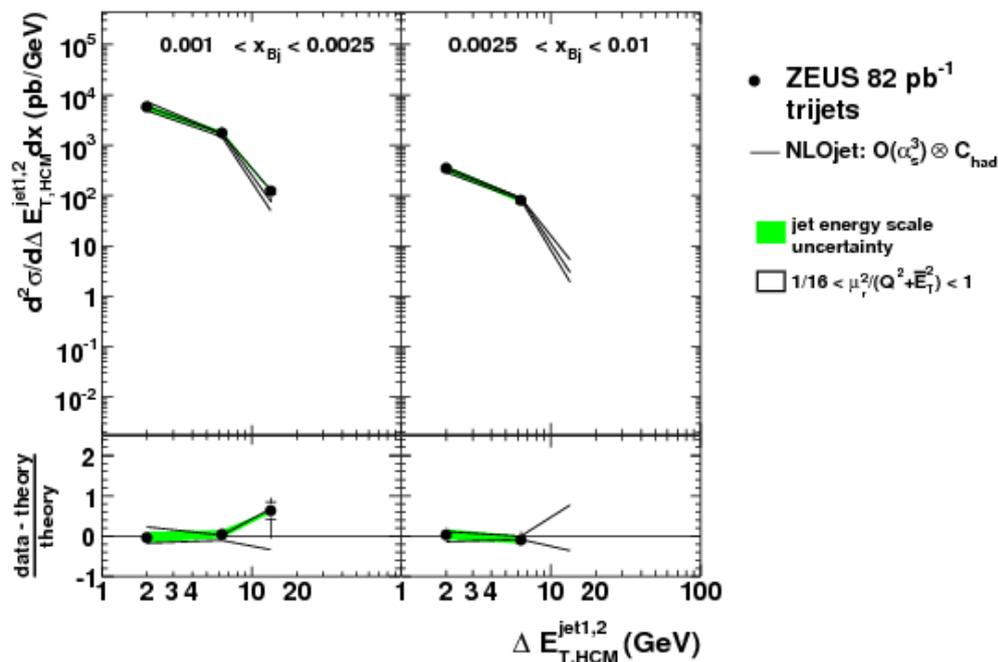
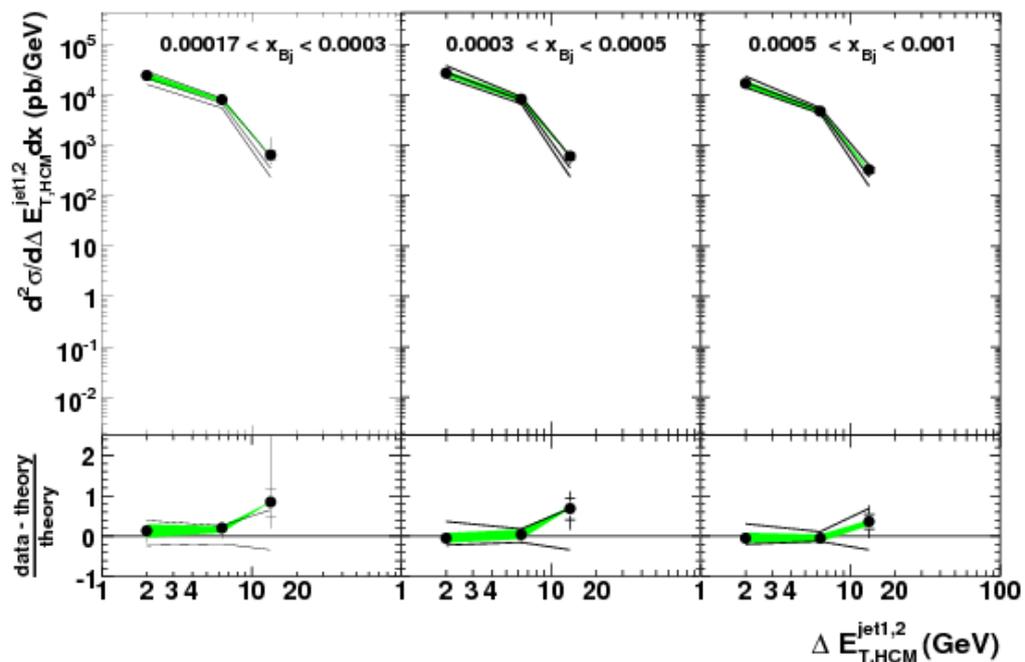
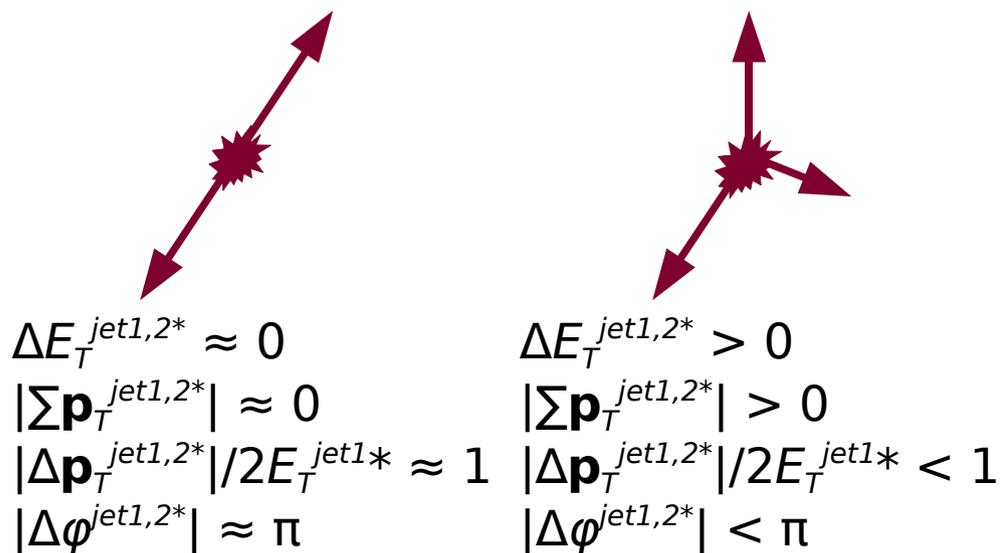




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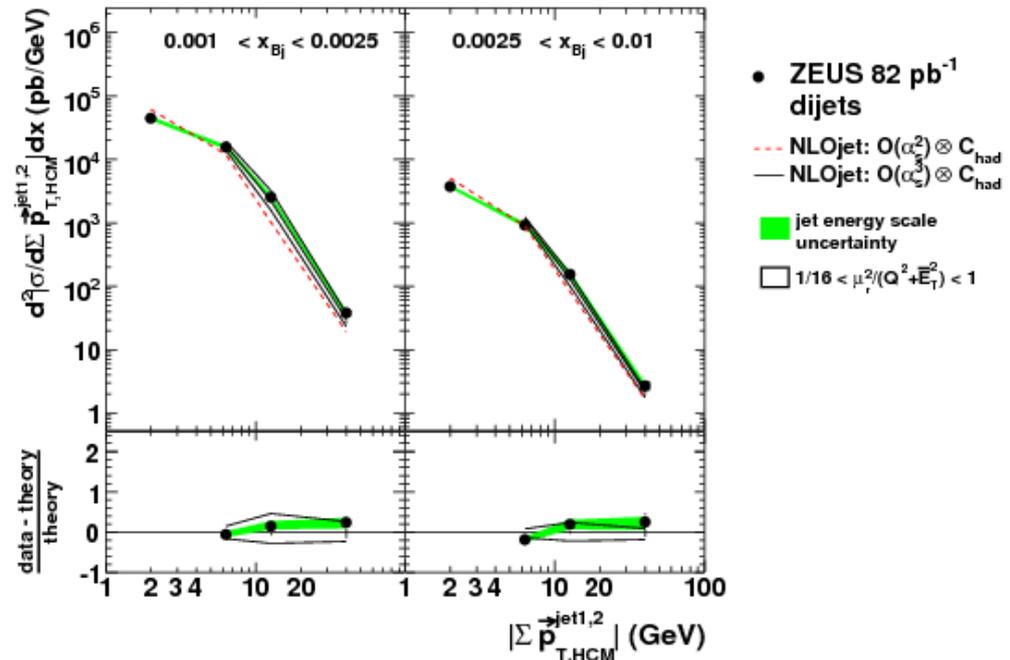
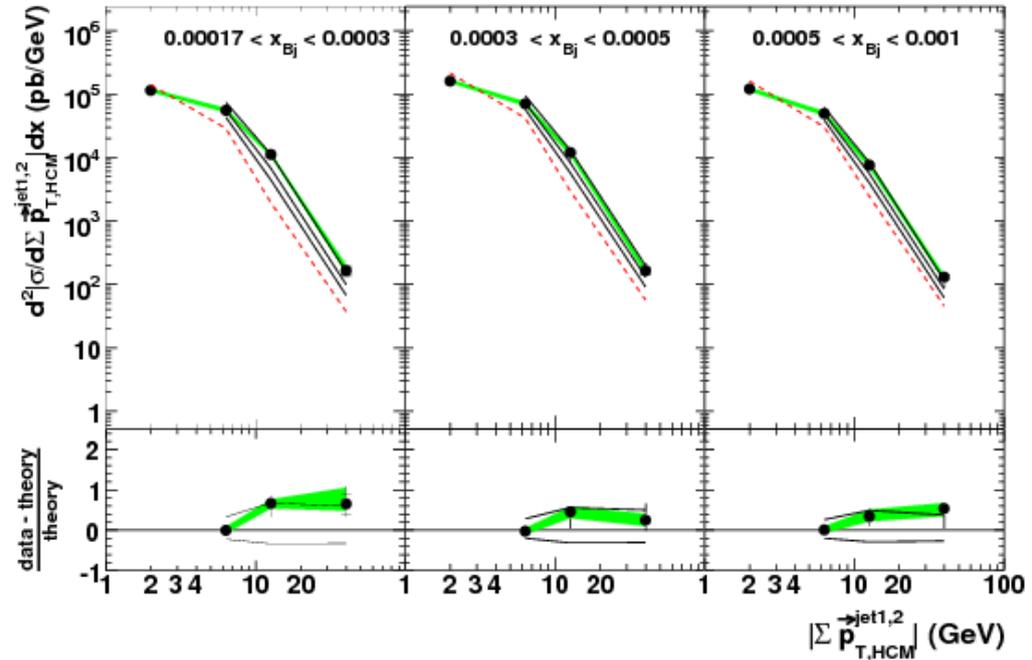
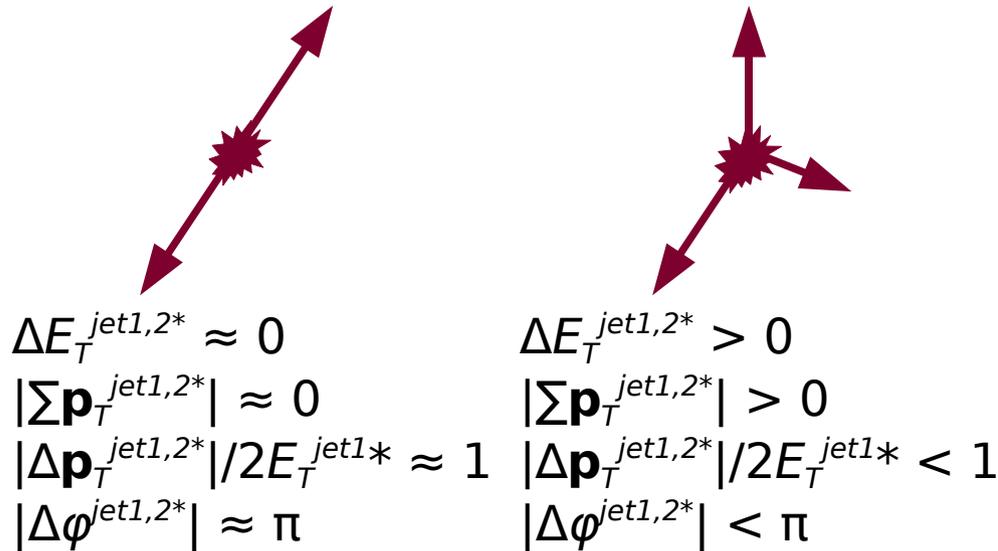




