

# Running of the Charm-Quark Mass from

## HERA Deep-Inelastic Scattering Data



$\alpha_s$

$m_b$

$m_c$

DIS 2017, Birmingham, UK, 5 April 2017

Achim Geiser  
DESY Hamburg

work partially done  
within scope of

PROSA, ZEUS and H1  
collaborations



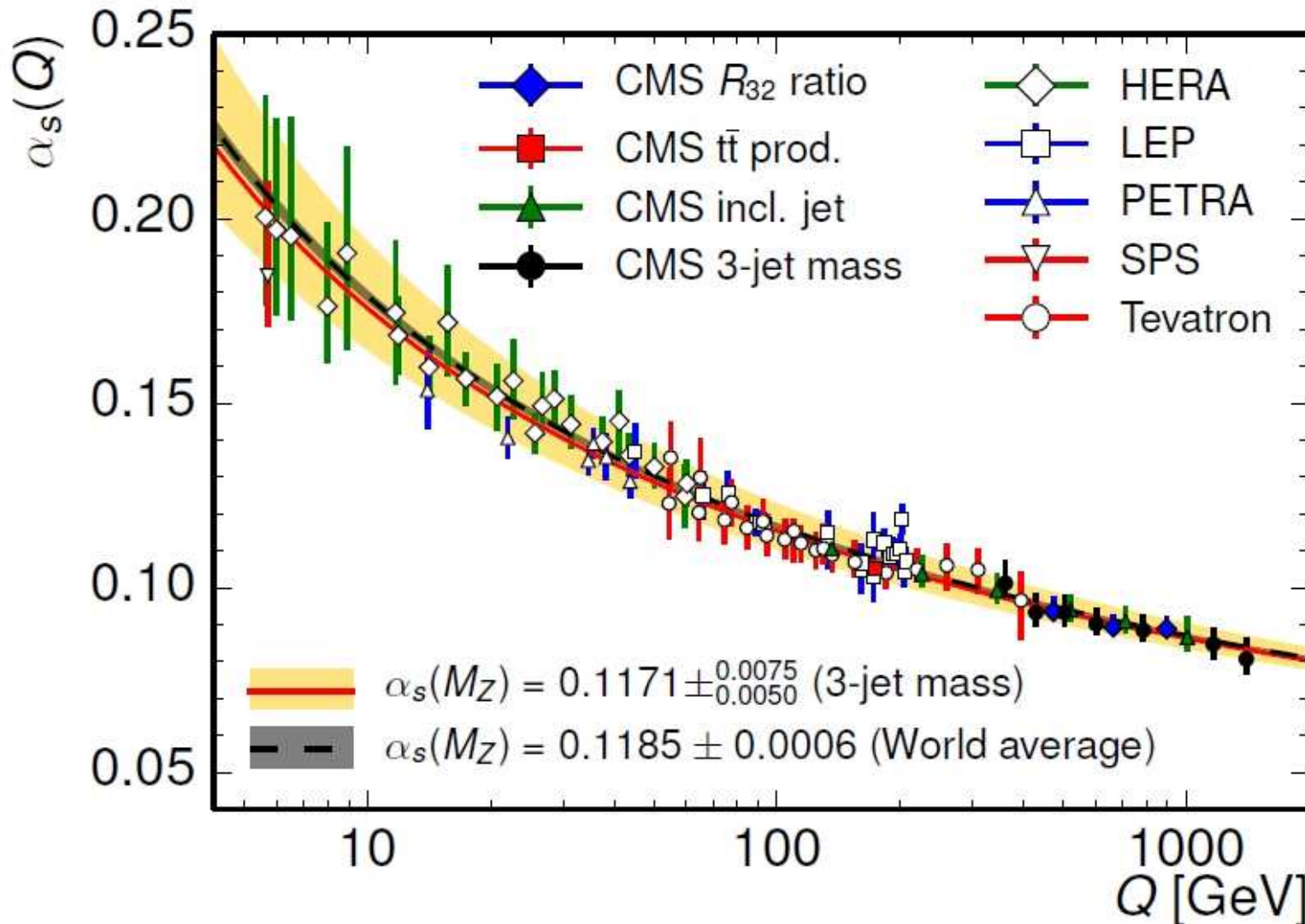
- Introduction: running of  $\alpha_s$  and  $m_b$
- Final results on charm mass running: DESY-17-048 (on arXiv soon)
- Interpretation in terms of Higgs Yukawa couplings: PoS CHARM2016 (2017) 012

# Running of strong coupling „constant“ $\alpha_s$

reminder

EPJC 75 (2015) 186

e.g. from jet production at  $e+e^-$ ,  $ep$ , and  $pp$  at DESY, Fermilab and CERN



updates see talks  
K. Rabbertz  
and D. Britzger

**Yes,  
it runs!**

# Running of $\alpha_s$ and quark masses $m_Q$

- $\alpha_s$  running depends on number of colours  $N_c$  and number of quark flavours  $n_f$

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s \times (11N_c - 2n_f)/12\pi \ln(\mu^2/\mu_0^2)}$$

- quark mass running depends on  $\alpha_s$ , e.g.

$$\begin{aligned} m_Q(\text{pole}) &= m_Q(m_Q) (1 + 4/3 \alpha_s/\pi) \\ &= m_Q(\mu) (1 + \alpha_s/\pi (4/3 + \ln(\mu^2/m_Q^2))) \end{aligned}$$

leading order QCD formulae

or

$$m_Q(\mu) = m_Q(m_Q) \times \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{c_0} \quad c_0 = 4/(11 - 2n_f/3) = 4/9 \quad \begin{matrix} N_c \\ n_f = 3 \end{matrix}$$

