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One-Pion Exchange and Deep-Inelastic Electron-Nucleon Scattering

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The role of one-pion exchange is examined in the deep-inelastic region for electron-nucleon scattering. Exclusive channels like πN, $\pi\Delta$ will contribute negligible, nonscaling contributions to σ_S. On the other hand, inclusive final states like N + "anything," where the detected final nucleon is slow in the lab system, afford the opportunity to experimentally determine the structure functions for electron-pion scattering provided the characteristic one-pion-exchange structure (dip or peak) is observed at small momentum transfer.

I. INTRODUCTION

Inelastic electron-proton experiments have been carried out recently in which hadrons are detected in coincidence with the scattered electron.[1] The region explored in these coincidence experiments does not yet overlap significantly with the region in which scaling has been established experimentally in inelastic electron-proton scattering.[2] Nevertheless these coincidence data are very interesting in their own right. Two results of particular relevance to our work here are the following[1]: (1) The charged-pion elastic form factor $F_\pi(q^2)$ is now well measured out to squared spacelike momentum transfers $q^2 \approx -1$ (GeV/c)^2; (2) A sizable contribution to the scalar cross section (σ_S) is seen in the πN and $\pi\Delta$ final states. These two items are in fact related because one-pion exchange (OPE) can contribute to these final states, and to the extent that OPE is large, one will have sizable contributions to σ_S and corresponding sensitivity to $F_\pi(q^2)$.

It is natural to ask, therefore, about the role OPE will play in the deep-inelastic (scaling) region.[3] In particular, what is the relation, if any, between the (small) value of σ_S observed in the deep-inelastic experiments and OPE contributions to exclusive final states (πN, $\pi\Delta$, etc.) or to the inclusive final state π + "anything"? Moreover, is there anything besides $F_\pi(q^2)$ that can be learned by isolating OPE contributions and measuring the residue of the pion pole?

In Sec. II we show that the answer to the first question is negative. Namely, the pion current contributions will not scale and will rapidly vanish in the deep-inelastic region. In Sec. III we answer the second question with a qualified yes. We show that the process e + nucleon $\rightarrow e'$ + nucleon$'$ + "anything" in an appropriate kinematic region has an OPE contribution which scales. Moreover, the residues of the pion pole term in the cross section for this process are proportional to $C_{1,2}^{(\pi)}$, the structure functions which describe inelastic electron-pion scattering. These objects are of obvious interest and deserve experimental measurement. It seems necessary, however, to pick processes for which the transition nucleon $\rightarrow$ nucleon$'$ implies nonzero quantum-number change in order to eliminate competing mechanisms which would otherwise overwhelm one-pion exchange. This complicates the experiment and is the reason that we have qualified our answer.

II. PION CURRENT CONTRIBUTIONS

Examples of processes to which the $\gamma_v\pi\pi$ vertex contributes are shown in Fig. 1. Throughout our work we will ignore the standard lepton factors and the photon propagator and talk in terms of the cross section for a virtual photon (γ_v) incident upon a nucleon. We choose the z axis along the incident photon direction and work in the rest frame of the target nucleon (lab system).

First consider process (a): $\gamma_v + p \rightarrow \pi^+ + n$ [Fig. 1(a)]. Throughout our work we consider the Bjorken limit[4] $-q^2 \equiv Q^2 \rightarrow \infty$, $m\nu = p\cdot q \rightarrow \infty$, with fixed $\omega = 2m\nu/Q^2$ (m = nucleon mass, μ = pion mass). In terms of these variables one has

$$s = (p+q)^2 = Q^2(\omega-1) + m^2 \rightarrow Q^2(\omega-1).$$

In addition to Q^2 and ω, one more variable is required; we choose it to be the invariant momentum transfer squared

$$t = (p'-p)^2 = 2m^2 - 2mE'.$$

Furthermore, since we are interested in the OPE contribution, we keep t fixed and small when carrying out the Bjorken limiting process. This means that the lab energy E' of the recoil proton, and hence all components of its momentum, remains small in the lab system as $Q^2 \rightarrow \infty$.