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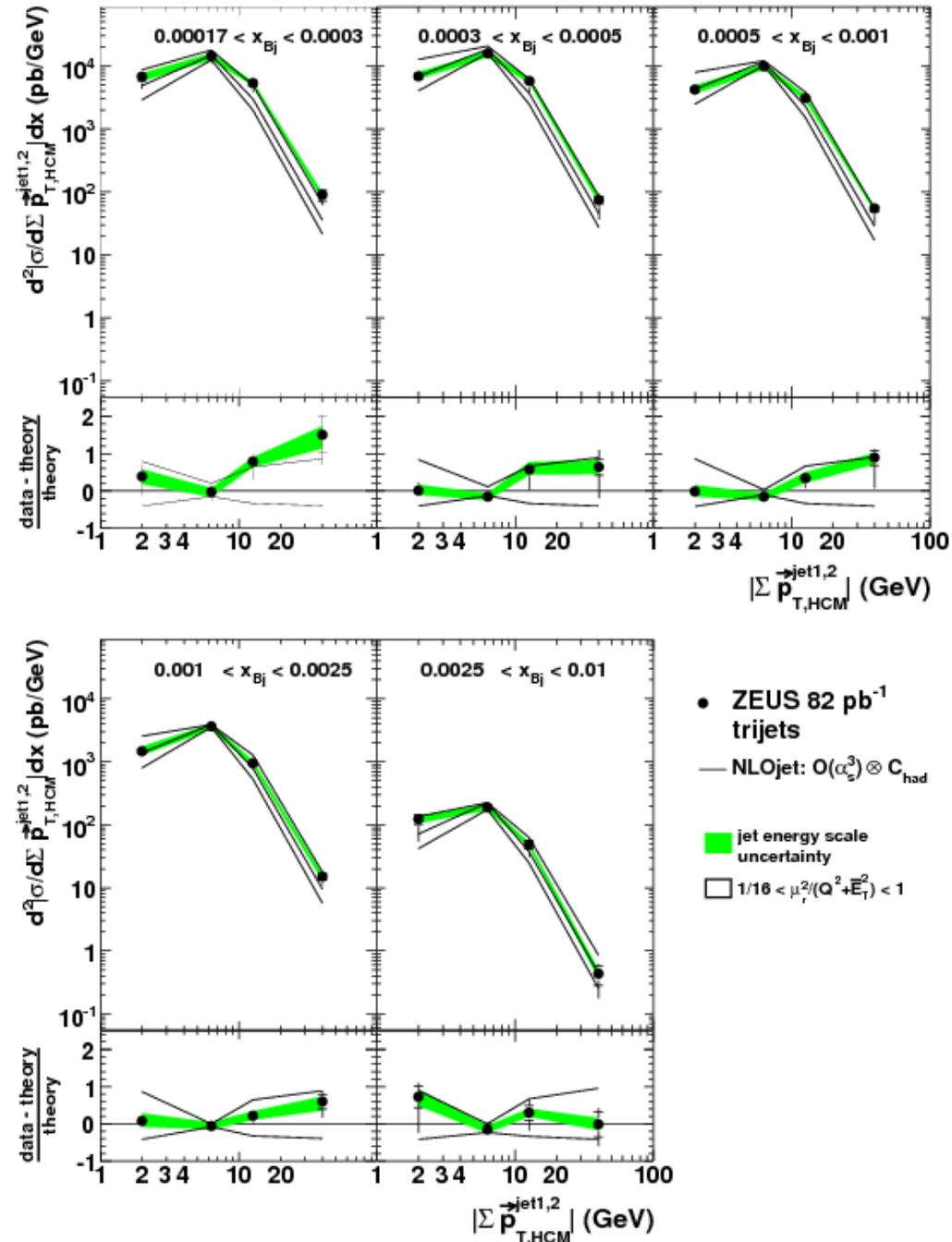
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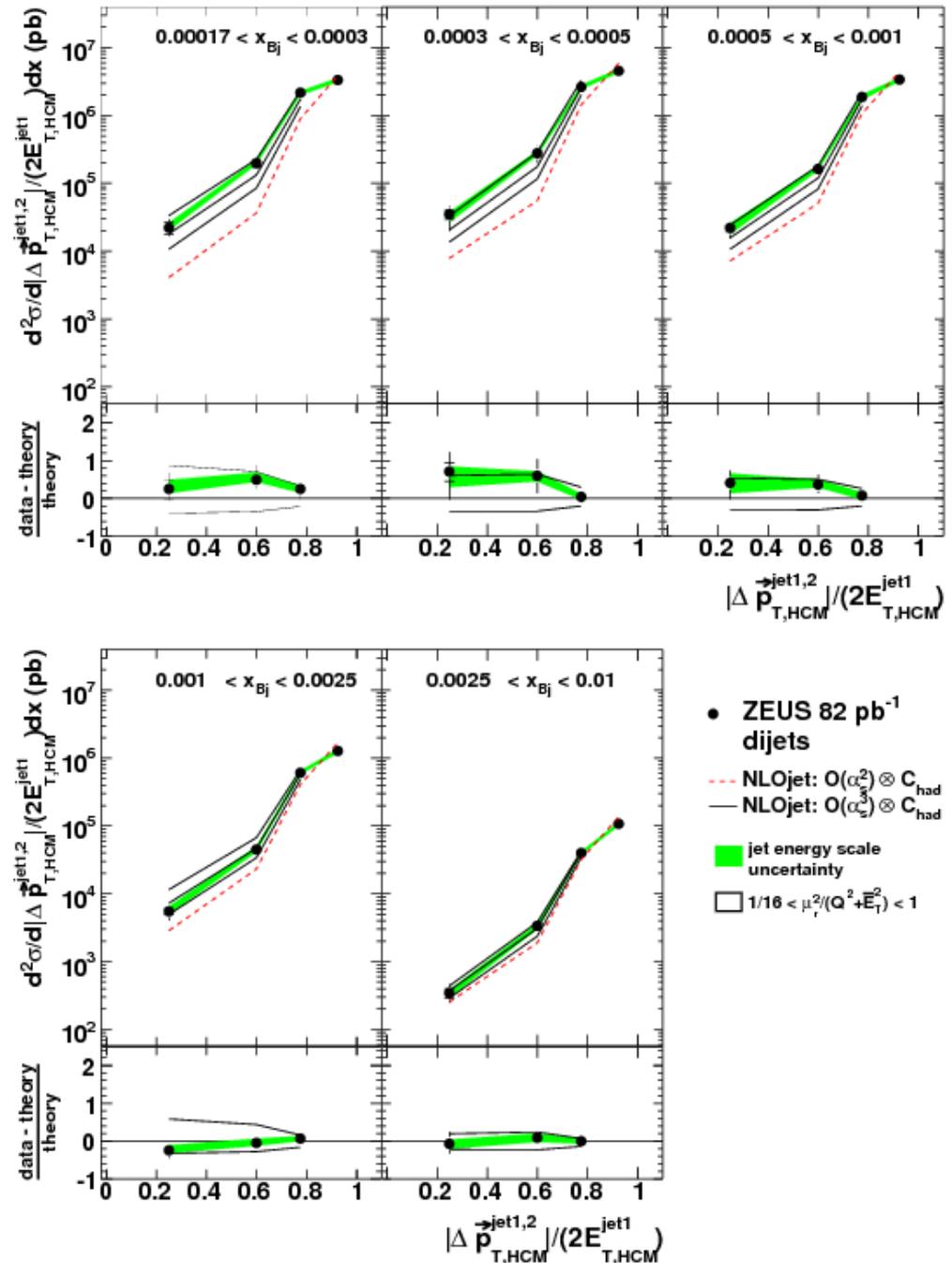
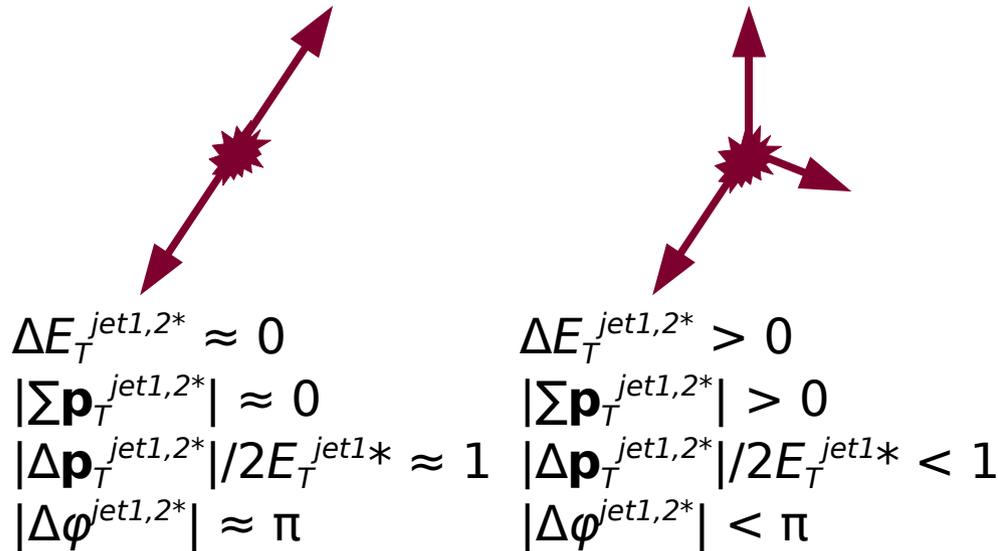




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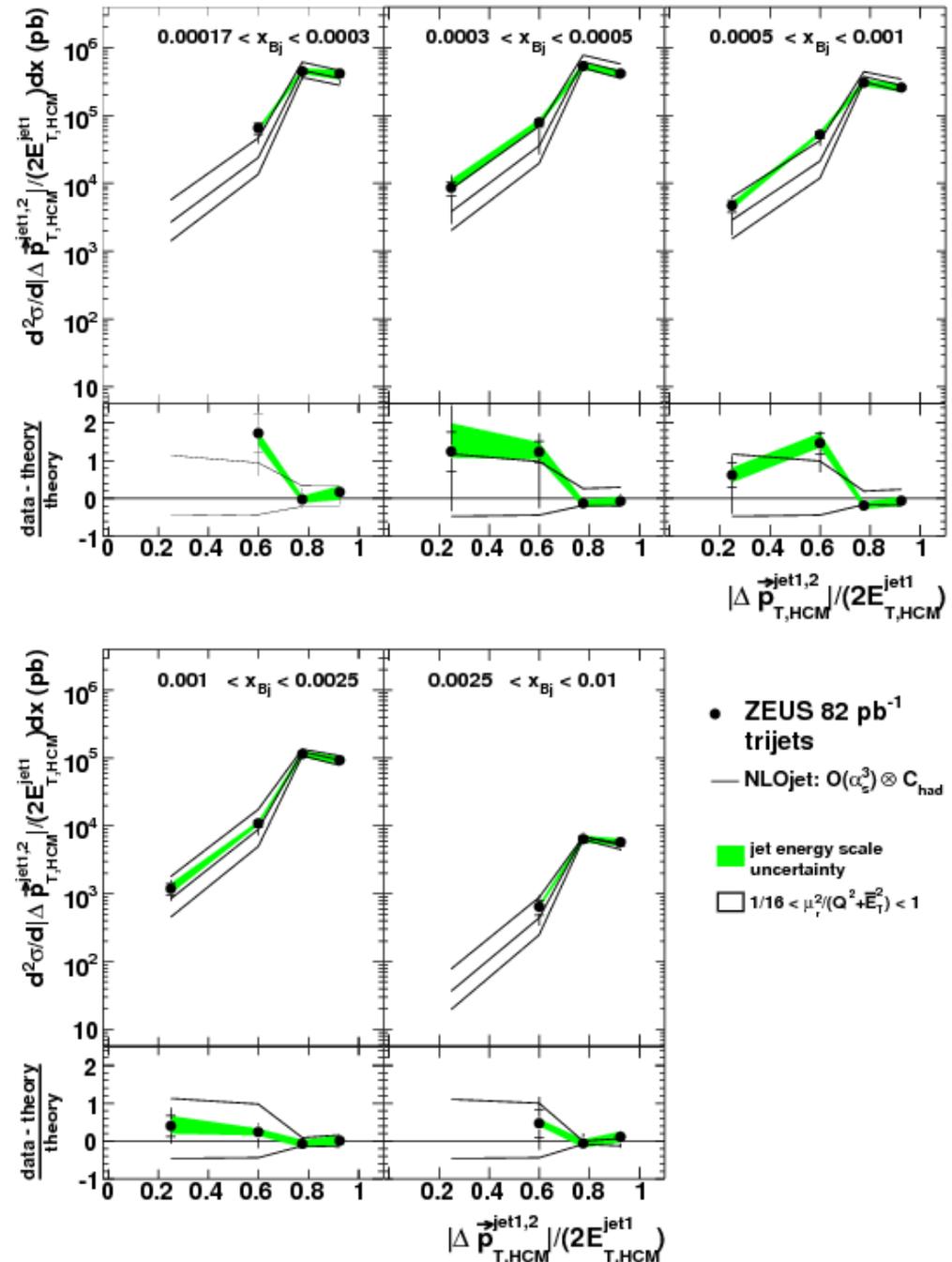
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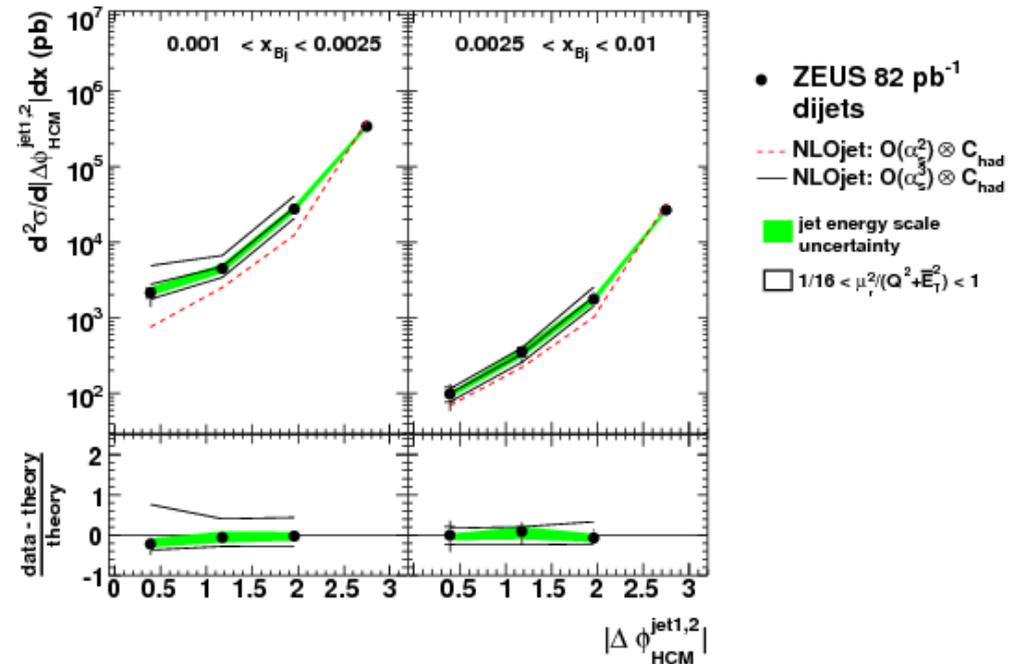
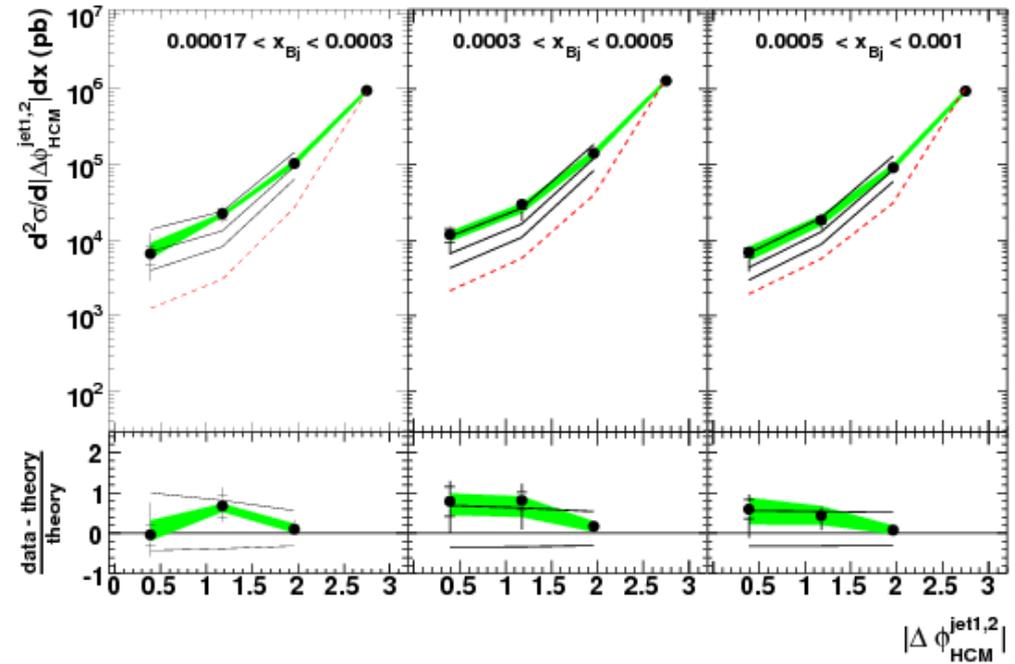
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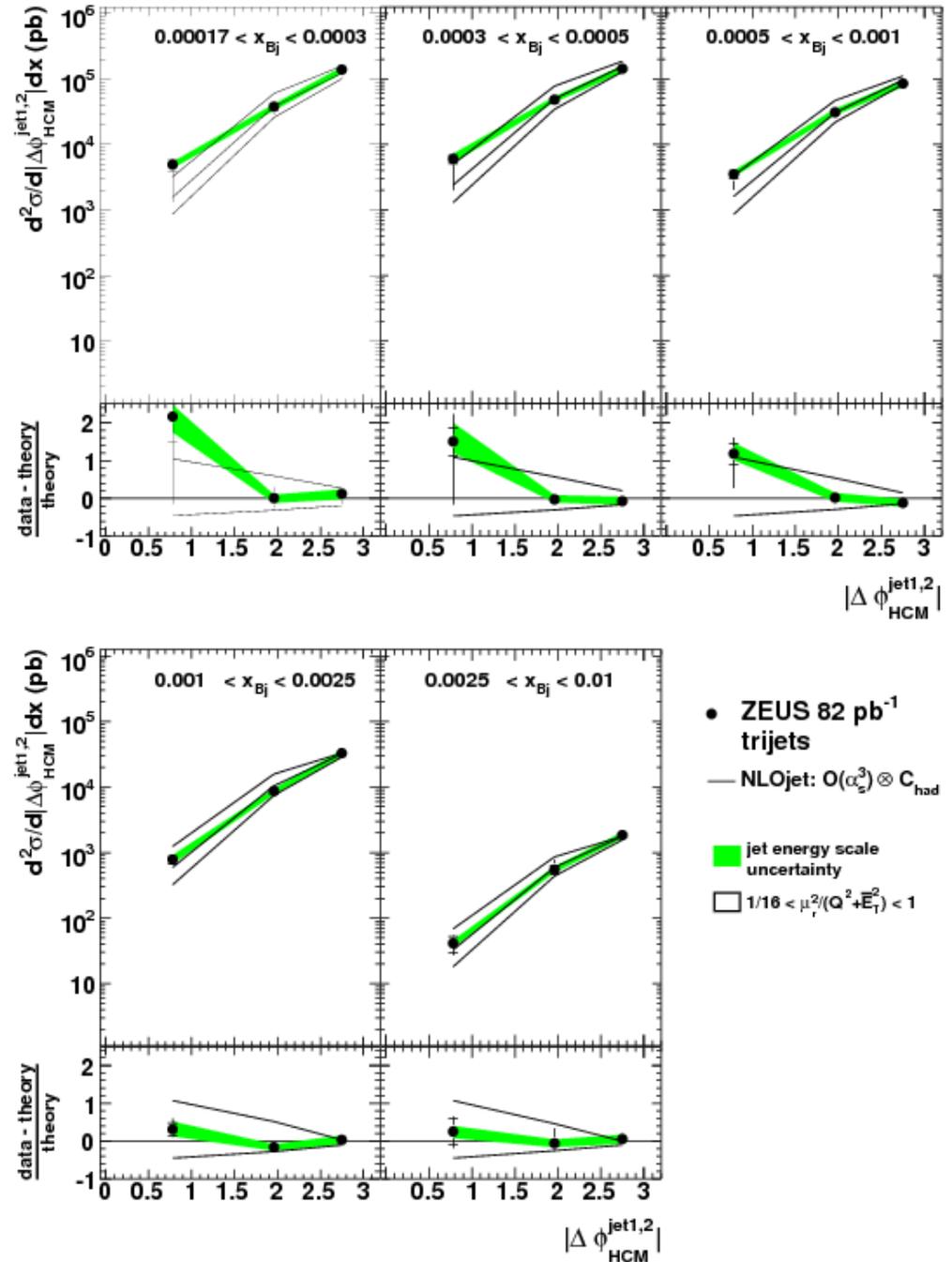
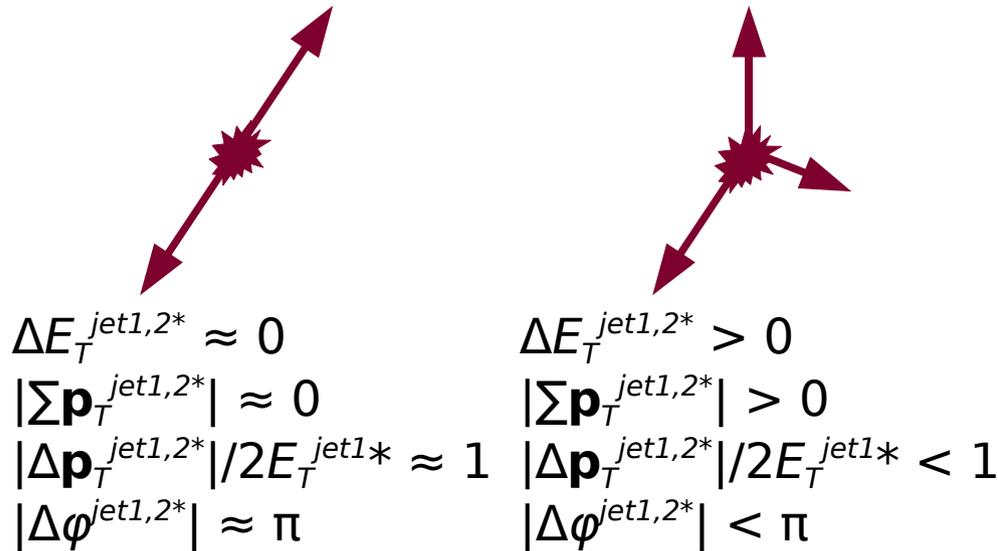




