# DETERMINATION OF THE PARTON DENSITIES OF THE PROTON AND $\alpha_S$ AT HERA

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**Abstract**

The inclusive cross section for electron-proton scattering has been measured to a high degree of accuracy at HERA in order to investigate the structure of the proton. Recent measurements by the ZEUS experiment at values of $x_{Bj}$ up to unity, as well as measurements with polarized lepton beams by H1 and ZEUS are discussed. The determination of the strong coupling constant from measurements of jets and from fits where the inclusive cross sections have been combined with additional data, to further constrain the parton density functions, is described.

## 1 Introduction

The proton structure functions have been extensively studied at the electron-proton collider HERA. The electron-proton scattering cross section can be obtained by convoluting the analytically calculable partonic cross section, $\sigma_{pQCD}$, with a parton density function (PDF), $f_i$, that provides the probability of scattering against a parton carrying a certain fraction $x$ of the proton momentum.

$$\sigma = \left[ \sum_{i=g,q,\overline{q}} \int dx f_i(x, \mu_f, \alpha_s(\mu_r)) \sigma_{pQCD}(x, \mu_f, \alpha_s(\mu_r)) \right] (1 + \delta_{had}) \quad (1)$$

The factorization scale $\mu_f$ defines at which value of the chosen scale the evolution stops, and $\mu_r$ is the renormalization scale, which is the scale variable used in the expansion of the strong coupling constant, $\alpha_S$. The term $\delta_{had}$ takes the hadronization corrections into account.

The parton density function can not be calculated completely but the distribution at a starting scale $\mu_o$ has to be extraced experimentally. This is possible through measurements of the proton structure functions, which in leading order QCD are related to the parton density functions. In the quark-parton model (QPM) the inclusive double differential neutral current electron-proton cross section can be expressed in terms of three independent generalised structure functions, $\tilde{F}_2, \tilde{F}_L$ and $x\tilde{F}_3$, according to:

$$\frac{d^2\sigma^{e\pm p}_{NC}}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ (1 + (1-y)^2)\tilde{F}_2(x, Q^2) - \frac{y^2}{2}\tilde{F}_L(x, Q^2) \mp (y - \frac{y^2}{2})x\tilde{F}_3(x, Q^2) \right] \quad (2)$$

with $\tilde{F}_L = \tilde{F}_2 - 2x\tilde{F}_1$ being the longitudinal structure function. The negative square of the boson four-momentum, $Q^2$, also called the *virtuality*, is a measure of the resolution power of the exchanged boson. $\alpha$ is the fine structure constant, and $y$ is the inelasticity of the process, defined in the proton rest frame as the energy fraction of the incoming electron transferred by the boson. The generalised structure functions $\tilde{F}_2$ and $x\tilde{F}_3$ can be decomposed into terms describing electromagnetic and weak interactions, and the interference between them, in the following way:

$$\tilde{F}_2^{\pm} = F_2 - (v_e \pm P_e a_e) \frac{\kappa Q^2}{(Q^2 + M_Z^2)} F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm P_e 2 v_e a_e) (\frac{\kappa Q^2}{Q^2 + M_Z^2})^2 F_2^Z \quad (3)$$

$$x\tilde{F}_3^{\pm} = -(a_e \pm P_e v_e) \frac{\kappa Q^2}{(Q^2 + M_Z^2)} x F_3^{\gamma Z} + (2 v_e a_e \pm P_e (v_e^2 + a_e^2)) (\frac{\kappa Q^2}{Q^2 + M_Z^2})^2 x F_3^Z \quad (4)$$

where the different signs correspond to electron and positron scattering, respectively. The relative amount of $Z$ and $\gamma$ exchange is given by $\kappa^{-1} = 4\frac{M_W^2}{M_Z^2}(1 - \frac{M_W^2}{M_Z^2})$, whereas $v_e$ and $a_e$ are the vector and axial-vector couplings of the electron to the $Z^o$. The lepton beam polarization is denoted $P_e$ and the terms containing $P_e$ express the parity violation.

