PION EXCHANGE AND INCLUSIVE SPECTRA †

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We discuss the implications of pion exchange for inclusive distributions in high energy pp collisions. An approximate spectrum for p + p → n + X is predicted and the possible corrections are pointed out. The role played by π-exchange, in building the asymptotic σ_T(pp), is studied. Other related processes, for which the same analysis applies, are briefly mentioned.

Inclusive experiments have been very useful for studying the production mechanisms in multiparticle processes. In particular the inclusive spectra near the kinematical boundary can be parametrized in terms of a properly defined "Reggeon"-particle total cross-section [1-7]. The relevance of such a parametrization to a bootstrap scheme has also been noted [8].

In this paper we shall consider the contribution of a low-lying trajectory, namely the pion Regge pole, to inclusive distributions. The possible importance of low-lying exchanges in building the inclusive spectra is not surprising at all. Indeed it is now a well-established fact that the triple pomeron [9] contribution is very small [4, 5, 10]. In other words the exchanges between the projectile and the fast detected particle (or between the target and the slow detected particle in the lab frame) are dominated by *normal* Reggeons and *not* by the pomeron. Moreover it has been shown [11] that the inclusive data, e.g., in p + p → p + X, not ** very near the phase space boundary, is consistent with an effective exchange having a small negative ** intercept. This may suggest that low-lying exchanges with small intercept can lead to a significant correction in the region very near the kinematical boundary where the dominating Reggeons have a half-unit intercept.

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** The analysis performed in ref. [3] has shown the importance of trajectories with half-unit intercept. However, this conclusion was drawn from data in the region $0.85 \lesssim x < 1$ where x is the Feynman variable (defined in the text). On the other hand the data analyzed in ref. [11] lie below x = 0.85.

Encouraged by the aforementioned remarks we first calculate the pion exchange contribution to a well measured inclusive reaction, namely p + p → p + X [12]. This measured reaction provides a *scale* for the importance of the pion exchange. Secondly an approximate spectrum is *predicted* for p + p → n + X where the pion trajectory is expected to dominate (see below). This prediction is the main result of the present paper. The difficult experiment of detecting the outgoing neutron is expected to be carried out in CERN laboratories. Thirdly in trying to build the total pp cross-section [7, 8, 13] from inclusive spectra (the so-called "inclusive bootstrap") the pion contribution is evaluated and turns out to be of considerable importance. Finally we shall discuss the implication of the pion exchange mechanism on previous attempts to evaluate amplitudes with external pomerons [4, 5, 14, 15].

Pion exchange in p + p → N + X (N = p, n): The kinematics and the relevant diagram is depicted in fig. 1. For concreteness we take the

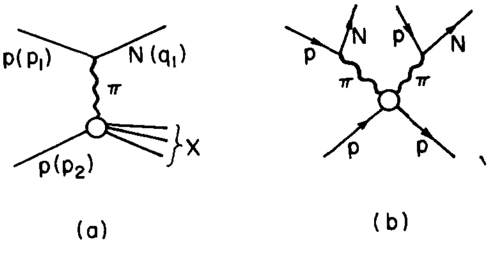

Fig. 1. (a) Kinematics and the pion exchange amplitude in p + p → N + X where N = p, n. (b) Contribution of the diagram in (a) to the inclusive differential cross-section of p + p → N + X.

detected nucleon to be emitted from the projectile vertex. The invariant variables to be used are defined as

$$s = (p_1 + p_2)^2 , \quad t = (p_1 - q)^2 ,$$

$$M^2 = (p_1 + p_2 - q)^2 . \tag{1}$$

These variables are related to the energy E and longitudinal momentum q_L of the outgoing particle in the center of mass, by

$$2E/\sqrt{s} = 1 + m^2/s - M^2/s \approx 1 - M^2/s \tag{2a}$$

and for large q_L

$$x = 2q_L/\sqrt{s} \approx 1 - M^2/s \tag{2b}$$

where for x not far from 1

$$t \approx 2m^2 - m^2(x + 1/x) - q_T^2/x . \tag{2c}$$

Here m, x, and q_T are, respectively, the nucleon mass, the well-known Feynman variable, and the transverse momentum of the emitted particle.