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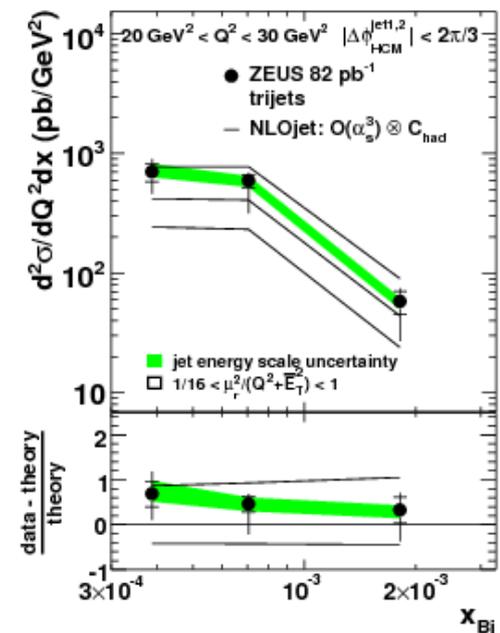
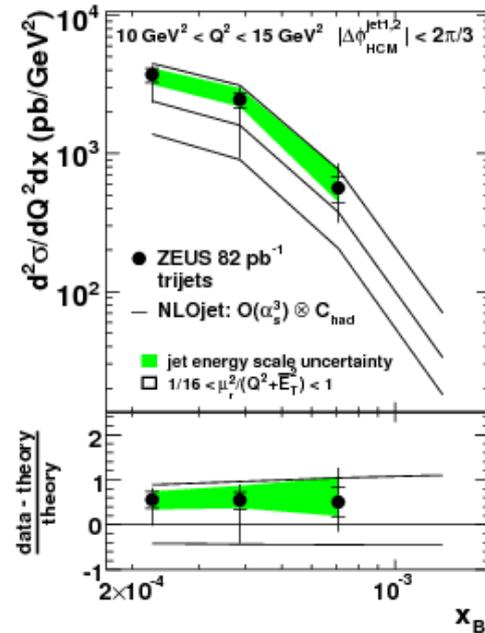
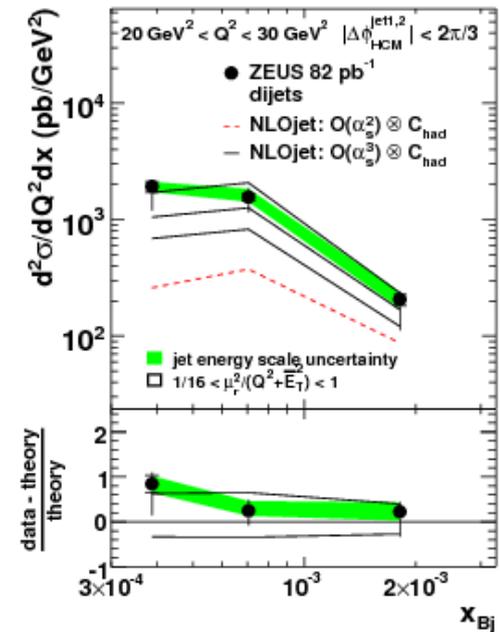
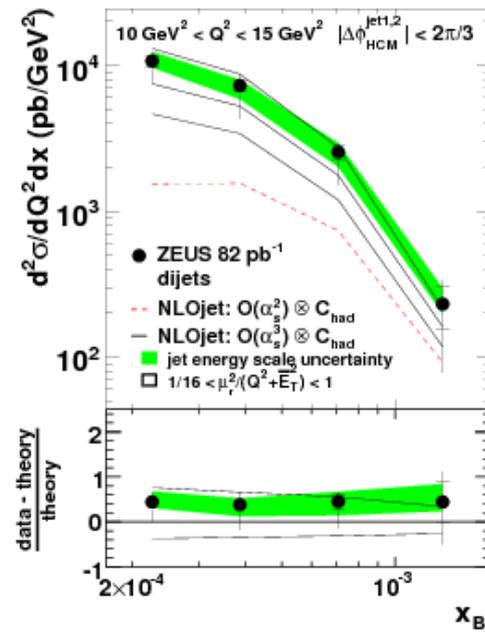
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Conclusion so far...

- 2-jet and 3-jet production at low x :
 - **aim:** check ME calculations for 2- and 3-jet production at NLO
 - **conclusion:** NLO ($O(\alpha\alpha_s^2)$, $O(\alpha\alpha_s^3)$) calculations work well; $O(\alpha\alpha_s^3)$ is especially needed for 2-jet production at low x when additional gluon radiation is highlighted

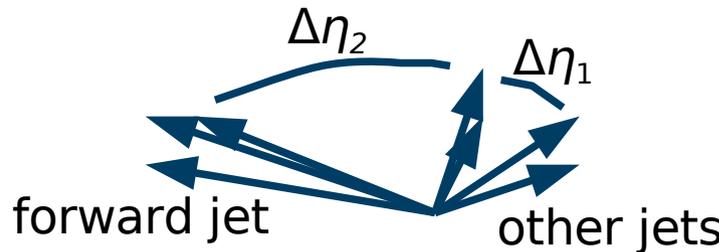
- **Aim:** check ME calculations at NLO and PS models in a region of phase space where additional gluon radiation and/or non-ordered PS are expected

- **Method:** enhance BFKL signal

- $(p_T^{jet})^2 \sim Q^2$: suppress strong k_T ordering
- $x^{jet} = E^{jet}/E^p \gg x$: select BFKL phase space
- in addition, a central dijet system can be required to highlight different parton dynamics

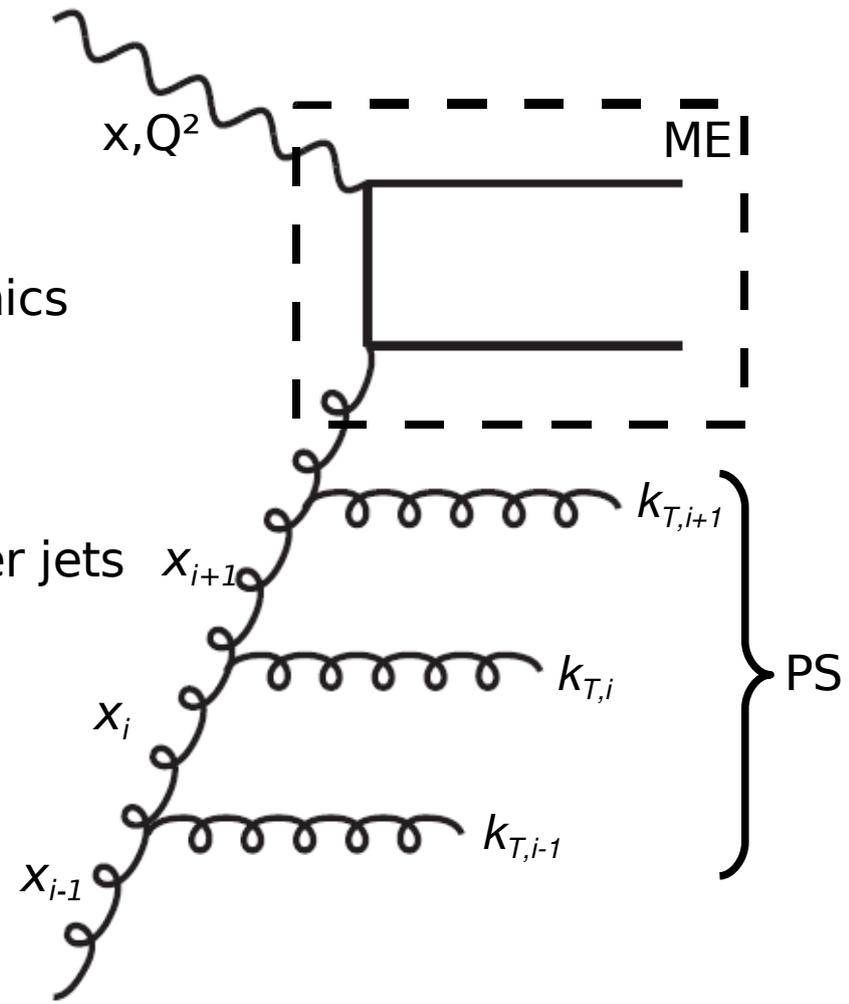
$$\Delta\eta_1 = \eta^{jet2} - \eta^{jet1}$$