The minimum value of $-t$ occurs for forward scattering and is

$$t^m \equiv -m^2/[\omega(\omega-1)]. \qquad (2.1)$$

Clearly then, the condition that one be near the pion pole, $-t^m \leqslant \mu^2$, can only be met at large

ω ($\omega \geq 7.2$). As we will see below, this fact tends to suppress the OPE amplitude since it contains a factor of ω^{-1}.

The OPE amplitude corresponding to Fig. 1(a) is

$$A^{\mu} = -(2k-q)^{\mu} F_{\pi}(q^2) \frac{i}{t-\mu^2} \bar{u}(p') i\gamma_5 u(p) \sqrt{2}\, g_{\pi NN}, \tag{2.2}$$

where F_{π} is the pion elastic form factor. As is well known, Eq. (2.2) alone is not gauge-invariant. We can correct this formally by the substitution

$$A^{\mu} \to \bar{A}^{\mu} = A^{\mu} - \frac{v^{\mu} q \cdot A}{q \cdot v}, \tag{2.3}$$

where v is any four-vector.[5] The particular choice of v^{μ} is not crucial since the quantity $q \cdot A$ has no pole at $t = \mu^2$, unlike A^{μ}. For convenience we choose $v^{\mu} = q^{\mu}$; this has the simple virtue that when we contract the amplitude (2.3) with a polarization vector for the virtual photon (ϵ_{μ}) the second term on the right-hand side of Eq. (2.3) makes no contribution since $\epsilon_{\mu} q^{\mu} = 0$.

Squaring the OPE contribution of Eq. (2.2), one finds

$$2mW^S_{\rm OPE} = 2\left(\frac{g_{\pi NN}^2}{4\pi}\right) [F_{\pi}(-Q^2)]^2 \frac{1}{4\pi\omega} \int_{t^c}^{t^m} dt \frac{-t}{(\mu^2 - t)^2}, \tag{2.4}$$

where

$$W^S = \epsilon^S_{\mu}{}^*(q) W^{\mu\nu} \epsilon^S_{\nu}(q) = W_2 \left(1 + \frac{\nu^2}{Q^2}\right) - W_1 \tag{2.5}$$

and where t^c is the square of the momentum transfer beyond which OPE is no longer credible due to absorption and/or form-factor effects at the πNN vertex. [Typically $-t^c \approx (3-6)\mu^2$.] The quantity W^S introduced in Eq. (2.5) is related to the familiar scalar cross section[6] by

$$\sigma_S = \frac{4\pi^2 \alpha}{K} W^S, \tag{2.6}$$

where

$$K = (s - m^2)/2m.$$

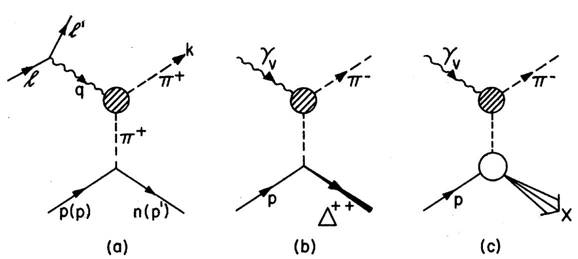

FIG. 1. (a) One-pion-exchange contribution to electroproduction of $\pi^+ n$. (b) Electroproduction of $\pi^- \Delta^{++}$ via π^- exchange. (c) Pion-current contributions to high-mass baryon states.

Clearly the expression (2.4) does not scale and instead is expected to vanish rapidly as $Q^2 \to \infty$ because of the presence of the pion form factor. Correspondingly $\sigma_S \sim F_{\pi}{}^2 Q^2 s^{-2}$. For orientation suppose that $F_{\pi}(-Q^2) = (1 + Q^2/m_{\rho}{}^2)^{-1}$. Then, for $Q^2 = 2$ (GeV/c)^2, $\omega = 8$, $-t^c = 5\mu^2$, Eq. (2.6) contributes

$$\sigma_S^{\rm OPE} \approx 0.09\ \mu{\rm b},$$

which is small in comparison to the cross sections $\sigma_S \approx 1-2\ \mu$b, $\sigma_T \approx 20\ \mu$b measured in this range in the inelastic electron-proton experiments. In Table I we give the contribution of Eq. (2.4) at other values of Q^2 and ω.