The structure functions can be given in terms of the cross sections $\sigma_T$ and $\sigma_L$, which correspond to the couplings to transversely and longitudinally polarized photons, respectively, and consequently $\sigma_{tot}^{\gamma p} = \sigma_L + \sigma_T$. At low $Q^2$, the exchange of $Z^o$ bosons is strongly suppressed compared to the photon exchange, and the cross section is therefore dominated by the contribution from $F_2$ (electromagnetic), which can be expressed as:

$$F_2(x, Q^2) = \frac{Q^2}{4\pi\alpha^2}(\sigma_L + \sigma_T) = \sum_q e_q^2(xq(x) + x\overline{q}(x)) \tag{5}$$

i.e. the sum of the momentum weighted quark and antiquark densities, $xq(x)$ and $x\overline{q}(x)$, and the electromagnetic couplings to the photon given by the electric charge of the quarks.

The longitudinal structure function, $\tilde{F}_L$, describes the interaction with longitudinally polarized photons, which vanishes in zeroth order $\alpha_S$ processes due to

$$\tilde{F}_L(x, Q^2) = \frac{Q^2}{4\pi\alpha^2}\sigma_L \sim xg \tag{6}$$

with $xg$ being the momentum weighted gluon distribution. From the ratio

$$R = \frac{\tilde{F}_L}{\tilde{F}_2 - \tilde{F}_L} = \frac{\tilde{F}_2 - 2x\tilde{F}_1}{2x\tilde{F}_1} = \frac{\sigma_L}{\sigma_T} \tag{7}$$

we get that $2x\tilde{F}_1 = \sigma_T$.

The structure function $x\tilde{F}_3(x, Q^2)$ is sensitive to the interference between photon and $Z^o$ exchange as well as pure $Z^o$ exchange, and is consequently only important at high $Q^2$. The sign of the contribution from $x\tilde{F}_3$ to the cross section expression (eq.2) is different for $e^-p$- and $e^+p$ scattering. Further, $xF_3^{\gamma Z}$ is sensitive to the difference in the quark and antiquark momentum distributions and can in leading order be written as:

$$xF_3^{\gamma Z} = 2x\sum_q (e_q a_q)(q - \overline{q}) = 2x(2u_v + d_v) \tag{8}$$

with $u_v$ and $d_v$ representing the valence quark distributions.

Via the relation to the PDF's the structure functions give us the probability to find a parton in the proton, carrying a fraction $x$ of the proton momentum, if the probe has a resolution power $Q^2$.

Intuitively one would expect that the probability of scattering against a pointlike parton would be independent of the momentum (resolution) of the probe. Thus, the structure function $F_2$ should not vary with $Q^2$, a phenomenon called *scaling*. Experimental data from HERA have, however, clearly demonstrated that scaling is violated. This is related to the resolution power of the exchanged virtual photon in the sense that a high momentum photon is more sensitive to partons carrying a small momentum fraction than a low momentum photon. Since now the structure function depends on $Q^2$ and the strong coupling, $\alpha_S$, the scaling violation effect can be used to extract the parton density function and the strength of the strong coupling.

The PDF's are determined through global fits to various experimental data at a smallest starting scale $\mu_o$, at which perturbative calculations are still expected to be valid. Parton evolution equations can then be used to give the PDF at an arbitrary scale. Partonic activites below the starting scale are included into the starting distribution of the PDF.

HERA has measured the proton structure over a large kinematic range, covering almost five orders of magnitude in $x_{Bj}$ and $Q^2$. One of the most spectacular observations is the strong rise of $F_2(x, Q^2)$ at small $x$-values. In order to explore this kinematic region special efforts have been made, like:

▷ running HERA with the collisions point shifted in the direction of the proton beam
▷ running HERA with lower electron beam energy
▷ special analysis of events where the incoming electron has suffered from photon radiation

All this leads to an increased detector acceptance for events with lower $Q^2$.

Due to the relation $Q^2 = x \cdot y \cdot s$, low $Q^2$ and very small $x$ give access to high $y$, which is the region where $\sigma_L$ makes a significant contribution to the cross section and thus $F_L$ may be measured.