$$\Delta\eta_2 = \eta^{fwdjet} - \eta^{jet2}$$

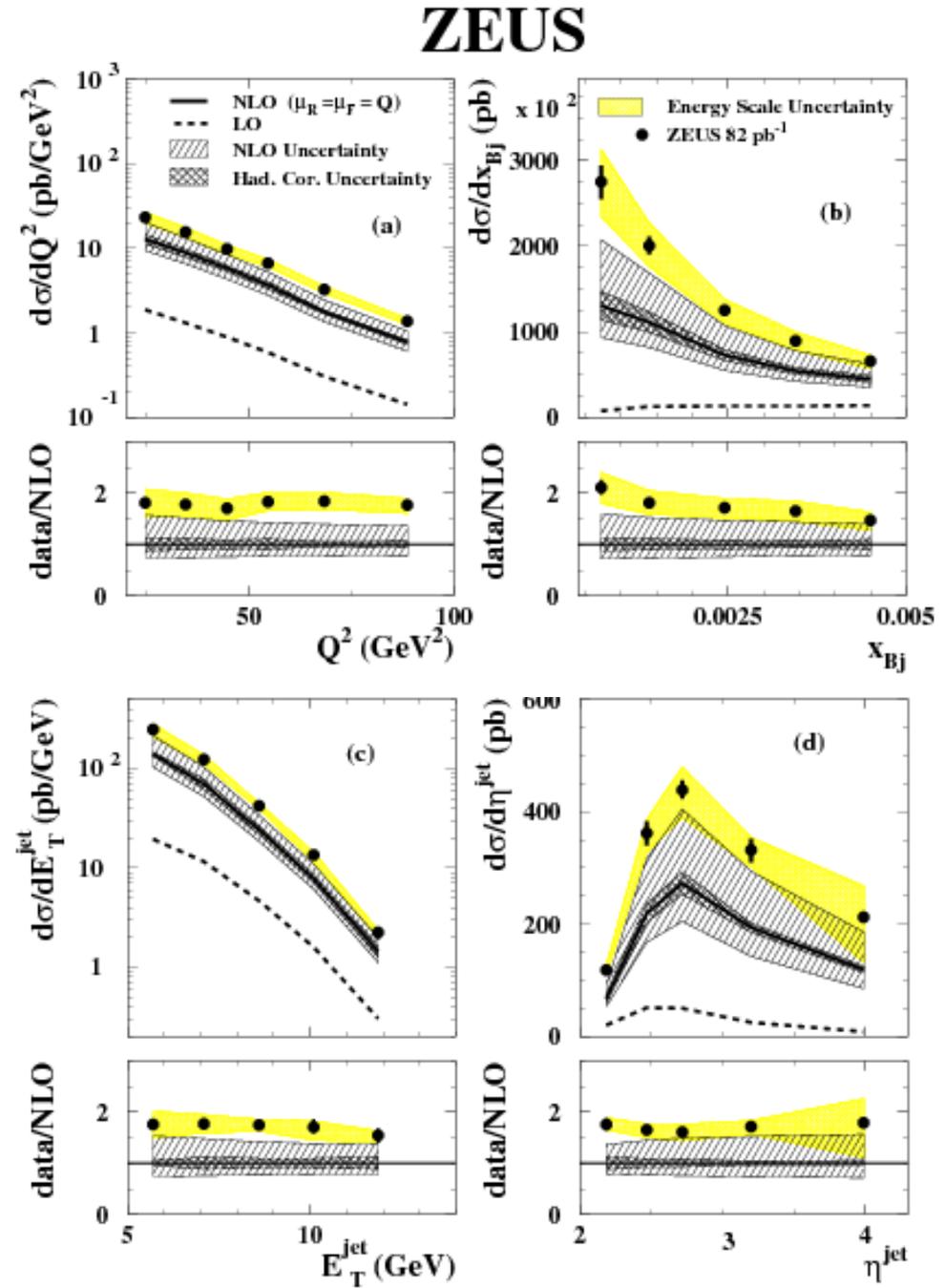


- **Kinematic selection:**

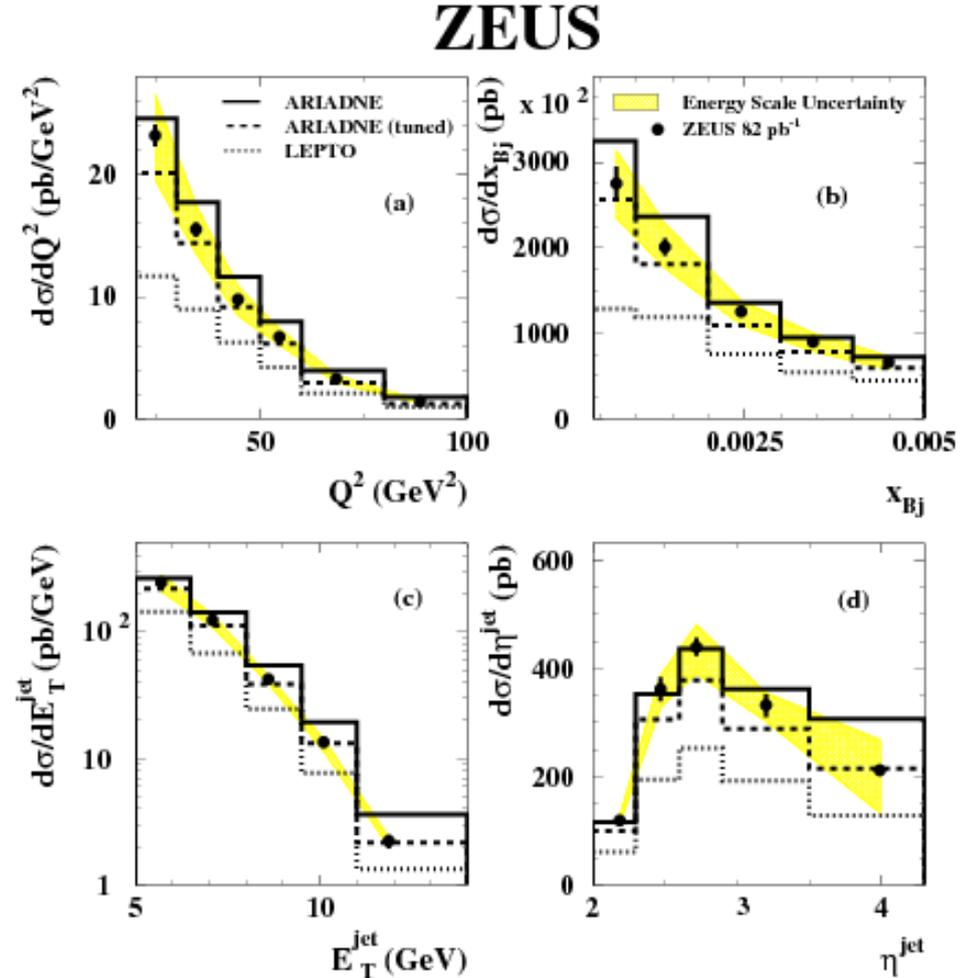
- $20 < Q^2 < 100 \text{ GeV}^2$,
- $0.04 < y < 0.7$,
- $0.0004 < x < 0.005$
- $E_T^{jet} > 5 \text{ GeV}$,
- $2 < \eta^{fwdjet} < 4.3, -1.5 < \eta^{jet1,2} < 4.3$
- $x^{fwdjet} > 0.036$
- $0.5 < (E_T^{jet})^2/Q^2 < 2$ (only for inclusive forward jet cross section)



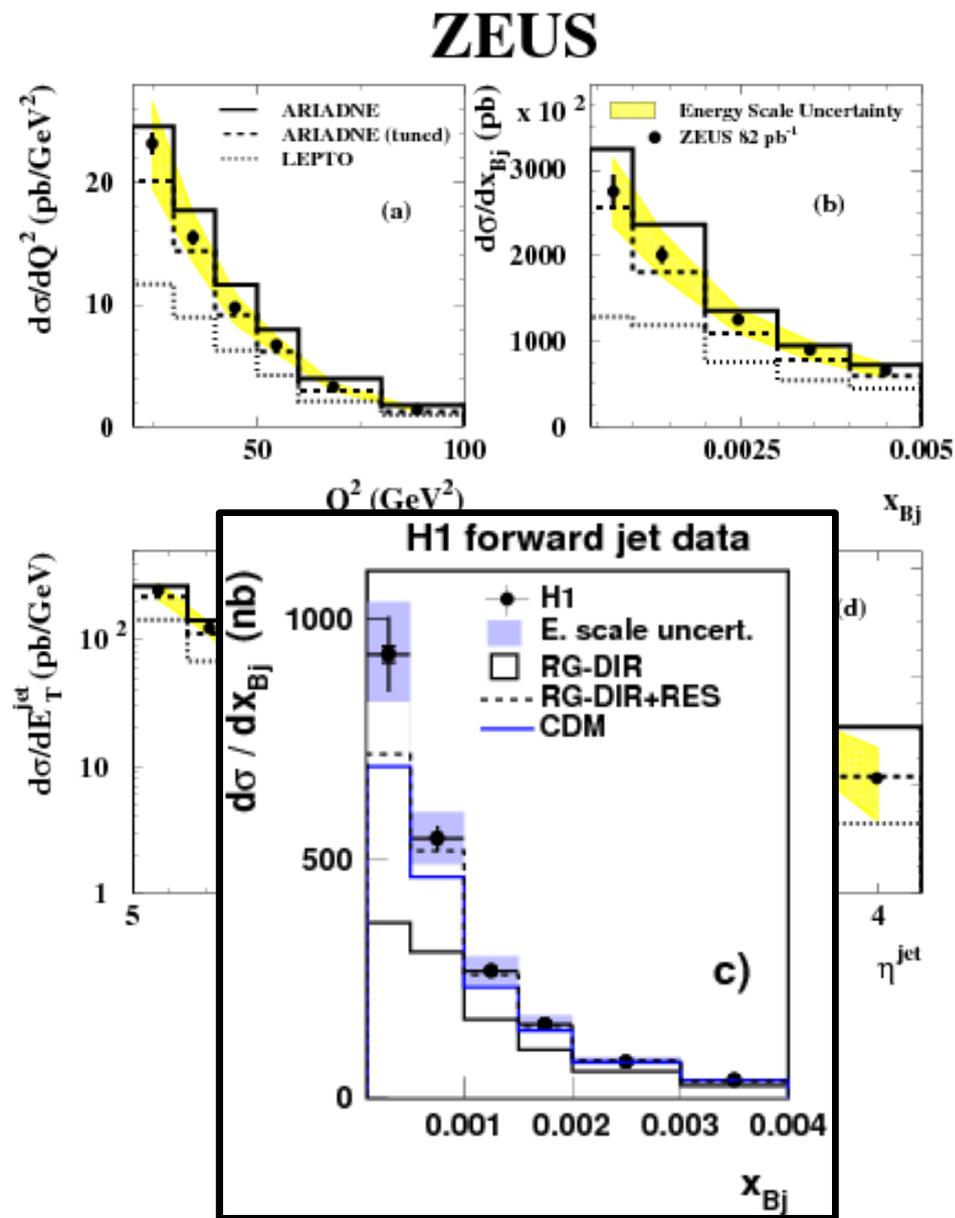
- Comparison to fixed order ME calculations (DISENT):
 - large correction from LO $O(\alpha_s)$ to NLO $O(\alpha_s^2)$
 - NLO $O(\alpha_s^2)$ still factor 2 below data



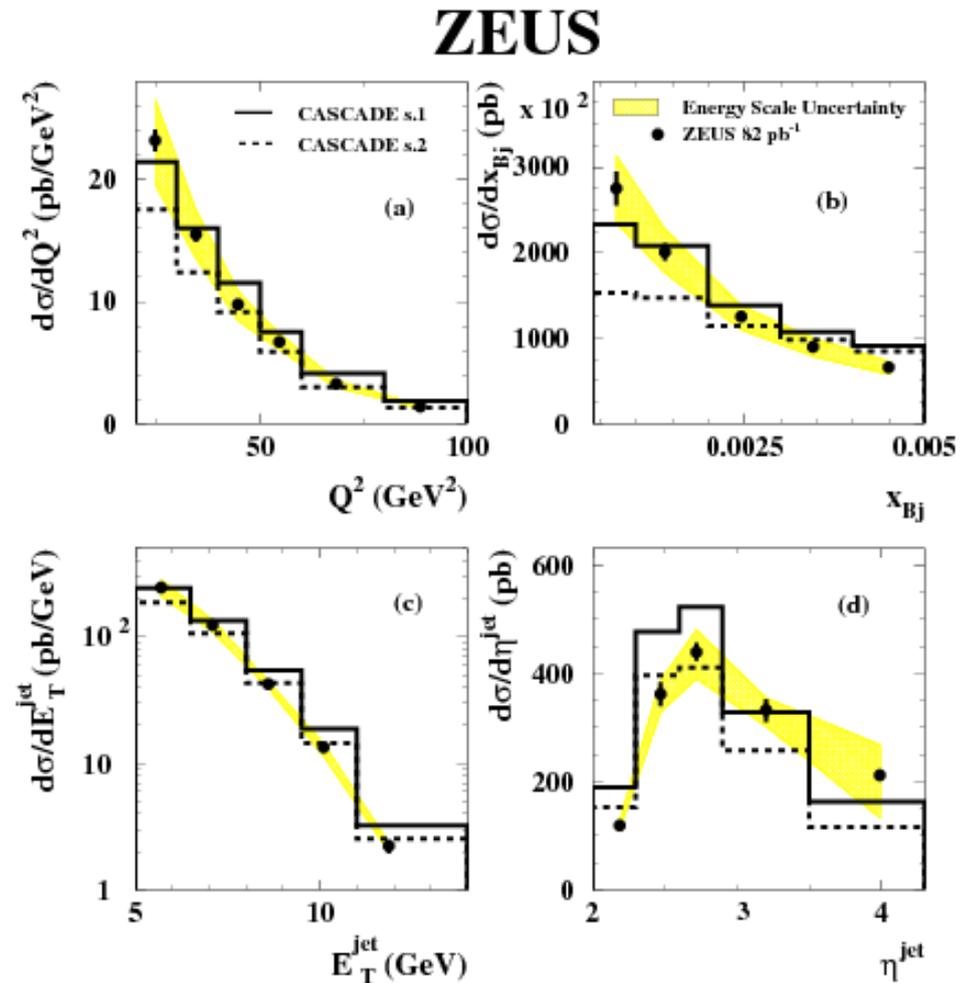
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- Comparison to LO ME+PS Monte Carlo models:
 - ARIADNE (CDM) gives a good description of the data
 - LEPTO (DGLAP) falls below the data by factor 2...



