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The pion flux and the OPE model

Aharon Levy

School of Physics and Astronomy

Raymond and Beverly Sackler Faculty of Exact Sciences

Tel–Aviv University, Tel–Aviv, Israel

Abstract

The pion flux is discussed in the assumption of an one-pion-exchange (OPE) model. The sensitivity of the pion flux to the parameters and to the different shapes for the form factor is presented and compared to the FNC measurements.

1 Introduction

One of the major aims of the forward neutron calorimeter (FNC) is to measure the pion structure function and the total $\gamma\pi$ cross section. The assumption is that the dominant process through which a forward neutron is produced is that of a pion exchange in the t channel, as described in figure 1.

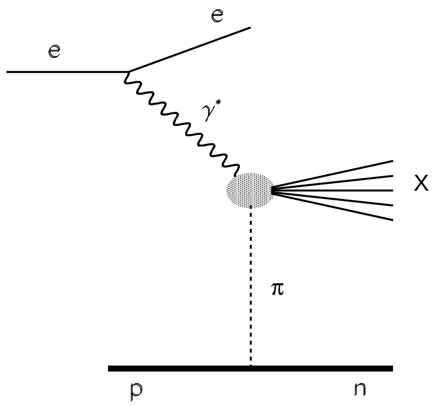

Figure 1: *An OPE diagram describing the process $ep \to enX$.*

Under this assumption one has in fact a deep inelastic scattering (DIS) off a pion 'target' probed by the virtual photon γ^* and thus can measure the pion structure function F_2^π. The semi-inclusive cross section is related to the structure function in the usual way,

$$\frac{d\sigma(ep \to enX)}{dx dQ^2 dx_L dt} = \frac{2\pi\alpha^2}{Q^2 x}\left(2 - 2y + \frac{y^2}{1+R}\right) F_2^{(4)}(x_L, t, \beta, Q^2), \qquad (1)$$

where x_L is the fraction of the proton beam momentum carried by the outgoing neutron, and all other variables are the standard DIS variables with $R = \sigma_L/\sigma_T$. The variable β here is given by $\beta = \frac{x}{1-x_L}$. Assuming factorization one can factorize the $F_2^{(4)}$ structure function into a factor $f_{\pi/p}(x_L, t)$ describing the flux of pions in the proton and the pion structure function $F_2^\pi(\beta, Q^2)$,

$$F_2^{(4)}(x_L, t, \beta, Q^2) = f_{\pi/p}(x_L, t) \cdot F_2^\pi(\beta, Q^2), \qquad (2)$$

where β is the Bjorken variable for DIS off the pion.

It is thus clear that for extracting the pion structure function it is essential to know the flux $f_{\pi/p}(x_L, t)$. The purpose of this note is to review the different forms of the pion flux in an OPE model and to see how much they differ in the kinematic range measured by the FNC. We study only the OPE case and will comment at the end on the possible contribution of other processes, like ρ exchange or Δ production.

The structure of the note is as follows. First there will be a description of the flux factors in a reggeized and a non-reggeized OPE model. Next, a study will be presented of the dependence of the flux on the parameters of the model, excluding the form factor. The dependence of the flux on the shape of the form factor, for a given choice of parameters will then follow. The use of the FNC data to discard some of the suggested form factor shapes is also discussed.

2 Reggeized OPE

The OPE diagram presented in figure 1 can be expressed for a Reggeized pion trajectory $\alpha_\pi(t) = \alpha'(t - m_\pi^2)$ by using the triple-regge diagram described in figure 2. One assumes that the energy of the $\gamma^*\pi$ system is big enough to consider only the Pomeron exchange at the γ^* vertex.