## 2 The high x-region

The high $x$-region has mainly been covered by previous fixed target experiments. The access to this region at HERA is restricted by acceptance limitations due to the fact that the scattered quark proceeds in the extreme forward direction, close to the beam pipe. Consequently, the jet produced is not well measured above a certain $x$-value that depends on $Q^2$. ZEUS [1)] has used a new method to measure the neutral current cross section up to $x$-values of one, by determining the $Q^2$ value of the event from the scattered electron and the $x$-value from the reconstructed jet. If no jet with a transverse energy above 10 GeV was found by the longitudinally invariant $k_t$ cluster algorithm, the event

was assigned to the $x$-bin not covered by the acceptance. Events with more than one jet were disregarded. The measured double differential cross section, $d^2\sigma/dxdQ^2$, was compared to predictions from Standard Model (SM) NLO calculations in which the CTEQ6M [2)] parametrization of the proton PDF's were used. An overall good agreement between data and the SM predictions was observed although the theory tends to fall below data in the highest $x$-bin. These measurements will have a significant impact on the determination of the valence quark distributions in the high $x$-region.

## 3 Cross section measurements with polarized lepton beams

### 3.1 Polarization asymmetry

Longitudinally polarized electron beams, delivered by HERA since 2002, have been used to measure the neutral current cross section, which according to the SM should depend on the polariztion of the incoming lepton. The degree of polarization is given by $P_e = (N_R - N_L)/(N_R + N_L)$, where $N_R$ and $N_L$ represents the number of right- and left handed leptons in the beam. The polarization effect is expected to be most significant at large $Q^2$ where $Z^o$ boson exchange becomes important and may be established by measuring the cross section asymmetry for right- and left handed $e^{\pm}p$ scattering, according to:

$$A^{\pm} = \frac{2}{P_R - P_L} \frac{\sigma^{\pm}(P_R) - \sigma^{\pm}(P_L)}{\sigma^{\pm}(P_R) + \sigma^{\pm}(P_L)} \tag{9}$$

where $P_R$ and $P_L$ stand for right- and lefthanded polarization, respectively. $A^+$ and $A^-$ will have opposite signs due to the different signs of the $x\tilde{F}_3$-term in the cross section expression (eq.2). The combined data from H1 and ZEUS on the asymmetries ($A^{\pm}$) as a function of $Q^2$ are shown in Fig.1 and are seen to be in good agreement with the SM predictions from QCD fits performed by H1 and ZEUS, respectively.

### 3.2 Determination of $x\tilde{F}_3$

A so called reduced cross section can be defined as:

$$\tilde{\sigma}^{\pm}(x, Q^2) = \frac{d^2\sigma^{\pm}_{NC}}{dxdQ^2} \frac{xQ^4}{2\pi\alpha^2} \frac{1}{Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+} x\tilde{F}_3 - \frac{y^2}{Y_+} \tilde{F}_L \tag{10}$$

where the helicity dependence is given by the terms $Y_{\pm} = 1 \pm (1 - y^2)$. The structure function $x\tilde{F}_3$ can be extracted from the difference in magnitude of the reduced cross sections for electron and positron scattering.

$$x\tilde{F}_3 = \frac{Y_+}{2Y_-} (\tilde{\sigma}^{e^-p} - \tilde{\sigma}^{e^+p}) \tag{11}$$

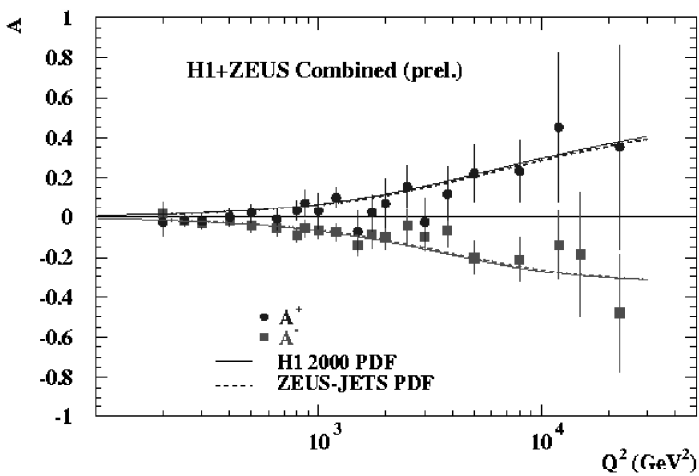


Figure 1: *The $Q^2$ dependence of the cross section asymmetry $A^{\pm}$ from combined H1 and ZEUS data. The curves represent SM QCD fits by H1 and ZEUS, respectively.*

in which the dominant contribution comes from the $\gamma Z$ interference. Since $x\tilde{F}_3$ depends only slightly on $Q^2$, the data from different $Q^2$ regions can be averaged by taking the weak $Q^2$ dependence into account. The results from H1 and ZEUS at an average $Q^2$ value of 1500 GeV$^2$ are shown in Fig.2. Data are reasonably well described, within the fairly large errors, by the NLO fits from both H1 [3] and ZEUS [4]. These results contribute to constrain the valence quark distributions at low $x_{Bj}$.