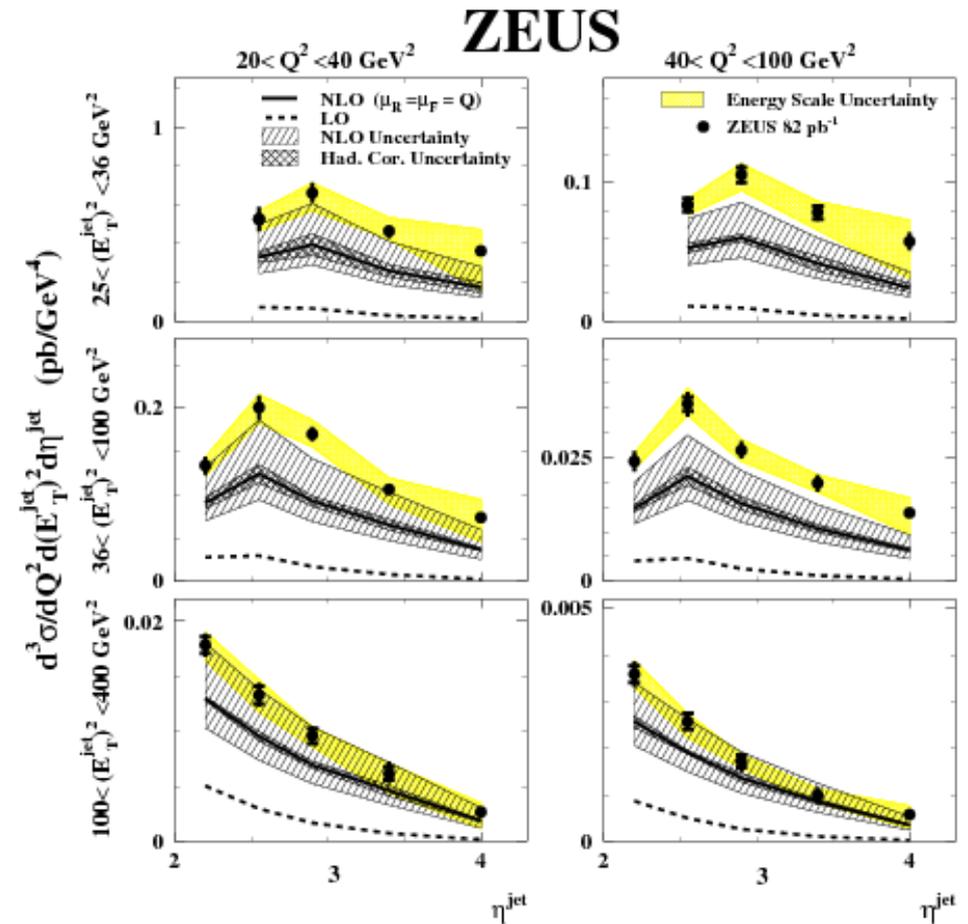
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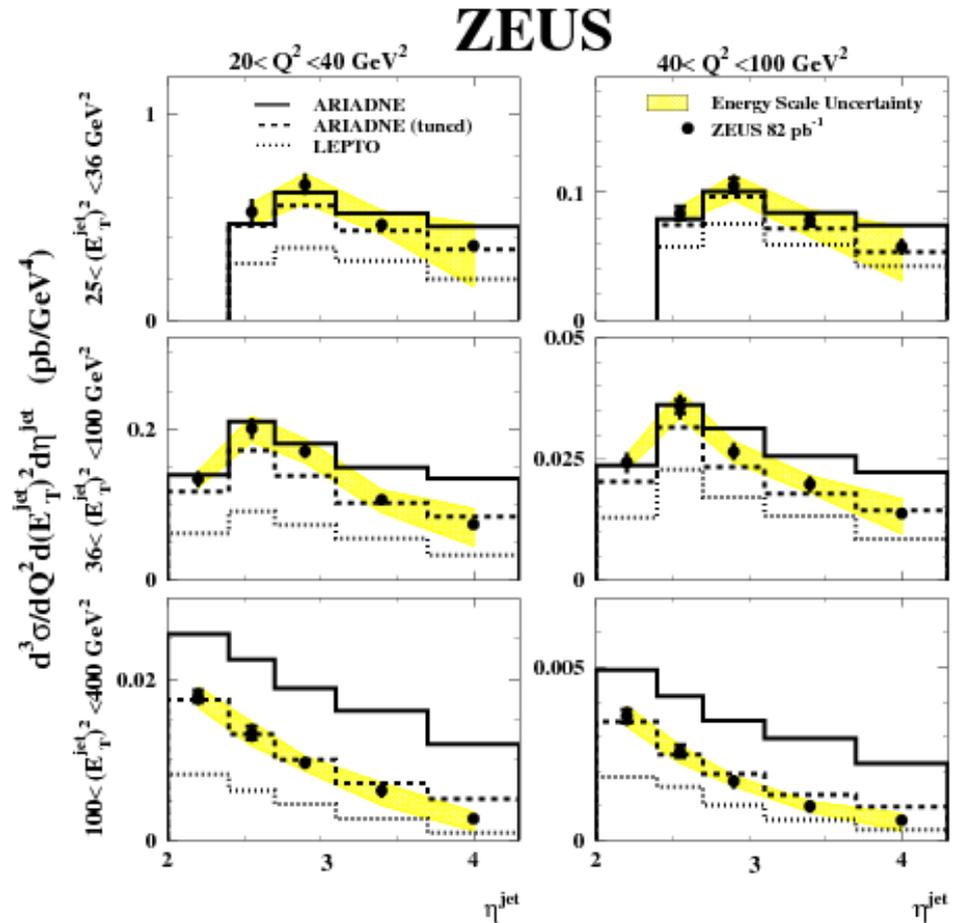
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 - CASCADE (CCFM) with two different sets of unintegrated PDFs does not describe the shape of the distributions



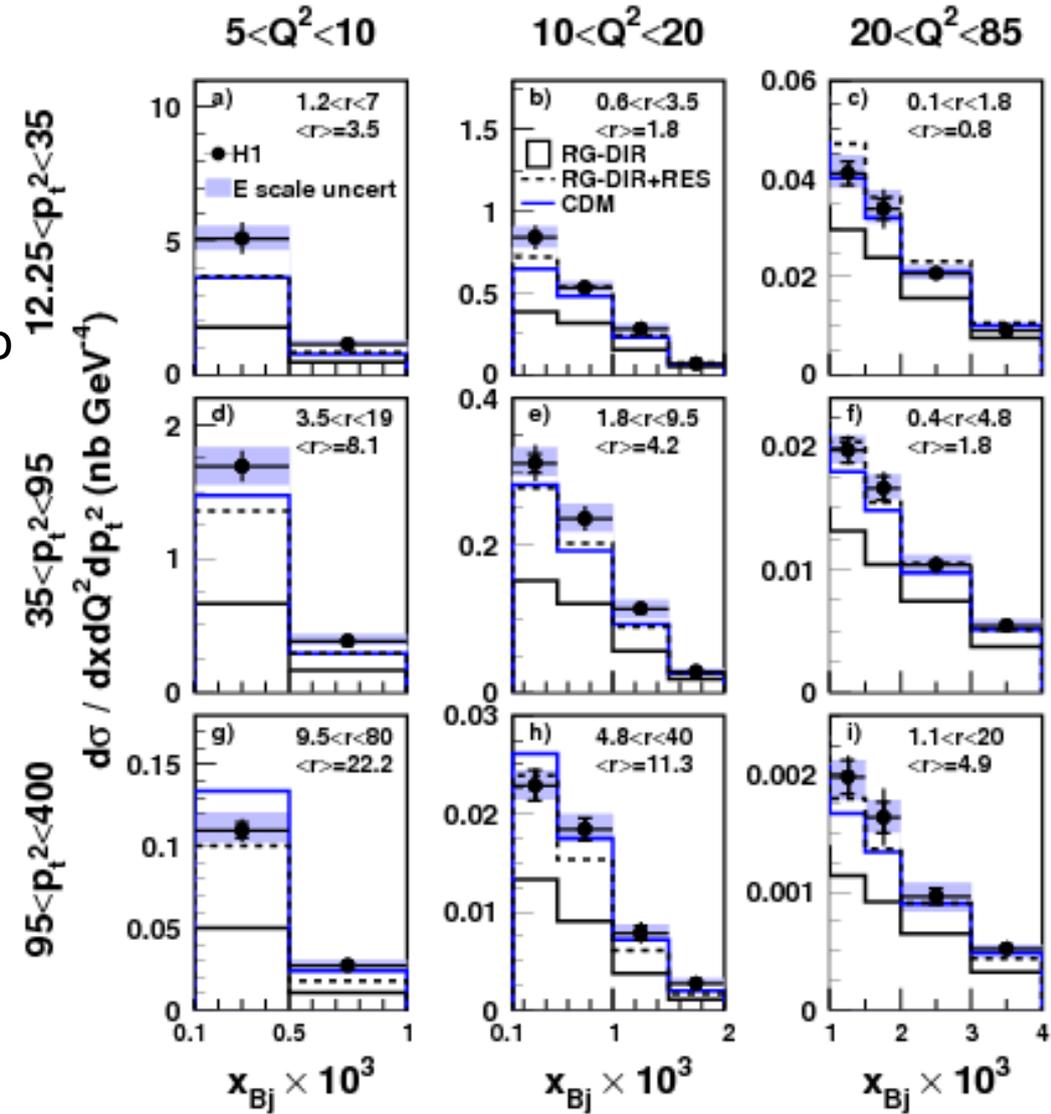
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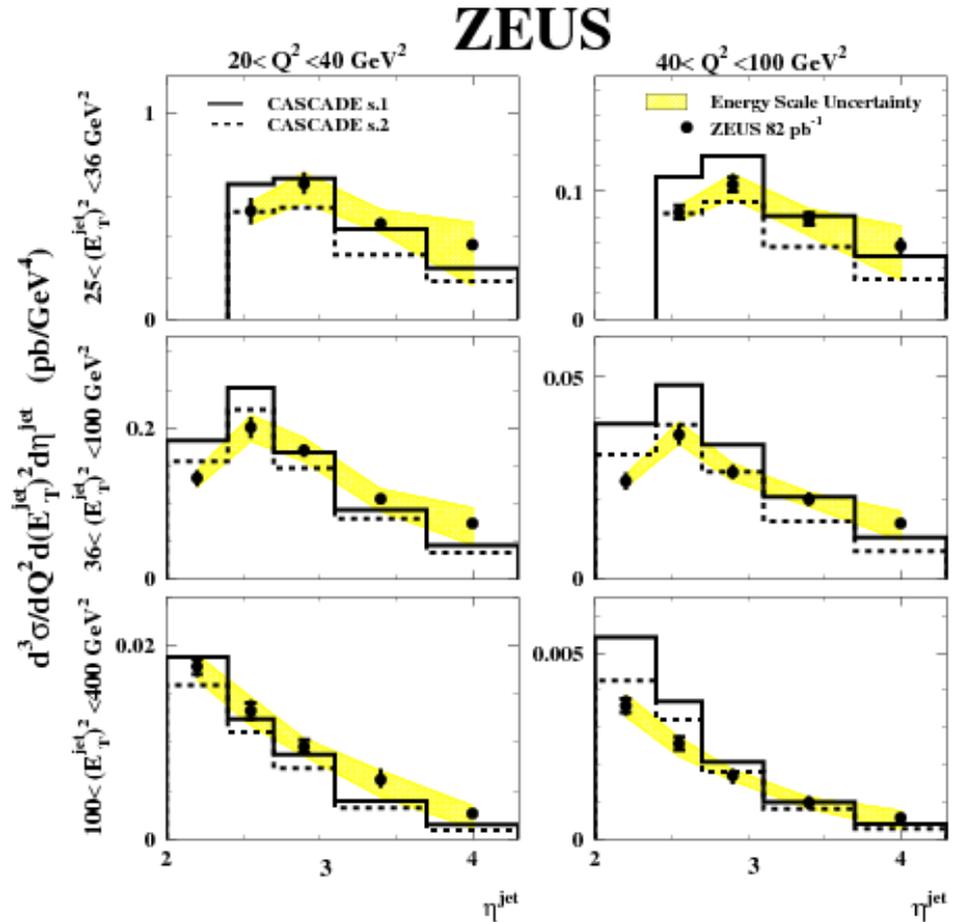
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- Comparison to LO ME + PS Monte Carlo models:
 - ARIADNE (tuned) (CDM) gives a good description of data
 - LEPTO (DGLAP) below data...