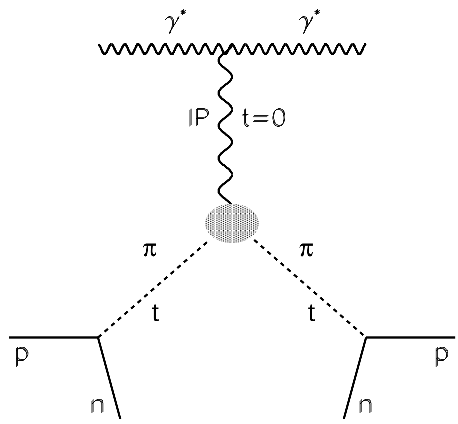

Figure 2: *A triple-regge diagram describing the process $ep \rightarrow enX$.*

Using the triple-regge formalism Bishari [1] wrote the cross section for the inclusive reaction $pp \rightarrow nX$ as follows:

$$\frac{d^2\sigma}{dtd(M^2/s)} = \frac{1}{4\pi} 2 \frac{g^2}{4\pi} \sigma_T(\pi p) F^2(t) \frac{(-t)}{(t - m_\pi^2)^2} \left(\frac{s}{M^2}\right)^{2\alpha_\pi(t) - \alpha_{I\!P}(0)} . \tag{3}$$

From this expression, and by using the relation $x_L = 1 - M^2/s$, one can write the pion flux in the form

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi} 2 \frac{g^2}{4\pi} \frac{(-t)}{(t - m_\pi^2)^2} (1 - x_L)^{\alpha_{I\!P}(0) - 2\alpha_\pi(t)} F^2(t). \tag{4}$$

Here $\alpha_{I\!P}(0)$ is the Pomeron intercept, $\frac{g^2}{4\pi}$ is the $pp\pi^0$ coupling constant and $F(t)$ is a form-factor to account for off mass-shell corrections, normalized to be unity at the pion pole, $F(m_\pi^2) = 1$. The factor 2 is the result of the relation $\frac{g^2_{pn\pi^+}}{4\pi} = 2\frac{g^2_{pp\pi^0}}{4\pi}$.

Two shapes of the form factor are considered for the reggeized OPE model, an exponential one [1, 2, 3] and a Gaussian one [3],

$$F(t) = \exp[b(t - m_\pi^2)] \qquad \text{exponential,} \tag{5}$$
$$F(t) = \exp[-R^2(t - m_\pi^2)^2] \qquad \text{Gaussian.} \tag{6}$$

The values of the slopes b and R^2 are obtained by comparison with data. Robinson et al. [4] use Fermilab data of the inclusive reaction $pn \rightarrow pX$, the Bishari formula and the triple-regge fits of Field and Fox [5] to show that the best fit is obtained for $b = 0$. Kopeliovich, Povh and Potashnikova (KPP) [2] claim in their paper that the results of comparison with data are unstable, yielding slopes which range from a value of 0 to 2 GeV^{-2}. They use the value $b = 0.3$ GeV^{-2}. Nikolaev, Schäfer, Szczurek, and Speth (NSSS) [3] use the NA27 data [6] on $pp \to \pi^0 X$ to show that the exponential form factor does not describe the data. They then use the recent E866 Drell Yan data [7] on the flavor asymmetry, $\overline{d} - \overline{u}$ and show that a Gaussian form factor with $R = 1$ or 1.5 GeV^{-2} is consistent with the data.

3 Non-reggeized OPE

The expression used in the non-reggeized OPE model for the flux of the pion is similar to that of the reggeized one (equation (4)), using a Pomeron intercept of $\alpha_{\mathbb{P}}(0) = 1$ and a pion trajectory $\alpha_\pi(t) = 0$. In addition the form factor F can be also a function of x_L,

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi} 2 \frac{g^2}{4\pi} \frac{(-t)}{(t - m_\pi^2)^2} (1 - x_L) F^2(t, x_L). \tag{7}$$