## 4 Determination of the strong coupling constant

The following procedure is used to determine the value of the strong coupling constant, $\alpha_S$. From a preselected parametrization of the proton parton density function, NLO calculations are performed for different values of $\alpha_S(M_Z)$. The $\alpha_S(M_Z)$-dependence of the experimentally measured variable, denoted $A$, is parametrized according to:

$$\frac{d\sigma}{dA} = C_1\alpha_S(M_Z) + C_2\alpha_S^2(M_Z) \tag{12}$$

where $C_1$ and $C_2$ are parameters in a fit to the calculated $d\sigma/dA$ values at various $\alpha_S(M_Z)$.

The experimentally measured value of $d\sigma/dA$ is then converted to an $\alpha_S(M_Z)$ value via the fitted curve. The error in the measurement is translated into an error of $\alpha_S(M_Z)$ via the slope of the curve. Finally, the Renormalization

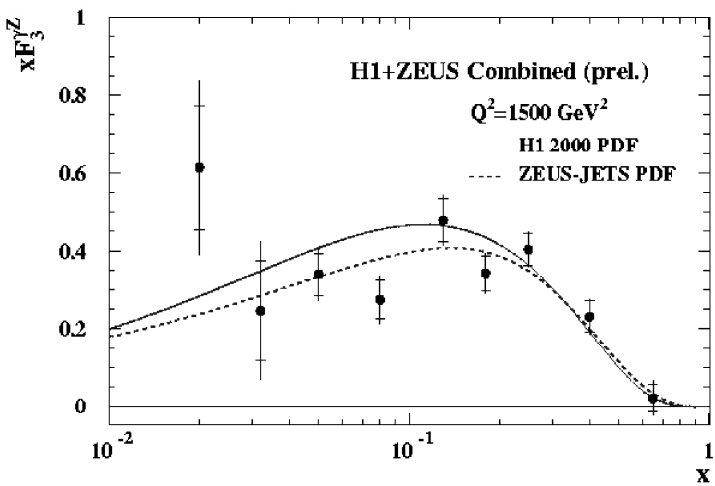


Figure 2: *The structure function $xF_3$ as a function of $x$ at an average $Q^2$ value of 1500 GeV$^2$ from the H1 and ZEUS experiments.*

Group Equation (RGE) is used to extract the $\alpha_S$ values at different scales and thereby the running of the coupling can be established. By using different parametrizations of the PDF's the parton density function dependence may be estimated.

### 4.1 Measurement of $\alpha_S$ from inclusive jet production at high $Q^2$

A number of different observables have been measured in order to determine the strong coupling constant. For example the QCD predictions on jet production depend on both the PDF's and $\alpha_S$. The measured cross sections can be used together with global fits of the proton PDF's to extract $\alpha_S$. Jets from order $\alpha_S$ processes i.e. QCD-Compton and BGF, provide direct sensitivity to the strong coupling and enable precision tests of QCD. In the Breit frame jets from zeroth order scattering processes produce no transverse energy whereas in leading order processes (BGF or QCD-Compton) transverse jets may be observed.

The longitudinally invariant $k_t$ algorithm [5] is the most frequently used method for reconstructing jets in ep collision events. A cone in the pseudo-rapidity ($\eta$) - azimuthal ($\phi$) space defines the jet radius as $R = \sqrt{\eta^2 + \phi^2}$. Most analyses so far have been performed with $R = 1$. It turns out that the inclusive jet production cross section provides the experimentally most precise determination of $\alpha_S$. The ZEUS experiment [6] has studied jet cross sections as a function of the jet radius in order to find an optimal value of $R$ for which the NLO calculations provide the highest precision. Jet radii of $R = 0.5$, 0.7 and 1 have been used to study the $Q^2$- and $E_T$-dependence of the jet cross

sections in events containing at least one jet with $E_T > 8$ GeV in the kinematic ranges $Q^2 > 125$ and $> 500$ GeV$^2$, respectively. The NLO calculations were performed by DISENT [7] in the $\overline{MS}$ scheme with five flavours, using the ZEUS parametrization of the proton PDF's. The variation of the cross section over several orders of magnitude is well reproduced by the theory. A linear increase of the measured cross section with increasing R is observed and can be understood from the simple fact that more transverse energy is contained in the cone as it opens up, resulting in an increased number of jets exceeding the minimum $E_T$ requirement. The theoretical uncertainties in the cross section integrated over the $Q^2$ range decreases in both cases slightly when going from $R = 0.5$ to 1, which is mainly attributed to higher order corrections and hadronization.