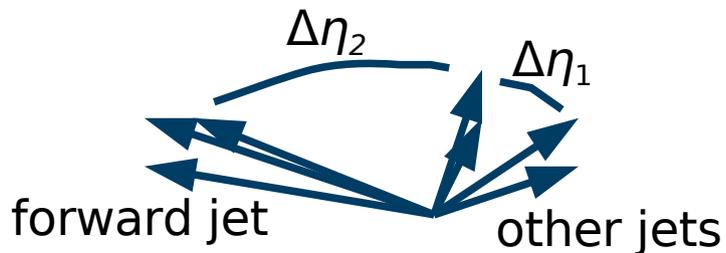
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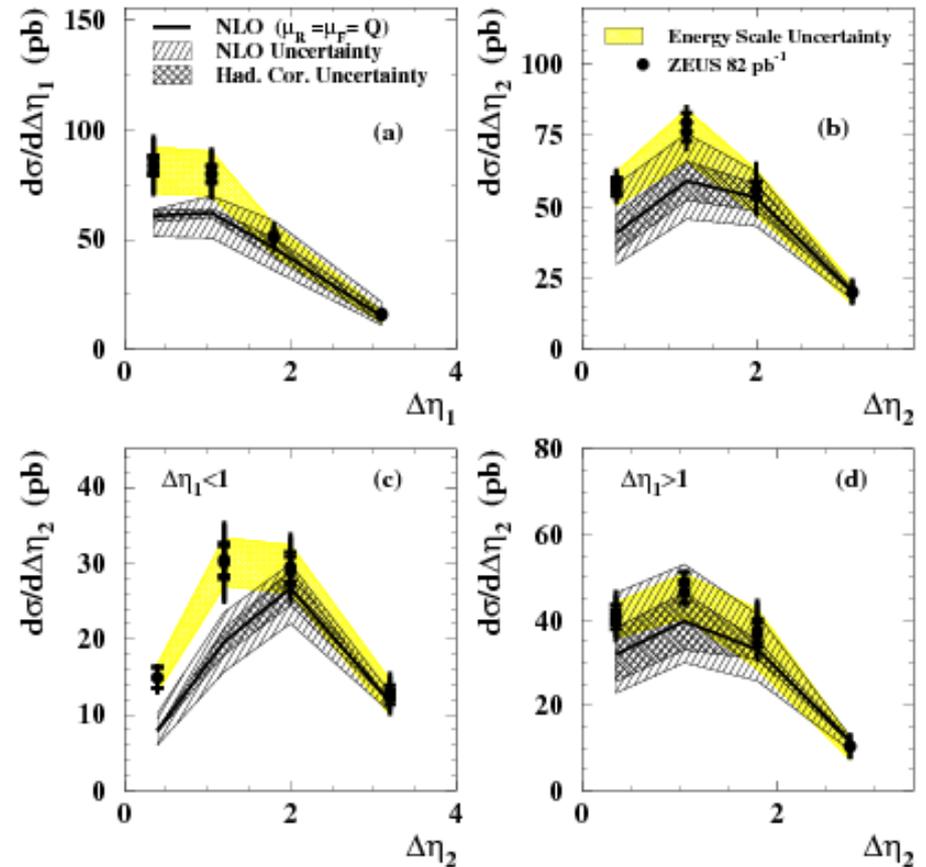
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 - none of unintegrated PDFs sets allow to accommodate all features of the data with CASCADE (CCFM)



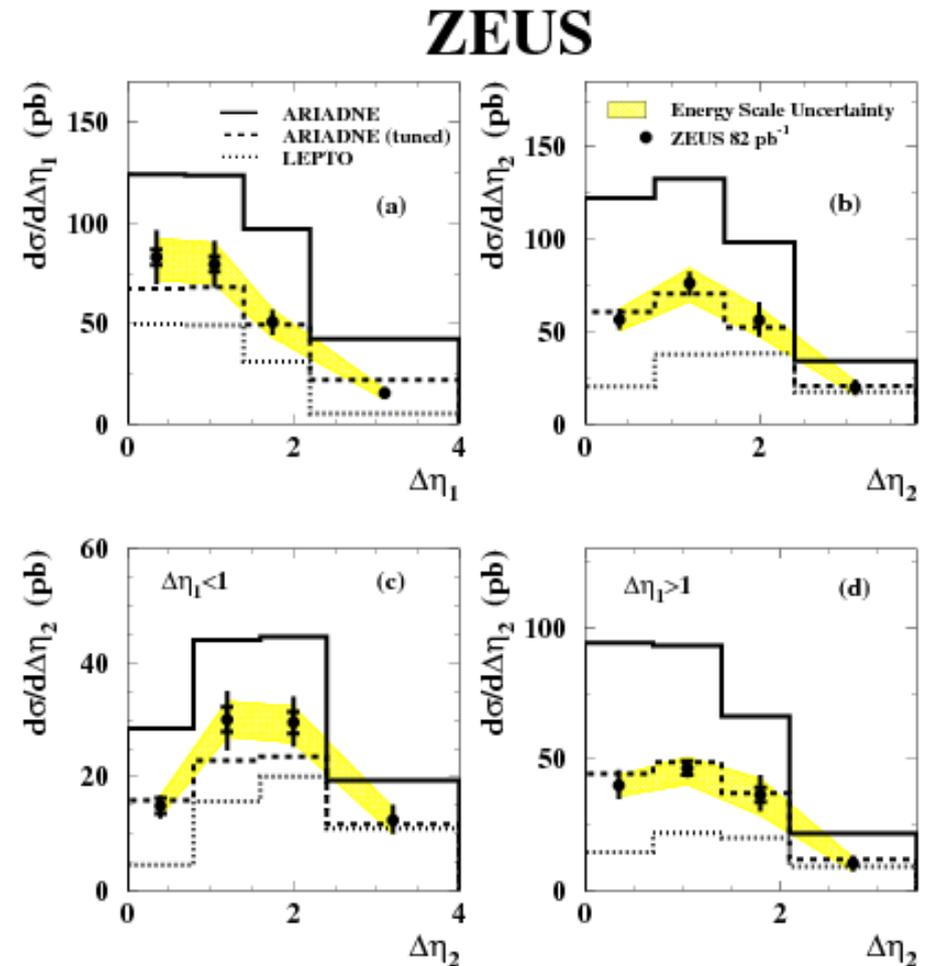
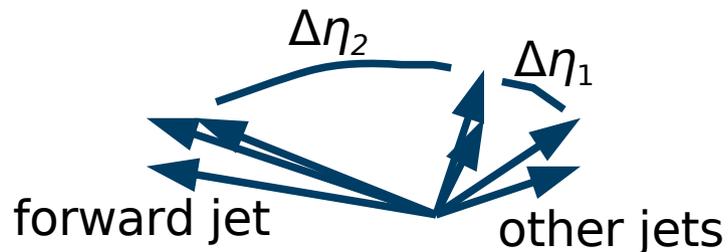
- Comparison to fixed order ME calculations (NLOJET++):
 - NLO $O(\alpha_s^3)$ describes the data well at large $\Delta\eta_2$
 - NLO $O(\alpha_s^3)$ fails when 2 or more jets go forward (small $\Delta\eta_2$)



ZEUS



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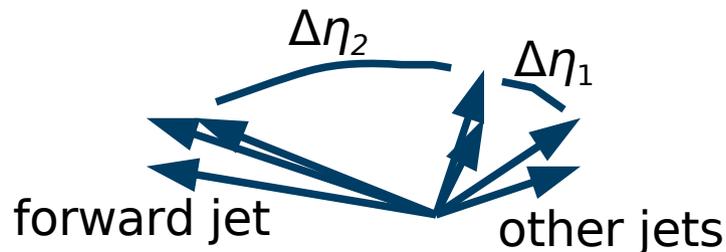
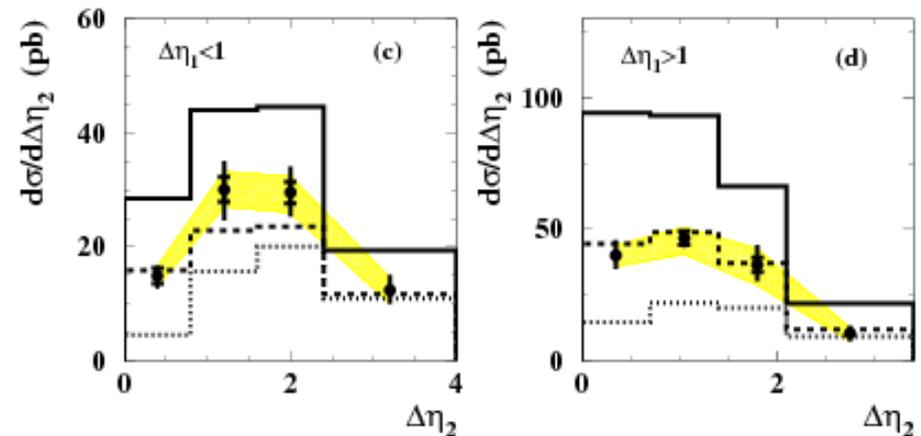
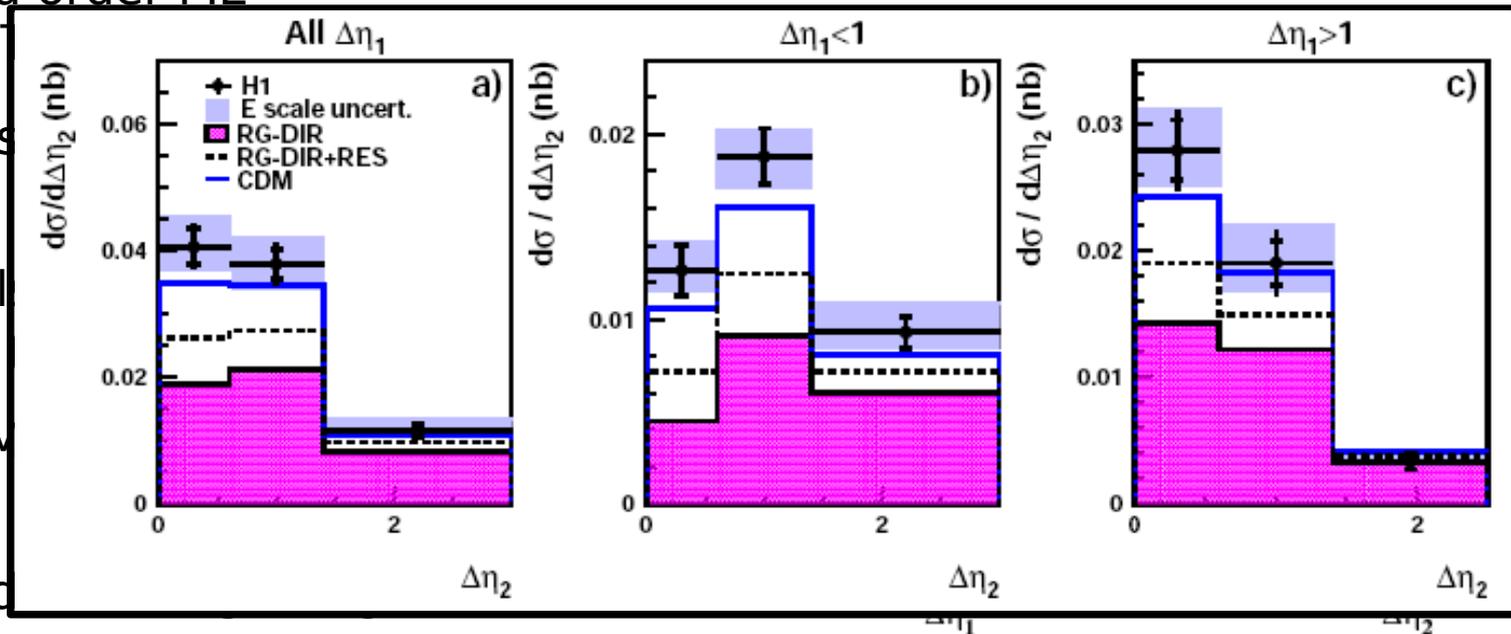


- Comparison to fixed order ME calculations (NLOJET)

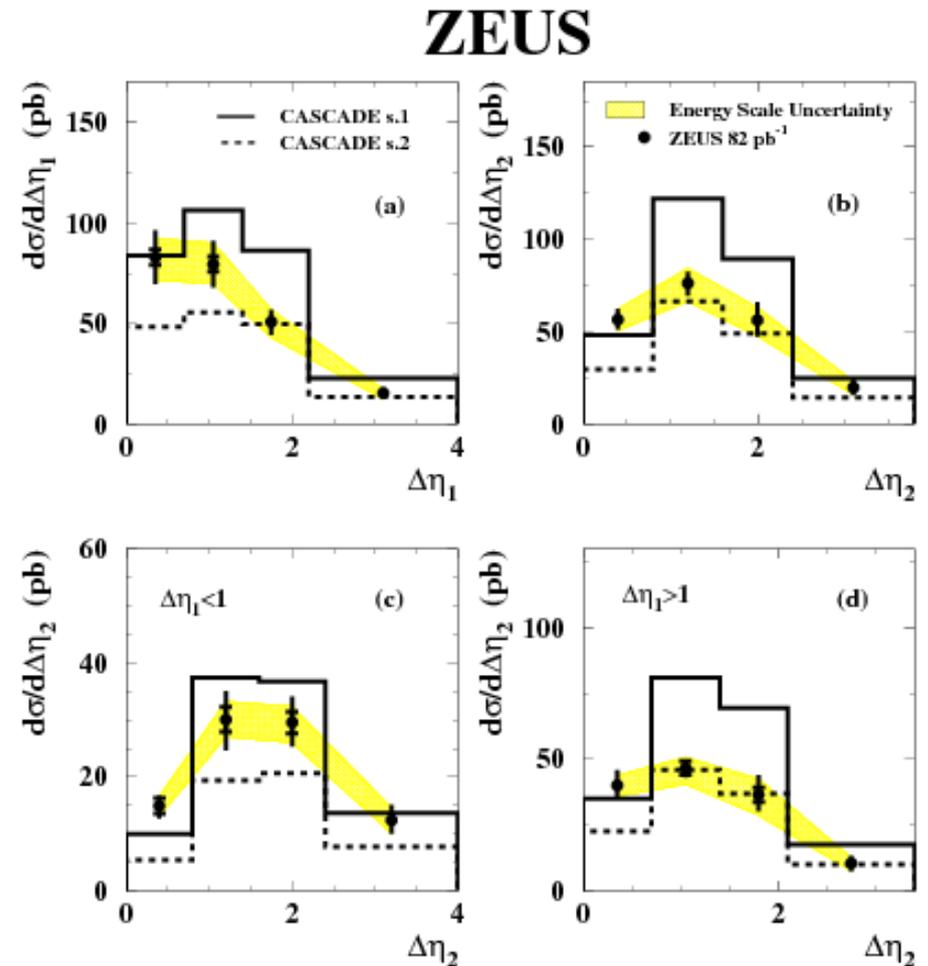
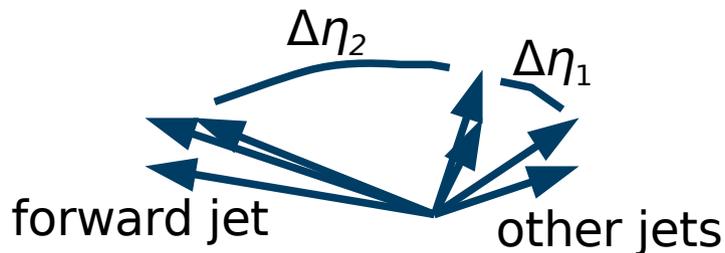
- NLO $O(\alpha\alpha_s^3)$ description fails at large $\Delta\eta_2$
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- Comparison to LO Monte Carlo models:

- ARIADNE (tuned) shows good agreement
- LEPTO (DGLAP) below data... and so is RAPGAP (DGLAP with a resolved photon) !!