In this model, 5 different shapes for the form-factor were used. A simple exponential one [8, 9], monopole and dipole forms [8], light-cone exponential [10, 11] and light-cone dipole [12],

$$F(t) = \exp[b(t - m_\pi^2)] \qquad \text{exponential,} \tag{8}$$

$$F(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t} \qquad \text{monopole,} \tag{9}$$

$$F(t) = \left(\frac{M^2 - m_\pi^2}{M^2 - t}\right)^2 \qquad \text{dipole,} \tag{10}$$

$$F(t) = \exp\left[b \frac{t - m_\pi^2}{1 - x_L}\right] \qquad \text{light} - \text{cone exp,} \tag{11}$$

$$F(t) = \left(\frac{\Lambda^2 + m_p^2}{\Lambda^2 + m_p^2 + \frac{m_\pi^2 - t}{1 - x_L}}\right)^2 \qquad \text{light} - \text{cone dipole.} \tag{12}$$

Frankfurt, Mankiewicz and Strikman (FMS) base their form-factor determination (eqs. (8), (9), (10)) on an earlier study of Thomas [13]. Thomas used the non-reggeized OPE expression together with some Bag Model considerations to limit the value of the exponent b by using early data from Fermilab which indicated a 30% excess of $\overline{d}$ over $\overline{u}$ [14]. FMS used better measurements from the CCFR collaboration [15] and found a value of $b = 1.8$ GeV^{-2}. For the monopole form they obtain $\Lambda^2 = 0.25$ GeV^2 and for the dipole form, $M^2 = 0.81$ GeV^2.

Golec-Biernat, Kwiecinski and Szczurek (GKS) use the simple exponential form and use the exponent as determined by Holtmann, Szczurek and Speth (HSS) [16], which is $b = 1.2$ GeV^{-2}.

The light-cone exponential form-factor is used by Przybycien, Szczurek and Ingelman (PSI) [10] as well as by Szczurek, Nikolaev and Speth (SNS) [11]. The reason given for using the light-cone approach [10] and for including the factor $(1 - x_L)$ in the exponent is that otherwise the form-factors can be a source of momentum sum rule violation. Both PSI and SNS use the work of HSS [16] where the exponent for the light-cone exponential form-factor was determined to be $b = 0.4$ GeV^{-2}.

The light-cone dipole form-factor was studied by Melnitchouk, Speth and Thomas (MST) [12]. By using the E866 data [7] of $\bar{d} - \bar{u}$ they find $\Lambda = 1.5$ GeV. Note however that this choice of form-factor with this parameter does not describe well the data on $\bar{d}/\bar{u}$ [12].

As a summary of these two sections, a table is presented with the expressions used for the form-factor by the different authors and the values of the parameters used for the study presented in the next sections.

Author	Form-factor (eq.#)	parameter	ref.
Bishari-0	exponential (5)	$b = 0$	[1]
Bishari-4	exponential (5)	$b = 2$ GeV^{-2}	[1]
KPP	exponential (5)	$b = 0.3$ GeV^{-2}	[1]
NSSS	Gaussian (6)	$R = 1.5$ GeV^{-2}	[3]
GKS	exponential (8)	$b = 1.2$ GeV^{-2}	[9]
FMS	exponential (8)	$b = 1.8$ GeV^{-2}	[8]
FMS	monopole (9)	$\Lambda^2 = 0.25$ GeV^2	[8]
FMS	dipole (10)	$M^2 = 0.81$ GeV^2	[8]
PSI, SNS	light-cone exp. (11)	$b = 0.4$ GeV^{-2}	[10, 11]
MST	light-cone dipole (12)	$\Lambda = 1.5$ GeV	[12]

Table 1: *The expressions used for the form-factor by the different authors and the values of the parameters used for the present study.*