The H1 experiment has compared cross section measurements of inclusive jets in the $Q^2$ range 150 - 15000 GeV$^2$ with NLO calculations provided by the NLOJET++ program [8], in the $\overline{MS}$ scheme with five massless quark flavours, using the CTEQ6.5M parametrization [2] of the proton PDF, and the value of the strong coupling constant has been extracted. The jets were reconstructed by the inclusive $k_t$ algorithm provided they had a transverse momentum greater than 7 GeV. The cross sections as a function of $Q^2$ and $E_T$ are well described by the NLO predictions within the dominating uncertainties given by the renormalizaion- and factorization scales. Twenty different measurements, subdivided into five $Q^2$-bins, of the double differential inclusive jet cross section, $d^2\sigma/dQ^2dE_T$, are compatible with the predicted scale dependence of $\alpha_S$. All the twenty measurements have been used in a combined fit from which the following value of $\alpha_S(M_Z)$ was obtained:

$$\alpha_S(M_Z) = 0.1187 \pm 0.0015(exp.)^{+0.0052}_{-0.0037}(theory)^{+0.0021}_{-0.0014}(PDF) \qquad (13)$$

### 4.2 Determination of $\alpha_S$ from multi jet production at high $Q^2$

A measurement of the ratio between three and two jet event cross sections can be used to determine the value of the strong coupling constant, provided the cross sections can be measured with high enough precision. The advantage with this method is that the correlated systematic errors and the renormalization scale uncertainties cancel to a large extent while the disadvantage is the low statistics available. The H1 and ZEUS experiments have performed this analysis in the Breit frame over a wide kinematic range covering $10 < Q^2 < 15000$ GeV$^2$. Jets with $E_T > 5$ GeV, as reconstructed by the inclusive $k_t$ algorithm, were used for the cross section measurements. In order to avoid problems with the phase space regions sensitive to infrared divergencies, the invariant mass of the jets with the highest transverse energies were required to be $M_{2jet} > 25$ GeV and $M_{3jet} > 25$ GeV, respectively. The reliablility of the NLO calculations are thereby ensured. These calculations are performed by

the NLOJET++ program in the $\overline{MS}$ scheme for five massless quark flavours and with various parametrization of the proton PDF. The $Q^2$ dependence of the measured cross sections and consequently of the ratio $R_{3/2}$ is in good agreement with the predictions of NLOJET++ in the phase space region where the electroweak effects can be neglected.

From fits to the measured ratios $R_{3/2}$, values of $\alpha_S(M_Z)$ were determined in five different $Q^2$ bins, but also in the complete $Q^2$-region giving the results below.
H1:

$$\alpha_S(M_Z) = 0.1175 \pm 0.0017(stat.) \pm 0.0050(syst.)^{+0.0054}_{-0.0068}(theory) \qquad (14)$$

ZEUS:

$$\alpha_S(M_Z) = 0.1179 \pm 0.0013(stat.)^{+0.0028}_{-0.0046}(exp.)^{+0.0064}_{-0.0046}(theory) \qquad (15)$$

### 4.3 Combined fits of $\alpha_S$ and the gluon density

Measurements of the $Q^2$ dependence of the structure function (scaling violation) provided by inclusive cross section measurements is the most commonly used method to determine the PDF's via the DGLAP evolution equations. However, fits to the inclusive cross sections alone suffer from strong correlations between the shape of the parton distribution and $\alpha_S$. These correlations may be reduced by including data which constrain the PDF's in an independent and complementary way.