The contribution of the diagram in fig. 1a to the inclusive cross-section is schematically described by fig. 1b. We shall consider *only* that term which leads to a *limiting* distribution [16, 17].

Taking into account the vanishing of the πNN vertex at $t = 0$ and the smallness of the pion mass, we parametrize the pion contribution to $d^2\sigma/dt\, d(M^2/s)$ as

$$\frac{d^2\sigma}{dt\, d(M^2/s)} = A\, e^{at} \frac{|-t|}{(t-\mu^2)^2} \left(\frac{s}{M^2}\right)^{2\alpha_\pi(t) - \alpha_P(0)} \tag{3}$$

where μ is the pion mass, $\alpha_\pi(t) = \alpha'(t - \mu^2)$, $\alpha' = 0.9\,\mathrm{GeV}^{-2}$, $\alpha_P(0) = 1$ and the cut-off parameter a is introduced to account for off mass-shell corrections. From the experimental transverse momentum distribution in $p + p \to p + X$ [12] one can estimate the parameter a. The overall normalization factor A will be determined below.

From eqs. (2b) and (2c) one can immediately see that eq. (3) leads to a limiting distribution.

We shall also use the following distribution function (written in the limiting form)

$$\rho(x, q_T) = \frac{E}{\sigma_T(pp)} \frac{d^3\sigma}{d^3 \boldsymbol{q}} = \frac{1}{\pi\, \sigma_T(pp)} \frac{d^2\sigma}{dt\, d(M^2/s)} \tag{4}$$

with $\sigma_T(pp) \approx 40$ mb.

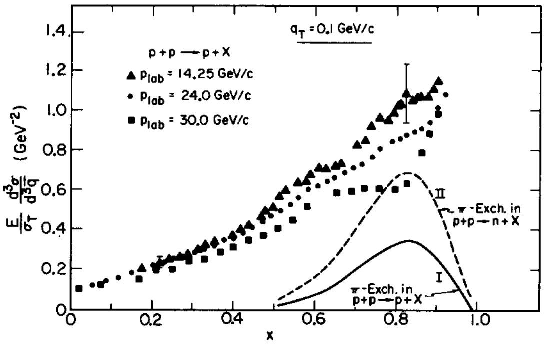

Fig. 2. Experimental data from p + p → p + X measurements and prediction for p + p → n + X (see curve II and in the text).

(i) p + p → p + X: The normalization factor A is determined by extrapolating eq. (3) to the pion mass-shell. Then the resulted pion contribution in p + p → p + X is

$$\frac{d^2\sigma}{dt\, d(M^2/s)} = \frac{1}{4\pi} \frac{g^2}{4\pi}\, \sigma_T(\pi p) \exp\{a(t - \mu^2)\} \frac{|-t|}{(t - \mu^2)^2}$$

$$\times (s/M^2)^{2\alpha_\pi(t) - \alpha_P(0)} \tag{5}$$

where the on mass-shell quantities, $g^2/4\pi \approx 15$ and $\sigma_T(\pi p) \approx 30$ mb are self-explanatory. The cut-off parameter we use * is $a \approx 4\,\mathrm{GeV}^{-2}$.

The experimental data in p + p → p + X is plotted ** in fig. 2 for $\rho(x, q_T)$ as a function of x for fixed q_T ($q_T = 0.1$ GeV). In the same figure curve I gives the pion contribution. As is expected the pion will not account for the whole spectrum since P' and ω exchanges are known to be important especially very near $x \lesssim 1$ where the pion term is vanishingly small. However, the pion exchange starts to be important at $x \approx 0.88$ where it gives ~ 35% of the spectrum and it is maximum at $x \approx 0.835$ where it accounts for ~ 50% of the distribution (at least as the 30 GeV/c data is concerned).

It is interesting to note that the π, P', and ω contributions are incoherent in the limiting distribution, namely because of iso-spin and G-parity the interference terms die out in the scaling limit. The ρ and A_2 trajectories are known to be weakly coupled to $N\bar{N}$ and we neglect them.

* The value of a is somewhere between 4 and 6 GeV^{-2}. For concreteness we take $a = 4\,\mathrm{GeV}^{-2}$.