For convenience we list in Table II the experimental value of $2mW^S$ computed by assuming $R = \sigma_S/\sigma_T = 0.18$ and $\nu W_2(\omega) = 0.3$ for $\omega \geq 3$. These values are presumably upper bounds to W^S. Of course, unlike the entries in Table I, the entries in Table II should not be multiplied by a form factor.

Finally the contribution of Fig. 1(a) to σ_T is, as expected from general arguments,[7] even smaller $[\sigma_T/\sigma_S = O(1/Q^2)]$.

Consider next the process of Fig. 1(b): $\gamma_v + p \to \pi^- + \Delta^{++}$. The minimum momentum transfer is now given by

$$t^m = -m_{\Delta}{}^2/(\omega - 1)^{-1} + m^2 \omega^{-1}, \tag{2.7}$$

where m_{Δ} is the mass of the final $\Delta(1236)$ resonance. Proximity to the pion pole, $t^m \leq -\mu^2$, now requires $\omega \geq 36$. This is a significant increase over that for the nucleon final state and is indicative of the trend should one consider higher-mass states.

We impose gauge invariance as above and find[8] for diagram 1(b)

$$2mW^S_{\rm OPE} = \left(\frac{g_{\Delta}{}^2}{4\pi}\right) \frac{[F_{\pi}(-Q^2)]^2}{24\pi\omega} \left(1 + \frac{2m^2}{m_{\Delta}{}^2}\right) (m_{\Delta}{}^2 - m^2) \times \int_{t^c}^{t^m} \frac{dt}{(t-\mu^2)^2}, \tag{2.8}$$

where g_{Δ} is the $\Delta N\pi$ coupling constant and is given in terms of the width by

$$\Gamma_{\Delta \to \pi N} = \left(\frac{g_{\Delta}{}^2}{4\pi}\right) \left(\frac{q}{m_{\Delta}}\right)^3 \tfrac{1}{3}(E_p + m), \tag{2.9}$$

TABLE I. Values of $2mW^S_{\rm OPE}(F_{\pi})^{-2}$ for the $\pi^+ n$ final state; Eq. (2.4).

$t^c \backslash \omega$	6	8	10	15	20	50	100
$-3\mu^2$	0.11	0.13	0.12	0.09	0.07	0.03	0.01
$-5\mu^2$	0.23	0.22	0.19	0.14	0.10	0.04	0.02
$-7\mu^2$	0.32	0.29	0.25	0.17	0.13	0.05	0.03

TABLE II. Experimental values of $2mW^S$ assuming $R=0.18$.

ω	6	8	10	15	20	50	100
$2mW^S$	0.28	0.37	0.46	0.69	0.92	2.29	4.58

with

$$q^2=\Delta(m_\Delta{}^2, m^2, \mu^2)/4m_\Delta{}^2,$$

$$E_p=(m_\Delta{}^2+m^2-\mu^2)/2m_\Delta,$$

where $\Delta(x, y, z)$ is the triangle function.

As one could anticipate, the result (2.8) for the $\pi\Delta$ final state has the same form as the πN final state, Eq. (2.4), and in the deep-inelastic region does not scale. For $\omega=40$ and all other parameters as listed above for the πN final state one finds a meager

$$\sigma_S(\pi\Delta)\approx 0.02\ \mu\text{b}.$$

The contribution of Eq. (2.8) at other Q^2, ω values is given in Table III.

Having failed to find interesting OPE contributions from the πN or $\pi\Delta$ final states we consider the "Drell process" illustrated in Fig. 1(c).[9] That is, we are considering final states $\pi+X$ where the square of the missing mass, $m_X{}^2$, grows linearly with Q^2. Namely we set

$$m_X{}^2=\lambda Q^2$$

and consider the limit $Q^2\to\infty$, λ fixed. [If we consider the case of $m_X{}^2$ fixed it is clear that one can only find results like Eqs. (2.4) and (2.8).] A simple calculation gives the minimum momentum transfer squared

$$t^m=(q-k)^2{}_{\min}=-\lambda Q^2(\omega-1)^{-1}\to-\infty \quad\text{as}\quad Q^2\to\infty. \tag{2.10}$$

Thus the requirement that we be near the pion pole at $t=\mu^2$ can never be satisfied. Processes of the type 1(c) play no role in the scaling region.[10]

Let us summarize the results of this section. Pion current contributions with low-mass baryon states like those in Figs. 1(a) and 1(b) give interestingly large contributions[1,3] to σ_S in the kinematic range $Q^2<1$ (GeV/c)^2, $s\approx 4$–5 GeV^2. As we move beyond this range (larger Q^2 and/or larger s) these contributions will decrease and be of little importance. High-mass baryon states, Fig. 1(c), are suppressed by the t^m effect.