Recent measurements of the structure function $F_2(x, Q^2)$ at low $Q^2$, performed by the H1 and ZEUS experiments, provide a considerable improvement compared to previous measurements. The kinematic range covered, $1.5 < Q^2 < 150$ GeV$^2$ and $3 \cdot 10^{-5} < x < 0.2$, means an extension in $x$ such that an overlap with previous fixed target data is obtained. In spite of this the precision of these data together with previous HERA data at large $x$ and high $Q^2$ is not sufficient to give enough constraints of both the gluon density and $\alpha_S$ to allow for a simultaneous determination. Instead the low $x$ data from H1, which essentially constrain the gluon distribution, have been combined with large $x$ data provided by the BCDMS $\mu p$ scattering experiment, in order to reduce the strong correlation between $\alpha_S$ and the gluon distribution, $xg(x, Q^2)$, and thereby enable an accurate simultaneous determination of the two.

The structure function $F_2(x, Q^2)$ has been decomposed into the gluon distribution, $xg(x, Q^2)$, and two independent parton density functions, one of which is defining the valence quark distribution whereas the other constrains the sea-quark distribution and provides a small valence quark correction in order to describe the low $x$ behaviour of $F_2$. The $x$ dependence of these parton distribution functions have been parametrized at a smallest scale $Q_o^2$ of 4 GeV$^2$.

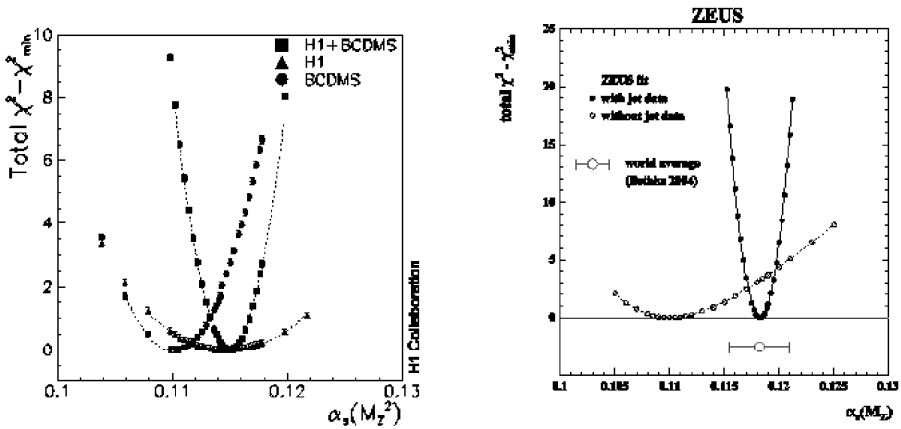


Figure 3: *The $\chi^2$ profiles as a function of $\alpha_S(M_Z)$ for a NLO DGLAP fit using the H1 ep and BCDMS $\mu p$ data (left) and the ZEUS data with and without the jet data included (right).*

In a NLO QCD fit to the combined H1 and BCDMS data, $\alpha_S$ is left as a free parameter, resulting in a value:

$$\alpha_S(M_Z^2) = 0.1150 \pm 0.0017(exp.)^{+0.0009}_{-0.0005}(model) \tag{16}$$

The fit to the data implies the minimization of a $\chi^2$ function, taking correlations between data points caused by systematic uncertainties into account. The results of the fits are shown in Fig.3(left).

The ZEUS experiment has performed a simultaneous fit to $\alpha_s(M_Z)$ and the proton PDF's in an analysis which includes neutral and charged current inclusive DIS cross sections from $e^+p$ and $e^-p$ scattering together with jet data. Two different jet samples were used in the fit. On one hand the DIS inclusive jet cross sections as a function of the transverse jet energy, $E_T$, in the Breit frame, for different $Q^2$ bins were included. On the other hand dijet cross sections in photoproduction as a function of $E_T$ of the most energetic jet, as measured in the laboratory system, for different rapidity regions were also considered. A parametrization of the PDF's at a smallest scale $Q_o^2 = 7$ GeV$^2$ provides a good fit of the jet cross sections over several orders of magnitude. The quality of the fit demonstrates that NLO QCD is able to simultaneously describe inclusive cross sections as well as jet cross sections.

The strong correlation between the gluon shape and the value of $\alpha_s(M_Z)$, which affects the fits to inclusive cross section data alone, is avoided by including the jet data, since their cross sections depend on the gluon PDF and the value of $\alpha_s(M_Z)$ in a different way compared to the total cross section. An

advantage in using different data sets from the same experiment compared to combining data from different experiments is that the contributions from correlated errors and normalization uncertainties are significantly reduced. The combined fit results in an $\alpha_S$ value of:

$$\alpha_S(M_Z) = 0.1183 \pm 0.0028(exp.) \pm 0.0008(model) \tag{17}$$

The $\chi^2$ profile as a function of $\alpha_S$, shown in Fig.3(right), illustrates the improved accuracy in the $\alpha_S$ determination from the data sample including the jet data compared to using the inclusive cross sections alone.