4 Flux dependence on general parameters

The expressions for the pion flux, be it based on the reggeized or the non-reggeized OPE, depend in addition to the shape of the form-factor also on some general parameters, like the $pp\pi$ coupling and the trajectory parameters. In order to study the effect of these parameters on the pion flux, we used the reggeized form of Bishari with no form-factor ($b = 0$, denoted as Bishari-0 in table 3). In this, and all the following studies, the flux is integrated over t in the kinematical region of the FNC, namely from t_{min} up to the angular acceptance of the FNC, which is 0.9 mrad. The values of t_{min} and t_{max} depend on x_L in the following way,

$$|t_{min}| = m_p^2 \frac{(1 - x_L)^2}{x_L} \tag{13}$$

$$|t_{max}| = 0.55 x_L + |t_{min}| \tag{14}$$

4.1 $g^2/4\pi$

In his original work in 1972, Bishari [1] used the value $\frac{g^2}{4\pi} = 15$. Though today the accepted value for this coupling is $\frac{g^2}{4\pi} = 13.6$, some authors prefer to use the value $\frac{g^2}{4\pi} = 13.75$. Figure 3(a) shows the pion flux as function of x_L for the above three values of the coupling constant. Since the flux is directly proportional to the value of the coupling, its magnitude changes accordingly.

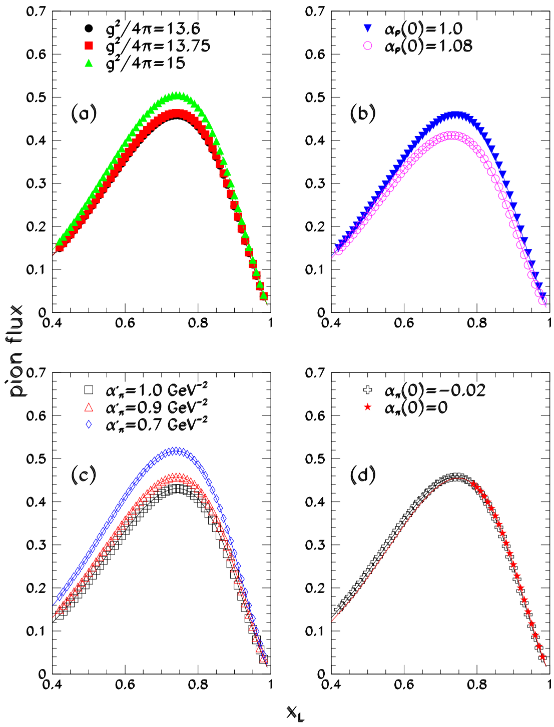

Figure 3: *The pion flux, using the Bishari-0 form, as function of x_L, for (a) different values of the $pp\pi^0$ coupling, (b) different values for the Pomeron intercept, (c) different values of the slope of the pion trajectory, and (d) different values of the pion trajectory inytercept.*

4.2 $\alpha_{\mathbb{P}}(0)$

All of the authors using the reggeized pion flux, use for the Pomeron intercept the value $\alpha_{\mathbb{P}}(0) = 1$. Figure 3(b) shows the pion flux for $\alpha_{\mathbb{P}}(0) = 1$ and for the Donnachie-Landshoff (DL) value [17] of $\alpha_{\mathbb{P}}(0) = 1.08$. As can be seen, using the DL value has quite a large effect on the pion flux, reducing it in magnitude by about 10% at its maximum.

4.3 α'_π

The slope of the pion trajectory, α'_π, is not well determined. Some of the authors use $\alpha'_\pi = 1$ GeV^{-2} [2], some use $\alpha'_\pi = 0.9$ GeV^{-2} [1], or even $\alpha'_\pi = 0.7$ GeV^{-2} [3]. The dependence of the pion flux on the choice of the value of the pion trajectory slope is shown in figure 3(c) as function of x_L. As one can see, the value of the flux can change by as much as about 25%, depending on the value used for α'_π.

4.4 $\alpha_\pi(0)$

In order to check the dependence of the pion flux on the value of $\alpha_\pi(0)$, the pion flux is shown for $\alpha_\pi(0)$ = -0.02 and $\alpha_\pi(0) = 0$, as function of x_L in figure 3(d). No significant dependence on this parameter is observed.