- Comparison to fixed order ME calculations (NLOJET++):
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 - ARIADNE (tuned) (CDM) gives good agreement
 - LEPTO (DGLAP) below data... and so is RAPGAP (DGLAP with a resolved photon) !!
 - CASCADE (CCFM) not satisfactory



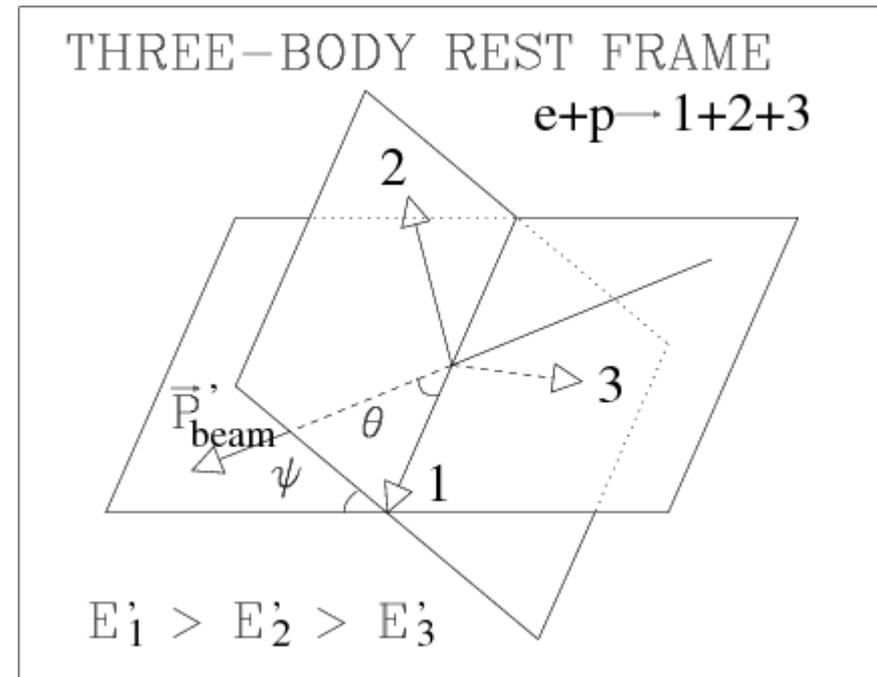


Conclusion so far...

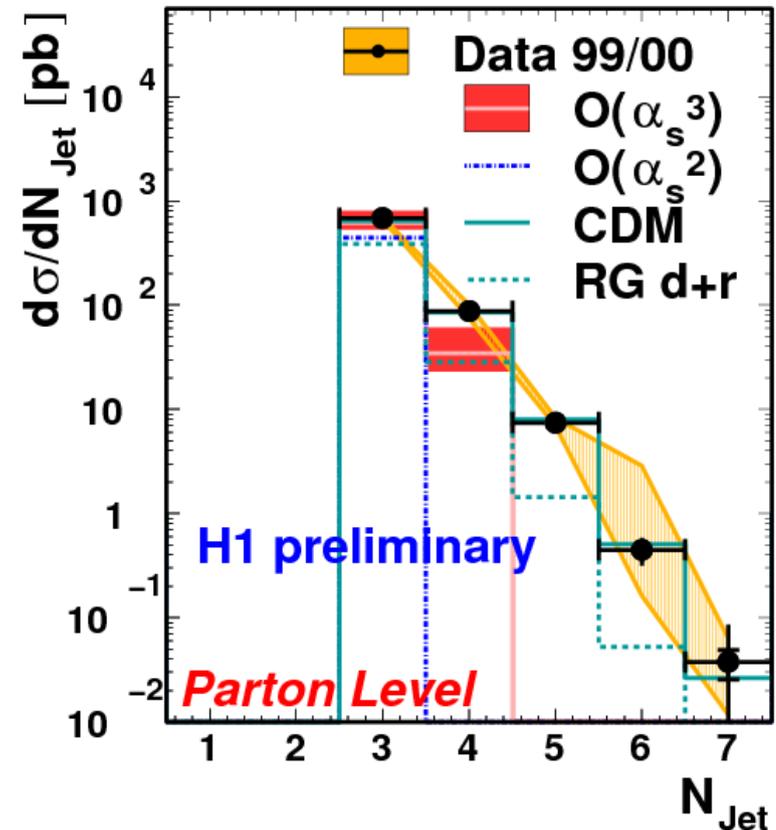
- 2-jet and 3-jet production at low x_{Bj} :
 - aim: check ME calculations for 2- and 3-jet production at NLO
 - conclusion: NLO ($O(\alpha\alpha_s^2)$, $O(\alpha\alpha_s^3)$) calculations work well and is especially needed at low x_{Bj} when additional gluon radiation is highlighted
- Forward-jet production:
 - aim: check ME calculations at NLO and PS models in a region of phase space where additional gluon radiation and/or non-ordered PS are expected
 - conclusion: NLO ($O(\alpha\alpha_s^2)$) below data, sometimes by factor 2; simple DGLAP fails but DGLAP with a resolved photon and CDM describe data well; CASCADE fails; when looking at forward-jet + dijet production, DGLAP with a resolved photon and CDM can be differentiated: CDM survives while DGLAP does not.

- **Aim:** check ME calculations at NLO and PS models by looking at 3-jet topologies in regions of phase space where additional gluon radiation and/or non-ordered PS are expected
- **Method:**
 - measure 3-jet cross sections
 - exploit three-jet topology
 - scaled energy in 3-jet rest frame

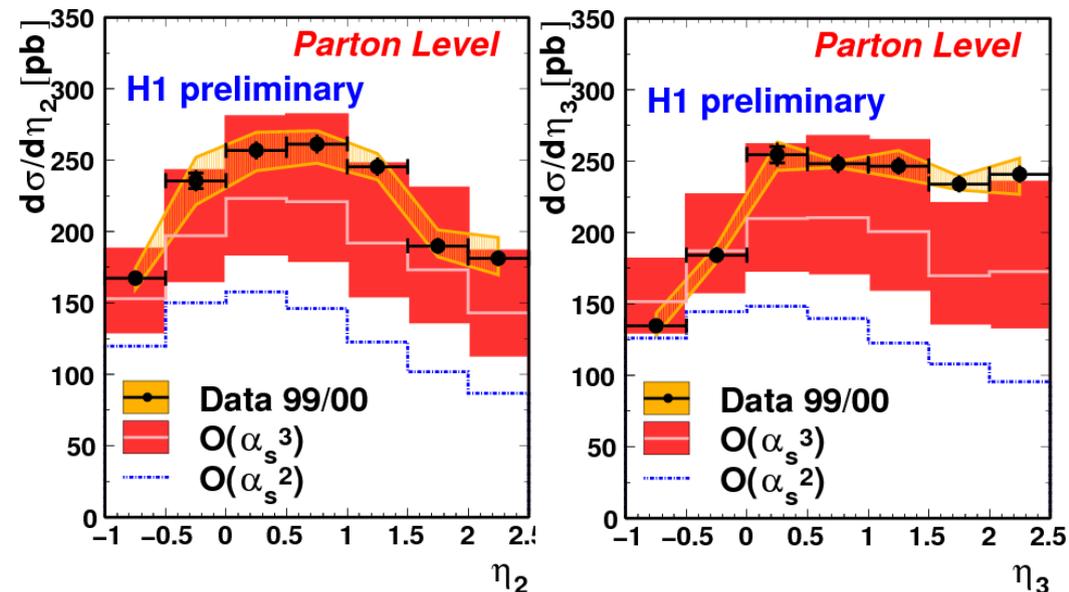
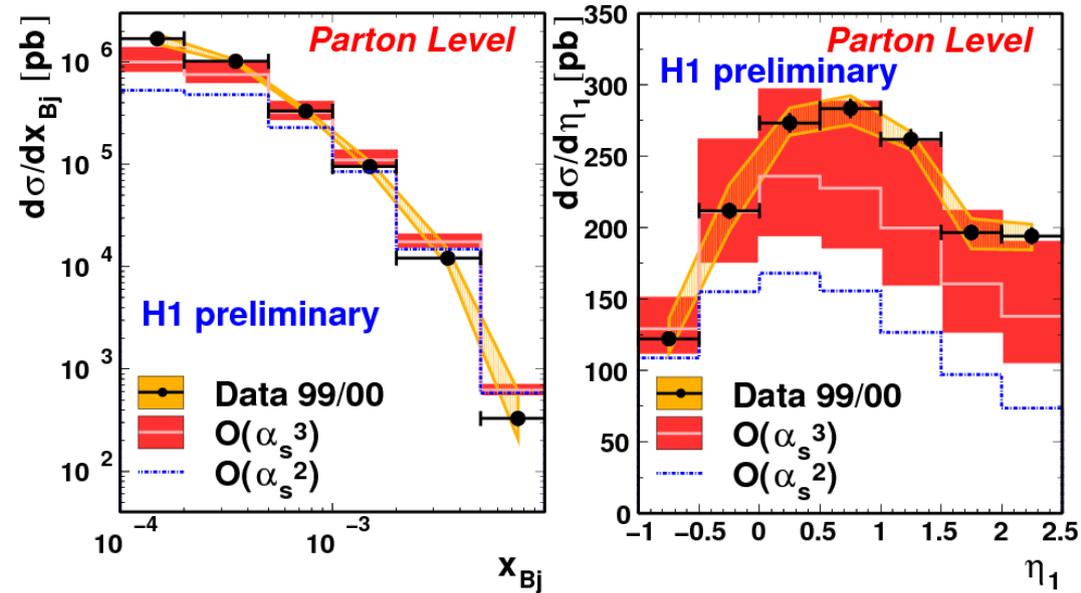
$$X_i = 2 E'_i / (E'_1 + E'_2 + E'_3)$$
 - angles θ', ψ' in 3-jet rest frame
 - look for events with at least 1 forward jet
 - $\Theta^{fwdjet} < 20^\circ, x^{fwdjet} = E^{fwdjet}/E_p > 0.035$
- **Kinematic selection:**
 - $5 < Q^2 < 80 \text{ GeV}^2, 0.1 < y < 0.7,$
 $10^{-4} < x < 10^{-2}, 156^\circ < \theta_e < 175^\circ, E_e > 9 \text{ GeV}$
 - $N_{jet} > 3, p_T^{jet} > 4 \text{ GeV}, p_T^{jet1} + p_T^{jet2} > 9 \text{ GeV},$
 $-1 < \eta^{jet} < 2.5$ (one jet with $-1 < \eta^{jet} < 1.3$)



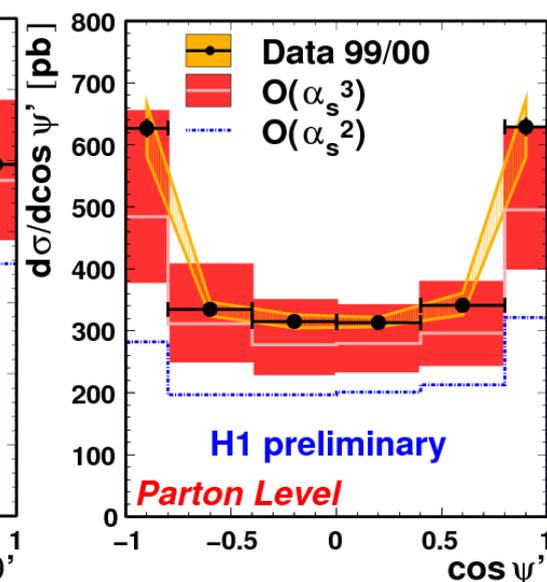
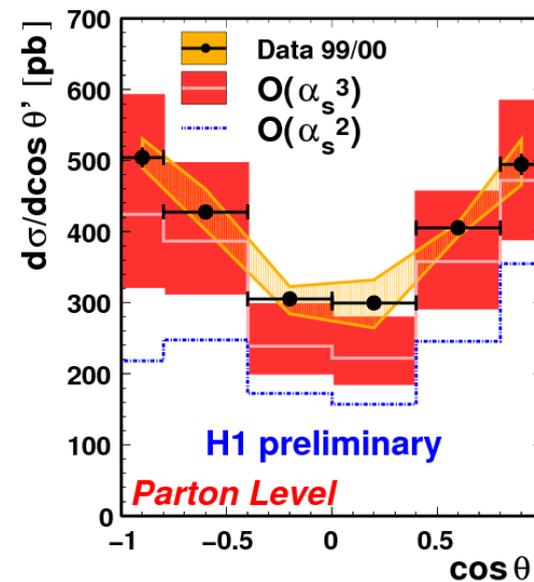
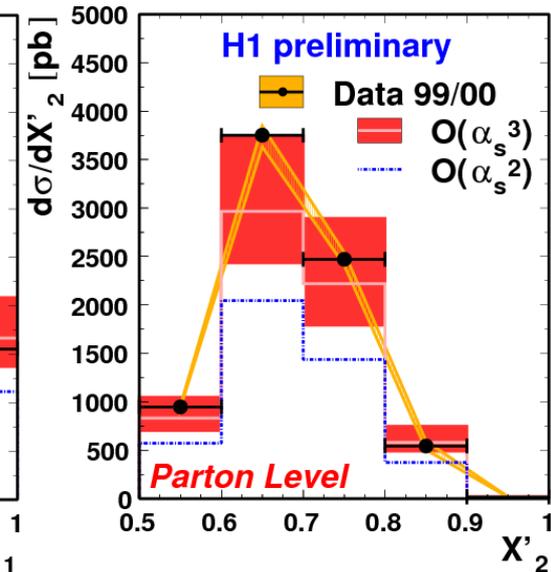
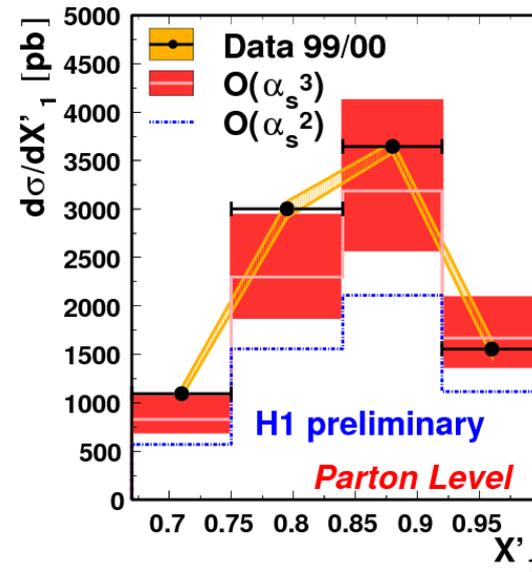
- Jet multiplicity
 - CDM gives excellent description; RAPGAP DIR+RES (DGLAP with a resolved photon) fails at high jet multiplicity
 - NLO $O(\alpha_s^3)$ agrees for $N_{jet} = 3$ but misses 18% of events with 4 or more jets