Fig.4 gives a compilation of various measurements of $\alpha_S(M_Z)$ at HERA. Within uncertainties all these results agree internally and with the world average.

$$\alpha_S(M_Z) = 0.1189 \pm 0.0010 \tag{18}$$

In Fig.5 the HERA results are complemented with measurements by other experiments to illustrate the running of $\alpha_S$ with the energy scale $Q$. The $Q$ dependence of $\alpha_S$ is well reproduced by QCD predictions, which makes it relevant to extrapolate all results to a common scale defined by the rest mass of the $Z^o$ boson, $M_Z$, using the Renormalization Group Equation.

## 5 Summary

A measurement of the inclusive $ep$-cross section in the high $x_{Bj}$ region by the ZEUS experiment is well described by recent parametrizations of the parton density function, except for the highest $x_{Bj}$-bin where the predictions undershoot the data. The first measurements from the H1 and ZEUS experiments with polarized lepton beams at HERA give cross sections which are consistent with the predictions of the Standard Model and the observed asymmetry clearly demonstrates the parity violation at small distances. The structure function $xF_3^{\gamma Z}$, which can be extracted from the polarization cross section asymmetries, is sensitive to the valence quark distributions and turns out to be in good agreement with Standard Model predictions.

The cross section of inclusive jet production as measured by the H1 and ZEUS experiments is well reproduced by NLO QCD calculations and fits to the data have been used to extract $\alpha_S(M_Z)$ and to demonstrate the scale dependence of the coupling constant. The ratio between trijet and dijet cross sections is also well described by NLO calculations over the measured $Q^2$ range where electroweak effects can be neglected. The strong coupling can be extracted with a precision that profits from cancellations of systematic errors. The inclusive cross section measurements have been combined with other data in order to further constrain the parton density functions and to extract $\alpha_S(M_Z)$ in combined fits. All results on $\alpha_S(M_Z)$ presented, are consistent and in good agreement with the world average value.