5 Flux dependence on the shape of the form-factor

5.1 Flux as function of x_L

In figure 4, the pion flux as function of x_L is displayed for the different shapes of the form-factor listed in table 3. As mentioned above, the flux has been integrated over t, from t_{min} up to the t_{max} determined by the angular acceptance of the FNC.

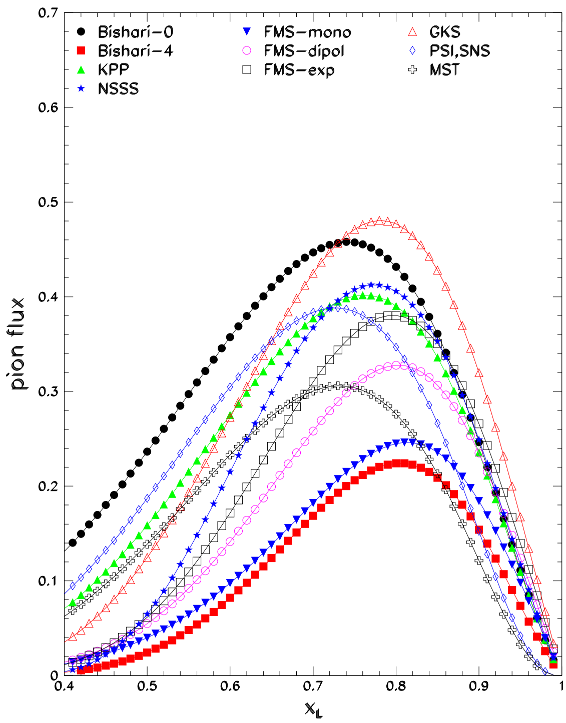

Figure 4: *The pion flux as function of x_L for different shapes of the form-factor (see text).*

One sees in general a strong dependence of the flux on the assumed model. The curves differ both in absolute values as well as in their shape. One clearly needs to confront the different models with experimental measurements in order to be able to find the one which could be used for the FNC data in order to extract the pion structure function. Some of the models have actually already been ruled out. As mentioned above, the Bishari model with b=0 is preferred over the one with b= 4 GeV^{-2}. However this was done with the relatively low energy data, with a Pomeron intercept of 1, and no comparison with data on $\overline{d}-\overline{u}$ has been done. The model NSSS [3], for instance, was compared only to $\overline{d}-\overline{u}$, but not to $\overline{d}/\overline{u}$. As was shown by MST [12], describing the difference does not necessarily mean that the ratio can be described.

5.2 Flux as function of t

Since the large difference between the model comes from the different shapes of the form-factors, in addition to whether one uses a reggeized or non-reggeized OPE model, one could study the t dependence of the models for different x_L values, and compare the expected slopes with measurements of the FNC.

In figure 5 the pion flux is plotted as function of t for fixed values of x_L, for the different models.

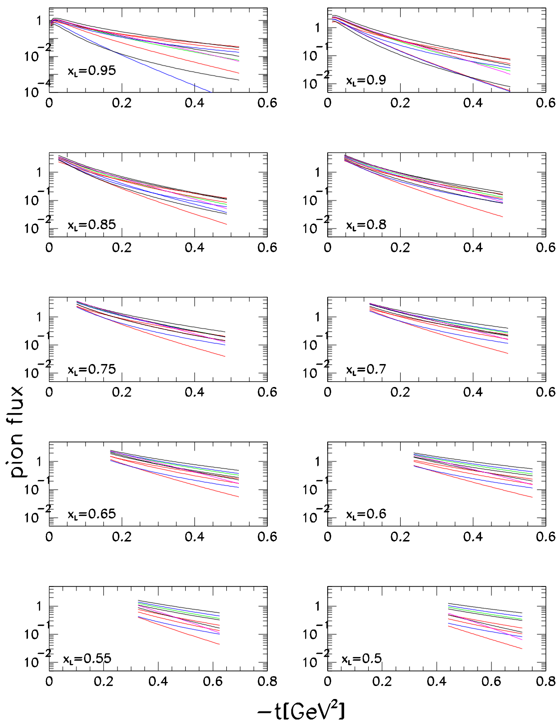

Figure 5: *The pion flux as function of t for fixed values of x_L as indicated in the figure, for different shapes of the form-factor (see text).*