** Thanks are due to K. Schlüpman for providing us the data of ref. [13], replotted in a normal scale.

Although the pion exchange was shown to be important in p + p → p + X, it is perhaps not crucial for the understanding of the spectrum structure †. On the other hand, the pion contribution is crucially important in p + p → n + X as discussed below.

(ii) p + p → n + X: Here the possible exchanges are ρ, A_2, and π. As was mentioned above the ρ and A_2 contributions will be neglected, as a first approximation, at least in the region where the π term has a *considerable* magnitude. Possible effects of ρ and A_2 exchanges will be mentioned later.

The reason for the crucial importance of the π-exchange in p + p → n + X as compared with p + p → p + X is simply due to isospin, i.e., $g^2_{\pi^+ pn}/4\pi = 2g^2_{\pi^0 pp}/4\pi \approx 30$. The pion contribution to the distribution of the outgoing neutron is plotted in fig. 2 and is given by curve II (curve I multiplied by 2 gives curve II). Excluding the region $0.94 \lesssim x \lesssim 1$, the distribution given by curve II *should give*, to a good approximation, the experimental spectrum of p + p → n + X. Our prediction for p + p → n + X shows in what region of phase space neutrons are expected to be *copiously produced* (relative to protons) in inelastic final states in pp collisions.

It is interesting to note that there is a possibility of a *cross-over* between the p + p → p + X and p + p → n + X distributions near $x \approx 0.83$ (see fig. 2).

The ρ and A_2 trajectories, having a half-unit intercept, lead to a constant contribution near $x \lesssim 1$, which can dominate the pion term in the region $0.94 \lesssim x \lesssim 1$. Therefore their effect in p + p → n + X is to fill up that small region near $x \lesssim 1$ in which the pion contribution is very small. However, the ρ and A_2 being weakly coupled to $N\bar{N}$, it is predicted that a significant *dipping* ‡‡ of the p + p → n + X distribution will occur toward $x \lesssim 1$. Such phenomenon is *not* present in p + p → p + X since here the constant behavior near $x \lesssim 1$ is obtained from P' and ω exchanges which couple strongly to $N\bar{N}$.

Inclusion of A_2 leads to an interference term (with the pion) which contributes in the limit of scaling. It vanishes at $x \lesssim 1$ and will slightly affect the p + p → n + X spectrum away from $x \lesssim 1$.

Composition of σ_T(pp): It is of interest to see in what way the total pp cross-section is built from the various inclusive distributions in high energy pp collision. Describing the inclusive spectra of the emitted nucleons as in fig. 1 ‡, it is of importance to examine the amount of the various exchanges in the asymptotic absorptive forward amplitude.

Conservation of energy [18] and the identity # of the colliding particles lead to the sum rule

$$\sigma_T(\mathrm{pp}) = \sum_i \int \frac{\mathrm{d}^2\sigma_i}{\mathrm{d}t\,\mathrm{d}(M^2/s)} (1 - M^2/s)\,\mathrm{d}t\,\mathrm{d}(M^2/s) \quad (6)$$

where the integration is taken over only *half* of the phase space and the summation is on the various *types* of the outgoing stable particles. The factor $(1 - M^2/s)$ stems from $2E/\sqrt{s}$) [see eq. (2a)]. Note that the elastic peak is included in the right-hand side of eq. (6).

Let us discuss the role played by π-exchange in sum rule (6). The pion trajectory contributes to i = p, n, namely to p + p → p + X and p + p → n + X. Substituting the pion term in p + p → p+X, given by (5), to (6) one obtains (the integration is from $t = 0$ to ∞ and $M^2/s \approx 0$ to 1);

$$\frac{1}{4\pi}\frac{g^2}{4\pi}\sigma_T(\pi\mathrm{p})\exp(-a\mu^2) \times \times \Big\{\tfrac{1}{6}[\exp(a\mu^2)(1+a\mu^2)E_1(a\mu^2) - 1] - \tfrac{1}{2}\exp\Big(\frac{a}{\alpha'}\Big)E_1\Big(\frac{a}{\alpha'}\Big) + \tfrac{1}{3}\exp\Big(\frac{3}{2}\frac{a}{\alpha'}\Big)E_1\Big(\frac{3}{2}\frac{a}{\alpha'}\Big)\Big\}$$

leading ‡‡‡ to a contribution of ~ 5 mb ##. Since π-exchange in p + p → n + X is *twice* stronger than in p + p → p + X, it is estimated to account for ~ (10.5-15) mb of the total cross-section. It

† For example, in the 30 GeV/c data the flat structure near $x \lesssim 0.81$ (see fig. 2) indicates the presence of exchanges with half-unit intercept, namely ω and P'. Also in view of the large experimental errors (a typical error-bar is shown in fig. 2) it is difficult to draw decisive conclusions on the various exchanges.