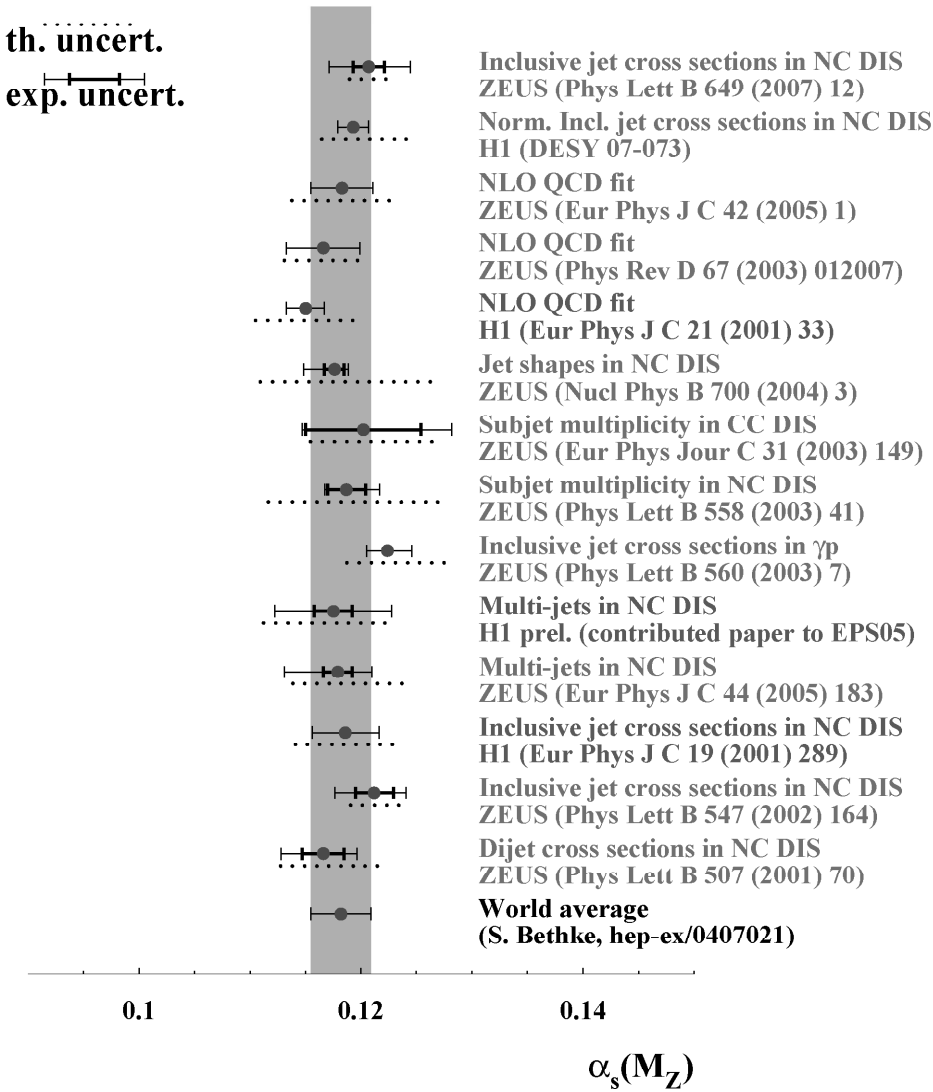


Figure 4: *A compilation of $\alpha_S(M_Z)$ measurements at HERA compared to the world average value.*

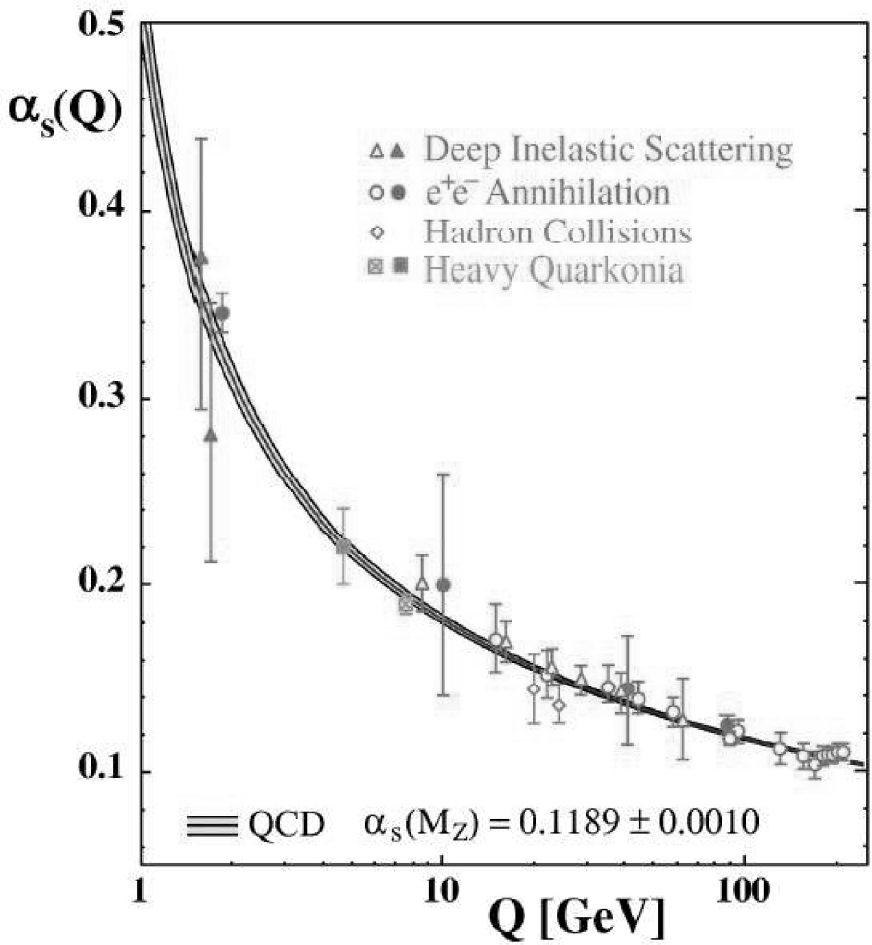


Figure 5: *The scale dependence of $\alpha_S(Q)$ including data from various measurements compared to the QCD prediction.*

## 6 Acknowledgements

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