It is difficult to indicate in the figure which line belongs to which model. However one can get the overall impression of the diversity of the distribution for the high x_L region. One also sees that at the low end of x_L, the t range is quite small and makes a reliable determination of the slope a very difficult task.

In each of the x_L bins, a simple exponential fit of the form $A \exp bt$ was performed to each of the models, in the range of t_{min} to t_{max} and the values of b for each x_L has been plotted in figure 6. A smooth line has been drawn through the points and this is shown in the figure. The lines are compared with the preliminary data of the FNC presented at the EPS conference in Jerusalem [18].

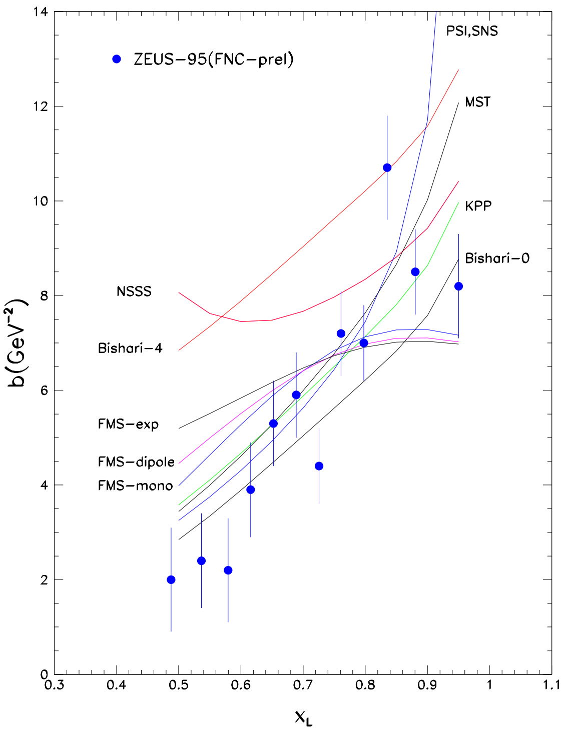

Figure 6: *The exponential slopes fitted through the t dependence of the different models, as function of x_L. The preliminary FNC measurements of the slopes are plotted as full dots with statistical errors only.*

The data, though preliminary, show that they can be very useful in ruling out some of the models. First look would immediately rule out Bishari-4, NSSS, PSI and SNS, and also GKS. Also MST does not do a good job in the largest x_L value.

For easier comparison, we plot in figure 7 only the reggeized OPE models which give a reasonable description of the FNC data. Note that the NSSS model with the Gaussian form using the parameter $R = 1.0$ GeV^{-2} (not favoured by the authors [3]) has been plotted here. In figure 8 we show the pion fluxes of these three models.

We thus see that if one is able to use the FNC data of the b measurement to rule out some of the models, the big uncertainty from the spread of the fluxes, shown in figure 4, can be reduced.