‡‡ Low-lying trajectory contributions will always dip toward $x \lesssim 1$. However, the very rapid fall off of curve II (fig. 2) toward $x \lesssim 1$ is a unique feature of pion exchange and is the result of the smallness of the pion mass and the relatively strong π^+pn vertex.

‡ Such an approximate approach will be applied only for the detected nucleons since they are produced mainly far from the central region (in the c.m. system). The possibility that the diagrams, as the one in fig. 1, will give on the average a good description make the approach even more reliable for predictions from integrated quantities (see text).

When the incident particles are not identical the sum-rule (6) can be derived by imposing also conservation of longitudinal momentum at $s \to \infty$.

‡‡‡ The exponential integral function $E_1(z)$ is defined by

$$E_1(z) = \int_z^\infty \frac{\exp(-v)}{v}\,\mathrm{d}v.$$

Footnote see next page.

is indeed a considerable fraction of the total inelastic cross-section.

Finkelstein and Rajaraman [7] estimated the P' and ω exchanges in p + p → p + X to give ~ 10 mb # to the right-hand side of eq. (6). Taking into account the elastic cross-section, $\sigma_{el}(pp) \approx (8\text{-}10)$ mb, the amount due to protons is ~ (18.5-25) mb and to neutrons it is ~ (7-10) mb making together ~ (25.5-35) mb. The experimental total cross-section is ~ 40 mb and the rest [~ (5-14.5) mb] is mainly from emitted pions.

Pomeron-particle amplitudes: Plotting [4, 5, 14, 15] $d^2\sigma/dt\,dM$ versus $1/s$ and extrapolating to infinite s the obtained intercept gives a measure for the absorptive forward Pomeron-particle amplitude. The present work shows that pion exchange can be important in p + p → p + X at very small values of t (e.g., $t \approx -0.04$ GeV^2). Therefore in order to have a more meaningful parametrization for $d^2\sigma/dt\,dM$ (s/M^2 not too large) one needs to include, in addition to a constant and $1/s$ terms, also $1/s^2$ behavior. More measurements at different incident energies are needed to clarify the predictions of such analysis.

To summarize, an approximate spectrum was *predicted* for p + p → n + X in the limit of scaling. Corrections to the spectrum very near $x \lesssim 1$ were discussed. Very near $x \lesssim 1$ more protons will be produced than neutrons in pp collision. However, away from the boundary, e.g., $x \approx 0.84$, their number can be *comparable* and the presence of a *cross-over* between p + p → p + X and p + p → n + X is well possible.

In trying to build $\sigma_T(pp)$ from inclusive spectra it turns out that pion exchange [19] contributes about ~(10.5-15) mb. This relatively large number is not surprising. Indeed every exchange, irrespective of its intercept, gives rise to a limiting distribution and therefore to a constant (independent of s) term in sum-rule (6). This constant term is particularly large for pion exchange due to the very small mass and relatively strong coupling to $N\bar{N}$.

We are also led to the expectation that, for $d^2\sigma/dt\,dM$, the background [20] in p + p → n + X will have the dominant behavior of $\sim M^3$ (t small), which is very different from the one in [20] p + p → p + X.

The contribution of ~ 5 mb should be considered as an overestimate since the integration range contains also high missing mass values where eq. (5) is not locally applicable. As the factor $(1 - M^2/s)$ in (6) de-emphasize high values of M, an uncertainty of $\lesssim 30\%$ is assumed.

Even preliminary data on p + p → n + X can test the validity of the present approach, in particular the prediction of the *very sharp rise* of the spectrum away from $x \lesssim 1$.

Finally we remark that similar conclusions are valid for processes of the form b + p → N + X where the nucleon N (=p, n) is slow in the lab. system and b = $K^{\pm}, \pi^{\pm}, \gamma$ (real or virtual) etc.

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