TABLE III. Values of $2mW^S_{\text{OPE}}(F_\pi)^{-2}$ for the $\pi^-\Delta^{++}$ final state; Eq. (2.8).

t^c \ ω	20	30	40	50	75	100	200
$-3\mu^2$	0.15	0.22	0.23	0.22	0.19	0.16	0.10
$-5\mu^2$	0.29	0.31	0.30	0.28	0.23	0.19	0.11
$-7\mu^2$	0.36	0.36	0.33	0.30	0.24	0.20	0.12

III. INELASTIC e-π SCATTERING

In this section we no longer insist that the exchanged pion remain a pion after it has absorbed the incoming virtual photon.[11] Instead we allow arbitrary final states at the upper vertex as indicated in Fig. 2. We keep the lower vertex simple, however. The simplest case is that in which the target nucleon remains a nucleon as illustrated in Fig. 2(a). Of course now that the final state at the upper vertex is no longer constrained to be spinless, OPE can contribute to both σ_S and σ_T. Indeed if the basic constituents of the pion are spin-$\frac{1}{2}$ objects one expects only σ_T to persist in the scaling limit.

The appropriate choice of variables is as follows. Since we are interested in staying close to the pion pole we want to keep

$$t=l^2=(p'-p)^2=2m^2-2mE'$$

fixed as $Q^2\to\infty$. As remarked already for the πN final state this means the recoil nucleon will be slow in the lab system. In inelastic e-N scattering the unobserved final state of the nucleon has mass squared $s=Q^2(\omega-1)+m^2$. Thus it is natural to introduce a variable ω' which is analogous to ω and in terms of which the mass squared of the final state of the virtual-photon–virtual-pion system may be written

$$s'=(p+q-p')^2=Q^2(\omega'-1)[1+O(1/Q^2)]\,. \tag{3.1}$$

As the remaining variables one may pick θ and Φ, the spherical angles of $\vec{p}'$ as seen in the lab system. (Recall the virtual photon is incident in the $+z$ direction.)

Finally we give the relation between ω', the variable defined by Eq. (3.1), and the vector $\vec{p}'$ as measured in the lab system. It is

$$\omega'-1=(\omega-1)(1-y), \tag{3.2}$$

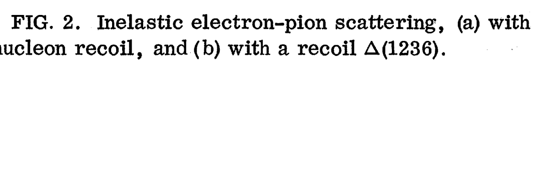

FIG. 2. Inelastic electron-pion scattering, (a) with a nucleon recoil, and (b) with a recoil $\Delta(1236)$.

where y is the fraction of the total minus momentum carried by p', namely

$$y=\frac{E'-p'\cos\theta}{m(1-1/\omega)}\ . \tag{3.3}$$

The minimum momentum transfer squared occurs for $\theta=0$ and is given by a simple generalization of Eq. (2.1),

$$t^m=-m^2[(\omega/\omega')(\omega/\omega'-1)]^{-1}. \tag{3.4}$$

We see that we can get near the pion pole provided ω/ω' is sufficiently large. In particular, $-t^m \leqslant \mu^2$ if $\omega/\omega' \geqslant 7.2$.

The amplitude corresponding to Fig. 2(a) is

$$A^\mu = g_{\pi NN}\frac{\bar{u}(p)i\gamma_5 u(p)}{t-\mu^2}\,(X|j^\mu|\pi). \tag{3.5}$$

[In Eq. (3.5) and in what follows $|\,)$ and $(\,|$ denote states with an invariant normalization of $2E$ particles per unit volume.] In writing Eq. (3.5) we have taken the upper vertex to be on-shell, i.e., $t=\mu^2$. We will return to this point later.