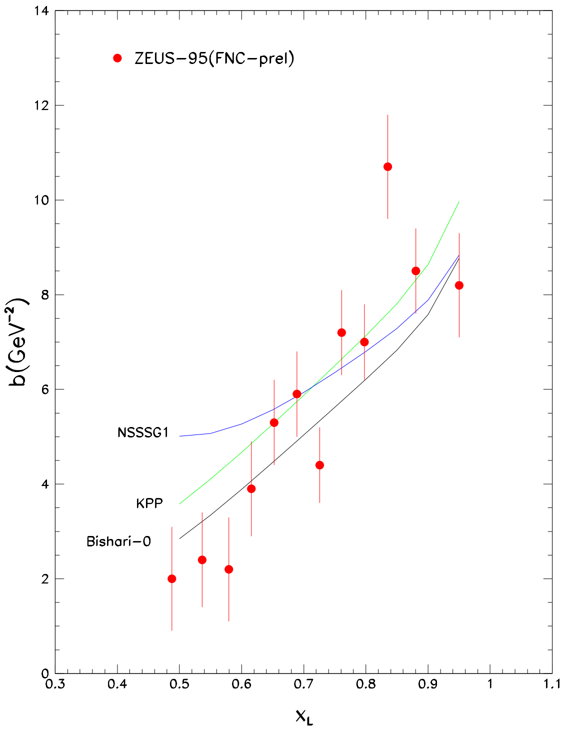

Figure 7: *The exponential slopes fitted through the t dependence of the reggeized OPE models, as function of x_L. The preliminary FNC measurements of the slopes are plotted as full dots with statistical errors only.*

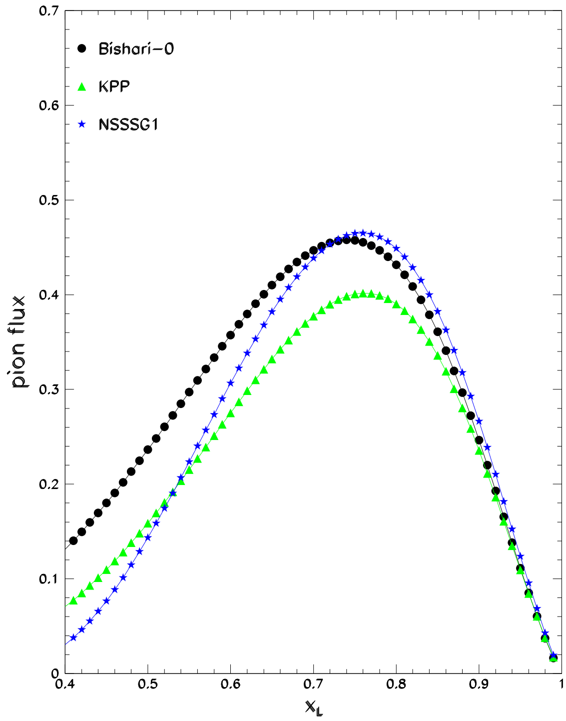

Figure 8: *The pion flux as function of x_L for different shapes of the form-factor (see text).*

6 Absorptive corrections

In all the above study we did not discuss the effect of absorptive corrections to the OPE model. These are discussed in [19] and in [20]. These corrections definitely complicate the extraction of the pion structure function, however it is shown [20] that the corrections are large only in the photoproduction region, while are negligible for $Q^2 > 10$ GeV^2.

7 ρ exchange and Δ production

A neutron in the final state can be also the result of a ρ exchange or that of a decay product originating from a Δ production at the proton vertex. The importance of these processes are discussed in [3]. It is concluded there that the pion exchange process is of the order of 80% for $|t|$ <0.2 GeV^2 and x_L >0.7. For larger values of $|t|$ and lower values of x_L the other processes can contribute as much as 50%.

8 Summary and conclusions

A review was presented of the different forms of the pion flux. It was shown that the magnitude and the shape of the pion flux depends strongly on the parameters and the type of form factors used. It is essential to be able to select only those fluxes which are in agreement with data from other experiments and from the FNC. In addition to using inclusive measurements of neutron production in pp and pn interactions at lower energies, one can use Drell-Yan data on $\overline{d} - \overline{u}$ to narrow down the choice of pion fluxes. In particular, one can use the FNC measurements of the t slope as function of x_L to exclude some of the suggested pion fluxes. In addition to all the above, one needs to consider other effects like ρ exchange, Δ production and absorptive corrections. The latter seems to be a small effect for $Q^2 > 10$ GeV^2.

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