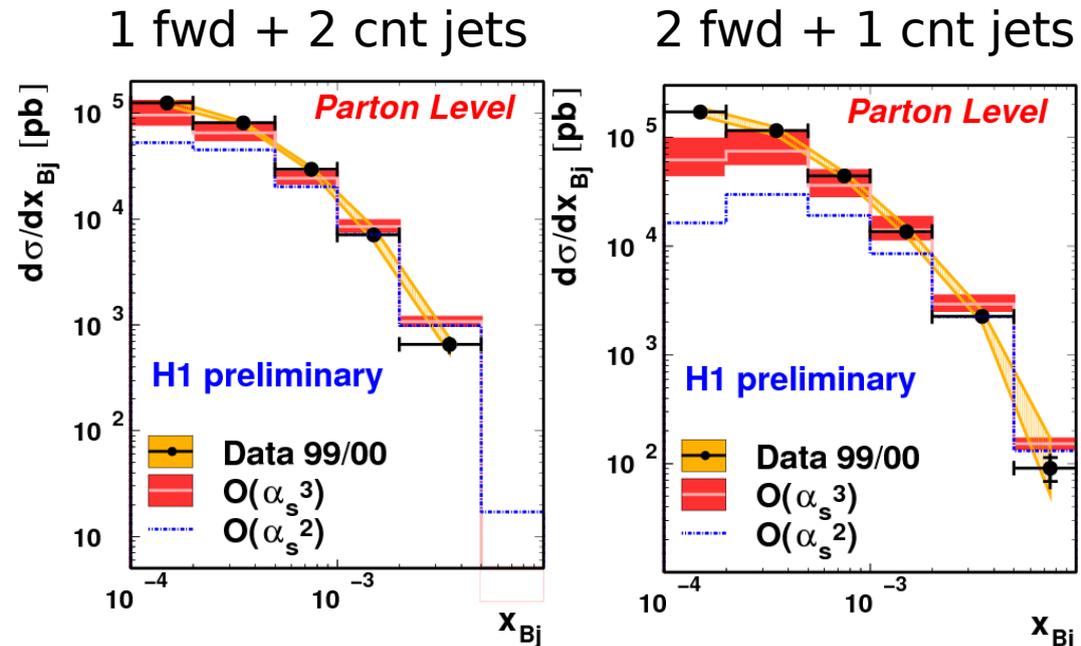
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- x and η^{jet} dependence
 - main discrepancies are seen at low x and forward η^{jet}
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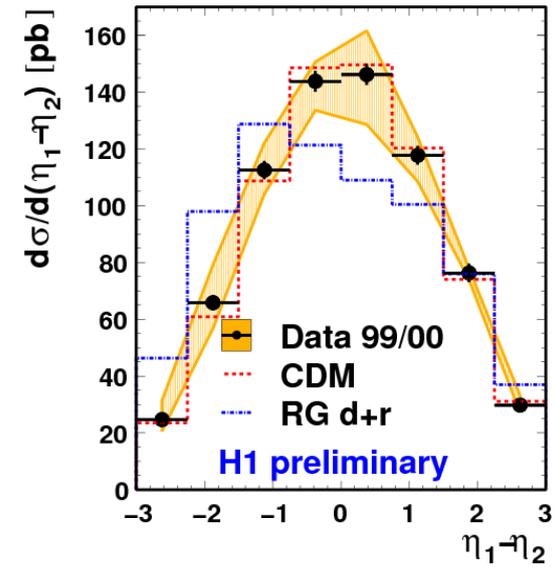
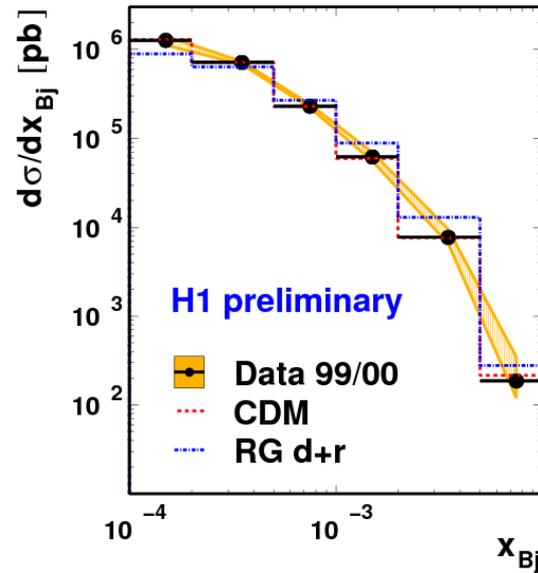
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- Jet topology
 - 3-jet topology is well described by NLO $O(\alpha_s^3)$, except for the 18% normalisation difference



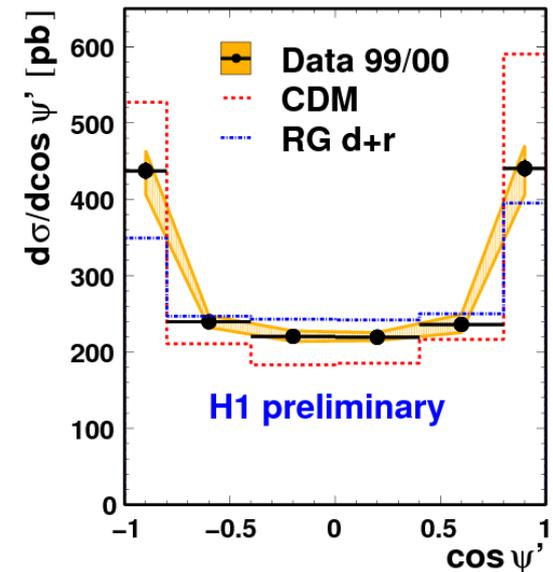
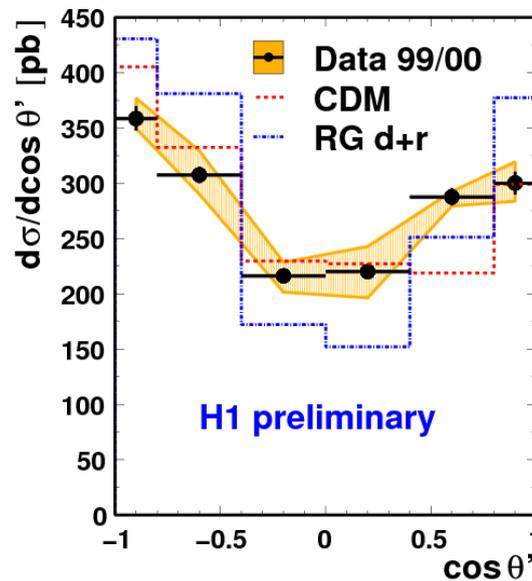
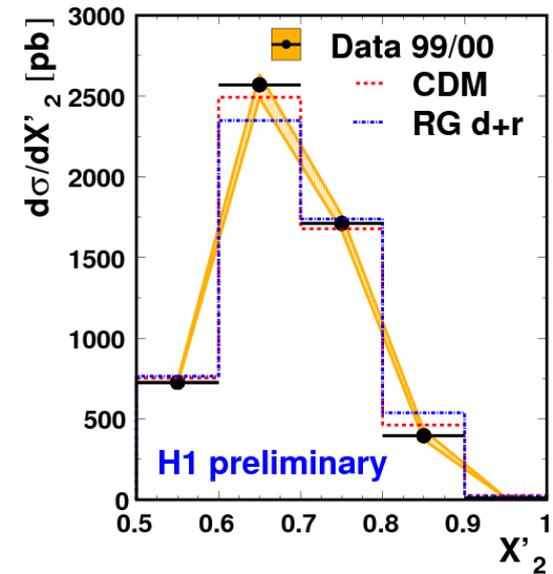
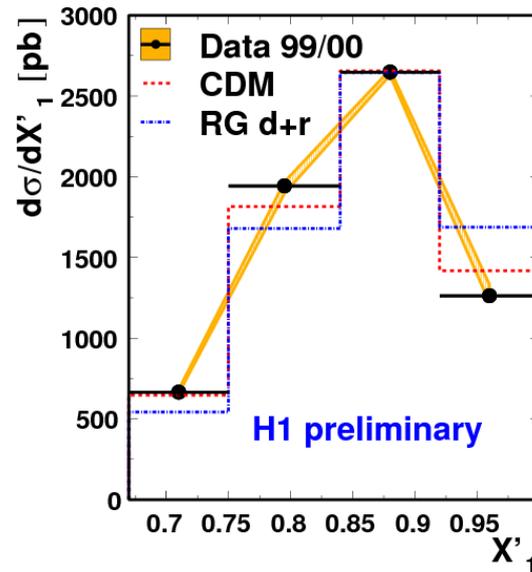
- x dependence
 - main discrepancy seen at low x with 2 forward jets



- 3-jet cross sections:
 - absolute normalisation too low
 - RAPGAP (DGLAP with a resolved photon) scaled by 174%
 - CDM scaled by 108%
 - RAPGAP fails in several aspects

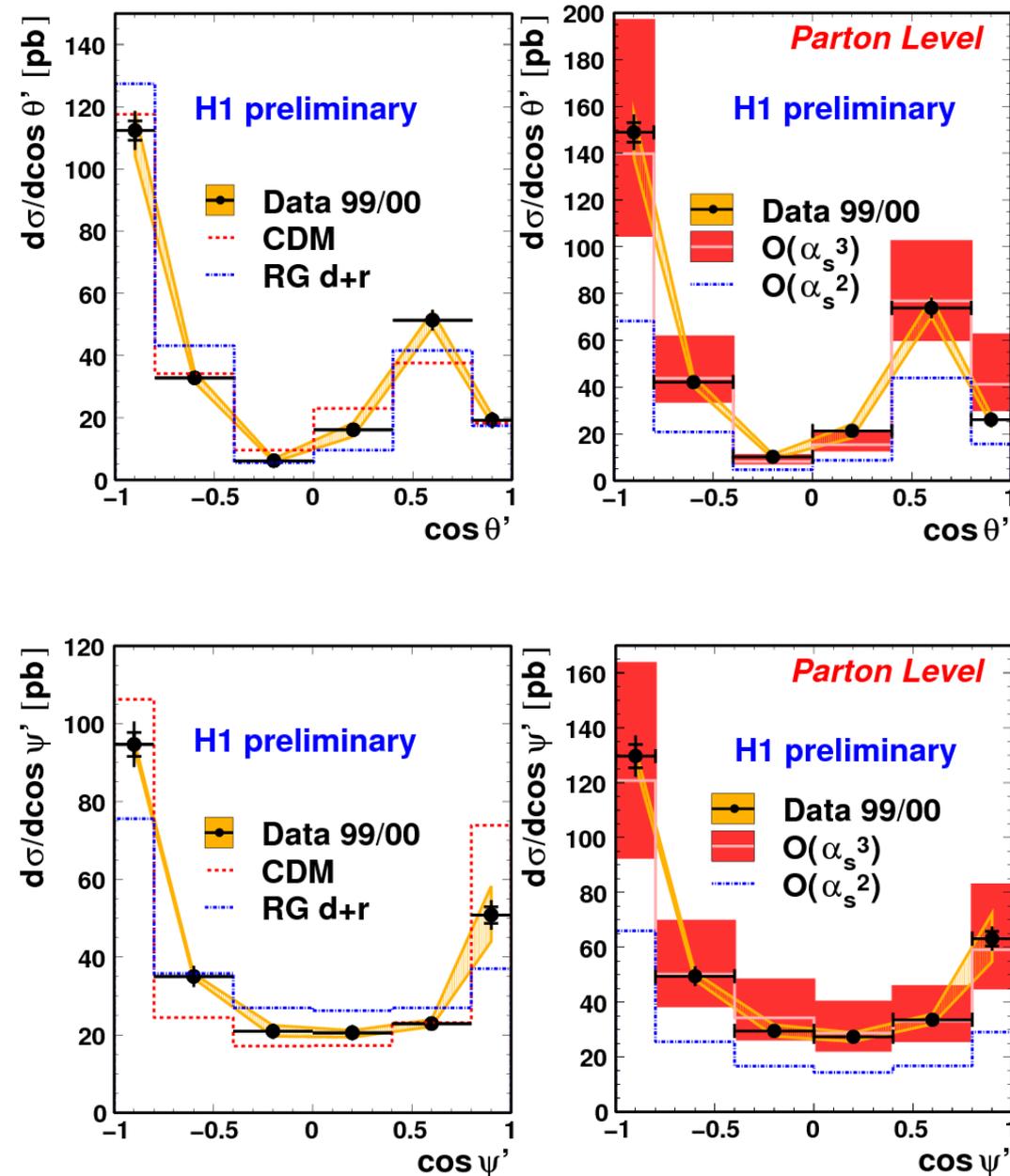


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 - absolute normalisation too low
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- 2 fwd + 1 cnt jet cross sections
 - Absolute normalisation too low
 - RAPGAP (DGLAP with a resolved photon) scaled by 385%
 - CDM scaled by 109%
 - NLO normalised to data
 - RAPGAP DIR+RES fails
 - CDM does well
 - NLO even better

2 forward + 1 central jets





General summary

- 2-jet and 3-jet production at low x_{Bj} :
 - aim: check ME calculations for 2- and 3-jet production at NLO
 - conclusion: NLO ($O(\alpha\alpha_s^2)$, $O(\alpha\alpha_s^3)$) calculations work well and is especially needed at low x_{Bj} when additional gluon radiation is highlighted
- Forward-jet production:
 - aim: check ME calculations at NLO and PS models in a region of phase space where additional gluon radiation and/or non-ordered PS are expected
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- 3- and 4-jet production at low x_{Bj} :
 - aim: check ME calculations at NLO and PS models by looking at jet topologies in regions of phase space where additional gluon radiation and/or non-ordered PS are expected
 - conclusion: NLO ($O(\alpha\alpha_s^3)$) describes jet topology surprisingly well, but misses 18% of events with 3 or more jets, especially when 2 jets are forward jets; CDM is very good to high jet multiplicity and also describes the jet topology well.



Questions for discussion

- Why is CDM so good?
 - Number of radiated gluons is not that high – any breaking of the ordering is fine (but RAPGAP DIR+RES doesn't work...!)
 - CDM has been tuned...
- Why is CCFM so bad?
 - CCFM needs better input (better unintegrated PDFs...). Can uPDFs be constrained by this data?
- What are the next steps in theory?
 - How far are we from a full BFKL calculation?
- What are the next steps in experiment?
 - Which analyses should still be done with HERA data?
 - What are the possibilities to measure forward jets at the LHC?

- x dependence
 - main discrepancy seen at low x with 2 forward jets
- η^{jet} and p_T^{jet} dependence

1 fwd + 2 cnt jets

2 fwd + 1 cnt jets