Squaring (3.5) and summing over nucleon spins, one finds in the limit $Q^2\to\infty$ with t, ω, ω' fixed

$$2m\frac{dW^{\mu\nu}}{dtd\omega'}(\omega;t,\omega')=\frac{1}{4\pi\omega}\left(\frac{g_{\pi NN}{}^2}{4\pi}\right)\frac{-t}{(t-\mu^2)^2}\left(\frac{1}{\pi}C_\pi^{\mu\nu}\right), \tag{3.6}$$

where $C_\pi^{\mu\nu}$ is the imaginary part of the invariant forward virtual-photon–pion scattering amplitude. ($W^{\mu\nu}$ is the virtual-photon–nucleon amplitude.) It can be expanded in terms of scalar invariants in an obvious manner:

$$\frac{1}{\pi}\,\mathrm{Im}C_\pi^{\mu\nu}=\sum_n(\pi(l)|j^\mu|n)(n|j^\nu|\pi(l))(2\pi)^3\delta^4(q+l-p_n)$$
$$=C_1(q^2,q\cdot l)\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)+\frac{C_2(q^2,q\cdot l)}{\mu^2}\left(l^\mu-\frac{l\cdot q q^\mu}{q^2}\right)\left(l^\nu-\frac{l\cdot q q^\nu}{q^2}\right). \tag{3.7}$$

The dimensionless amplitudes $C_{1,2}$ appearing in Eq. (3.7) differ from the familiar $W_{1,2}$ amplitudes defined for nucleon targets in normalization. Because of the (unfortunate) convention of extracting factors of $(m/E)^{-1/2}$ for spin-$\frac{1}{2}$, -$\frac{3}{2}$, ... particles in contrast to factors of $(2E)^{-1/2}$ for spin-0, -1, ... particles when writing the S-matrix element in terms of the Feynman invariant amplitude, the amplitudes $C^{\mu\nu}$ and $W^{\mu\nu}$ are dimensionally different. The correspondence is

$$2mW_1 \leftrightarrow C_1,$$
$$2m\frac{W_2}{m^2}\leftrightarrow\frac{C_2}{\mu^2}\ .$$

Note that if the pion structure functions C_1, C_2 obey Bjorken scaling then Eq. (3.6) is a special case of the generalized scaling laws of Ellis[12] and Stack.[13] If we contract Eq. (3.6) with polarization vectors for the virtual photon and supply trivial kinematic factors we can rewrite (3.6) in an equivalent form:

$$(\omega-1)\frac{d\sigma_N^i}{dtd\omega'}(s,Q^2;t,s')=\frac{(\omega'-1)}{4\pi\omega}\left(\frac{g_{\pi NN}{}^2}{4\pi}\right)\frac{-t}{(\mu^2-t)^2}\times\sigma_\pi^i(s',Q^2), \tag{3.8}$$

where $i=T,S$.

The fact that there is no mixing between the transverse (T) and scalar (S) terms in Eq. (3.8) is special to the kinematic region $\omega'/\omega \ll 1$ which we study. [Recall Eq. (3.4).]

For completeness let us record the relation between the cross section differential in t, ω' to the cross section differential in the laboratory momentum of the recoil nucleon:

$$\frac{d\sigma_N^i}{dtd\omega'}=\frac{\pi E'}{\omega}\frac{d\sigma_N^i}{d^3p'}\ . \tag{3.9}$$

Let us estimate what part of the nucleon scaling functions W_1 and νW_2 might be contributed by the OPE mechanism. Suppose at first the target nucleon and the detected nucleon are both protons. The exchanged pion will be a π^0. Assume

$$\sigma_T(\gamma_v\pi^0)=\tfrac{2}{3}\sigma_T(\gamma_v p),$$
$$\sigma_S(\gamma_v\pi^0)\ll\sigma_T(\gamma_v\pi^0)$$

when compared at the same s and Q^2 values. In terms of $F_2(\omega)=\nu W_2$ we can write the total contribution of Eq. (3.6) as

$$F_2^{\mathrm{OPE}}(\omega)=\frac{2}{3}\left(\frac{g_{\pi NN}{}^2}{4\pi}\right)\frac{1}{4\pi}\int_1^{\omega_M'}\frac{d\omega'\omega'}{\omega^2}\int_{t^c}^{t^m}\frac{dt(-t)}{(t-\mu^2)^2}\times F_{2p}(\omega'), \tag{3.10}$$

where t^m is given by Eq. (3.4), t^c is the cutoff discussed in Sec. II, and ω_M', the maximum value of ω', is fixed by the condition

$$t^m=t^c.$$

If we approximate the experimental curve for $F_{2p}(\omega)$ by

TABLE IV. The one-pion-exchange contribution to the scaling function $F_2(\omega)$. The entries listed are values of $F_2^{\mathrm{OPE}}(\omega)/(0.3)$ computed for the p final state; Eq. (3.10).

t^c \ ω	20	30	40	50	100	200
$-\mu^2$	0.000 31	0.000 59	0.000 72	0.000 78	0.000 88	0.000 91
$-2\mu^2$	0.0022	0.0030	0.0033	0.0035	0.0037	0.0037
$-5\mu^2$	0.0143	0.0163	0.0170	0.0173	0.0178	0.0179
$-10\mu^2$	0.0430	0.0461	0.0472	0.0478	0.0485	0.0487

$$F_{2p}(\omega)=0.3(\omega-1)/2, \quad 1\leq\omega\leq 3$$
$$=0.3, \quad 3\leq\omega$$

we find that Eq. (3.10) generates values of $F_2^{\mathrm{OPE}}(\omega)$ which are listed in Table IV. While it is clear that F_2^{OPE} is not a large fraction of the total F_{2p}, Table IV indicates that it is not hopelessly small either.

If we change the target to a neutron but still require a recoil proton (or conversely a proton target and recoil neutron) we will be studying π^- (π^+) exchange. Assuming $\sigma_T(\gamma_v\pi^\pm)=\frac{2}{3}\sigma_T(\gamma_v p)$, the OPE contribution is *double* that indicated in Table IV.

The figure of merit for detecting the OPE term is not the fraction of the total F_2 that it contributes; rather one should look at the background in the small-t region for which OPE can be applied. In the kinematic regime under study ($s\gg s'$, $\omega\gg 1$, t small) Regge-exchange mechanisms are applicable; see Fig. 3. The background will be smaller if we require a charge exchange so we continue to discuss this case.

It seems reasonable to assume that $N\bar{N}$ pair production may be neglected until super high values of s are reached. The final states will thus consist of a single baryon (p or n) accompanied by mesons. As a rough guess let us suppose that at most 50% of the total $\sigma(\gamma_v n)$ comes from final states having $|t|=|(p'-p)^2|<|t^c|\approx 5\mu^2$.

Existing data on the total cross-section difference $\sigma(\gamma_v p)-\sigma(\gamma_v n)$ indicate that at large values of ω (say, $\omega\approx 20$) the t-channel isospin-1 exchange is $\approx$10% and falling with ω. Thus aside from Clebsch-Gordan coefficients (which are of order of magnitude 1) we estimate that the channel $\gamma_v+n\to X+p$ in the interval $|t|<5\mu^2$ contributes at most 5% of the total cross section for $\gamma_v+n\to$ "anything." As Table IV indicates, the OPE is of comparable size and hence ought to be detectable.[14]

So far we have said nothing about "off-mass-shell" effects. For example, sometimes the lower vertex in Eq. (2a) is modified by the addition of a form factor $F_{\pi NN}(t)$ at the pion-nucleon vertex: $F_{\pi NN}(\mu^2)=1$. Although $F_{\pi NN}$ has a branch point at $t=(3\mu)^2$, the threshold for the three-pion state, one does not expect significant variation of $F_{\pi NN}$ until values $|t|\approx(m_p+\mu)^2$ or $m_{A_1}{}^2$ are reached. Thus if we expand $F_{\pi NN}(t)=1+(t-\mu^2)R(t)$ we can be confident that in the range $0\leq -t\leq 5\mu^2$ $R(t)$ is effectively constant. Thus the $R(t)$ term contributes only a background amplitude which is flat in t. Similar remarks apply to off-shell corrections at the upper vertex, contributions from the exchange of particles and Regge trajectories other than the pion, as well as multiple-exchange mechanisms. Only the OPE term, Eq. (3.6), has a significant variation over the range $0\leq -t\leq 5\mu^2$. This is the experimental signature which one must see to unambiguously identify the OPE. In particular it is *necessary* to have data at values of t at and inside of $-t=\mu^2$. Provided the background amplitude with which OPE interferes (destructively or constructively) is not overwhelming in comparison to the OPE amplitude, one will see a dip or peak inside of $-t\approx\mu^2$ and thus be able to find the pion structure functions C_1 and C_2. Our crude estimates above indicate that the background will not be overwhelming provided we look at the charge-exchange case

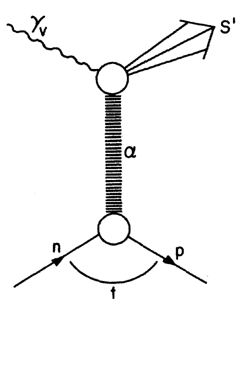

FIG. 3. Regge-exchange mechanisms which generate the background amplitude with which one-pion exchange will interfere.

TABLE V. The one-pion-exchange contribution to the scaling function $F_2(\omega)$. The entries are values of $F_2^{\mathrm{OPE}}(\omega)/(0.3)$ computed for the Δ^{++} final state; Eq. (3.11).

t^c \ ω	30	40	50	100	200
$-\mu^2$	...	...	...	0.014	0.032
$-2\mu^2$	...	0.000 84	0.020	0.050	0.069
$-5\mu^2$	0.064	0.086	0.100	0.135	0.155
$-10\mu^2$	0.013	0.201	0.217	0.254	0.275

$\gamma_v + n \rightarrow X + p$ (deuteron targets).

An alternative way to force isospin-1 exchange in the t channel is to use a proton target and require a $\Delta(1236)$ at the lower vertex; see Fig. 2(b). The cross section for this can be written down immediately from examination of Eqs. (2.4), (2.8), and (3.6) and is

$$2m \frac{dW^{\mu\nu}}{dt d\omega'} = \frac{1}{24\pi\omega} \left(\frac{g_\Delta^2}{4\pi}\right)\left(1 + \frac{2m^2}{m_\Delta^2}\right)(m_\Delta^2 - m^2) \times \frac{1}{(t-\mu^2)^2}\left(\frac{1}{\pi} C_\pi^{\mu\nu}\right) . \tag{3.11}$$

The minimum transfer squared is

$$t^m = \frac{-m_\Delta^2}{\omega/\omega' - 1} + \frac{m^2}{\omega/\omega'} . \tag{3.12}$$

In Table V we give values obtained by integrating Eq. (3.11) under the same assumptions as for Table IV.

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[1]For a review and summary of these experiments see K. Berkelman, in *Proceedings of International Symposium on Electron and Photon Interactions, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N.Y., 1972), p. 263.

[2]See H. Kendall, in *Proceedings of International Symposium on Electron and Photon Interactions, 1971,* Ref. 1, p. 247.

[3]See the discussion of these points in H. Harari, in *Proceedings of International Symposium on Electron and Photon Interactions, 1971,* Ref. 1, p. 299.

[4]J. Bjorken, Phys. Rev. 179, 1547 (1969).

[5]Obviously there are good and bad choices. A bad choice would be one for which $q \cdot v$ vanished inside or close to the physical region for our process.

[6]We use the Hand normalization convention; see L. Hand, Phys. Rev. 129, 1834 (1963).

[7]C. Callan and D. Gross, Phys. Rev. Letters 22, 156 (1969).

[8]In writing Eq. (2.8) we have made the small-t approximations $2m^2 + m_\Delta^2 - t \approx 2m^2 + m_\Delta^2$ and $m_\Delta^2 - m^2 - t \approx m_\Delta^2 - m^2$.

[9]The kinematics of this regime are also studied in G. West, Phys. Rev. Letters 24, 1206 (1970); D. M. Ritson, Phys. Rev. D 3, 1267 (1971).

[10]This answers the question raised in Ref. 3.

[11]Scaling laws in electroproduction with one detected hadron were first considered in S. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970).

[12]J. Ellis, Phys. Letters 35B, 537 (1971).

[13]J. Stack, Phys. Rev. Letters 28, 57 (1972).

[14]Obviously this experiment can also be done with real photons in order to measure $\sigma(\gamma\pi)$, the real-photon–pion cross section.