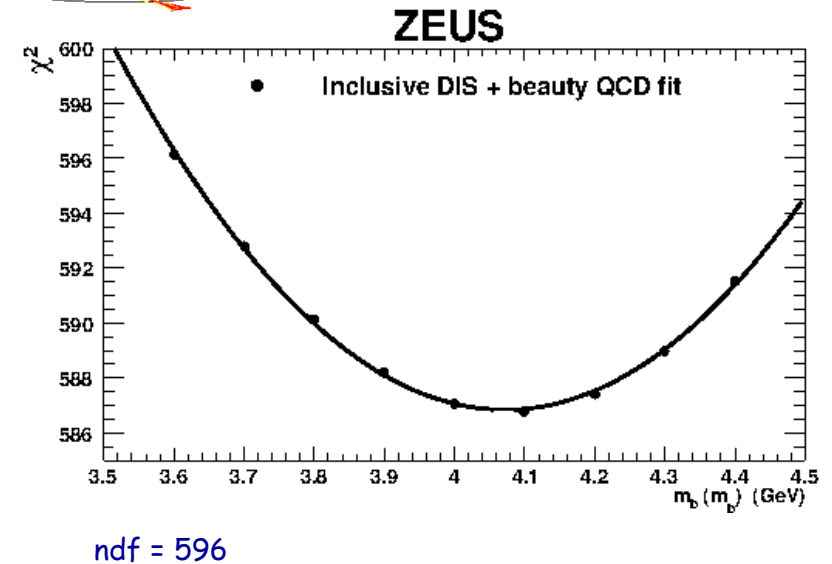
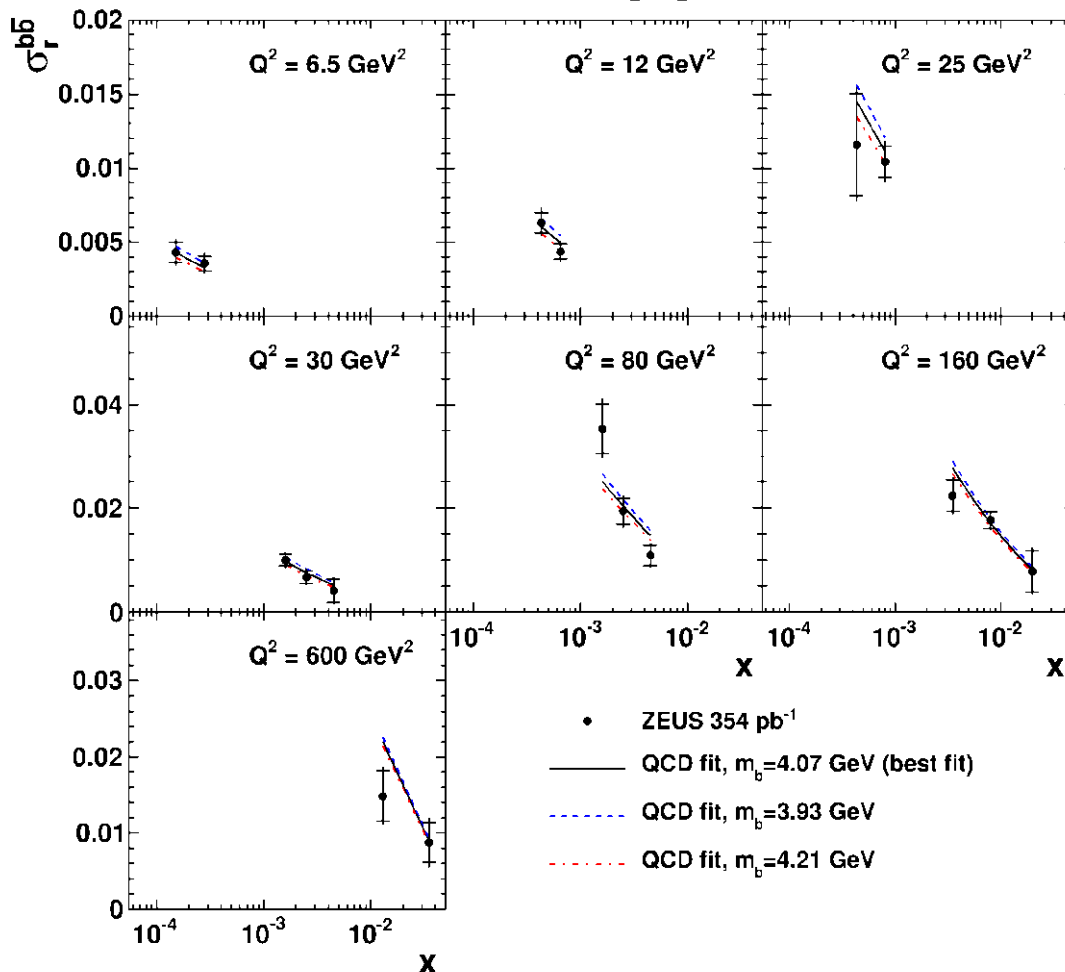
- part of gluon field around quark not 'visible' any more when 'looking' at smaller distances/larger energy scales  
-> **effective quark mass decreases**

# $m_b$ from reduced beauty cross section in DIS

reminder

JHEP 1409 (2014) 127

## ZEUS



update see talk  
O. Zenaiev

$$m_b(m_b) = 4.07 \pm 0.14_{\text{fit}} \quad +0.01 \quad -0.07_{\text{mod}} \quad +0.05 \quad -0.00_{\text{par}} \quad +0.08 \quad -0.05_{\text{th}} \quad \text{GeV}$$

PDG:  $4.18 \pm 0.03 \text{ GeV}$  (lattice QCD + time-like processes)

# The running beauty quark mass



ZEUS, JHEP 1409 (2014) 7;

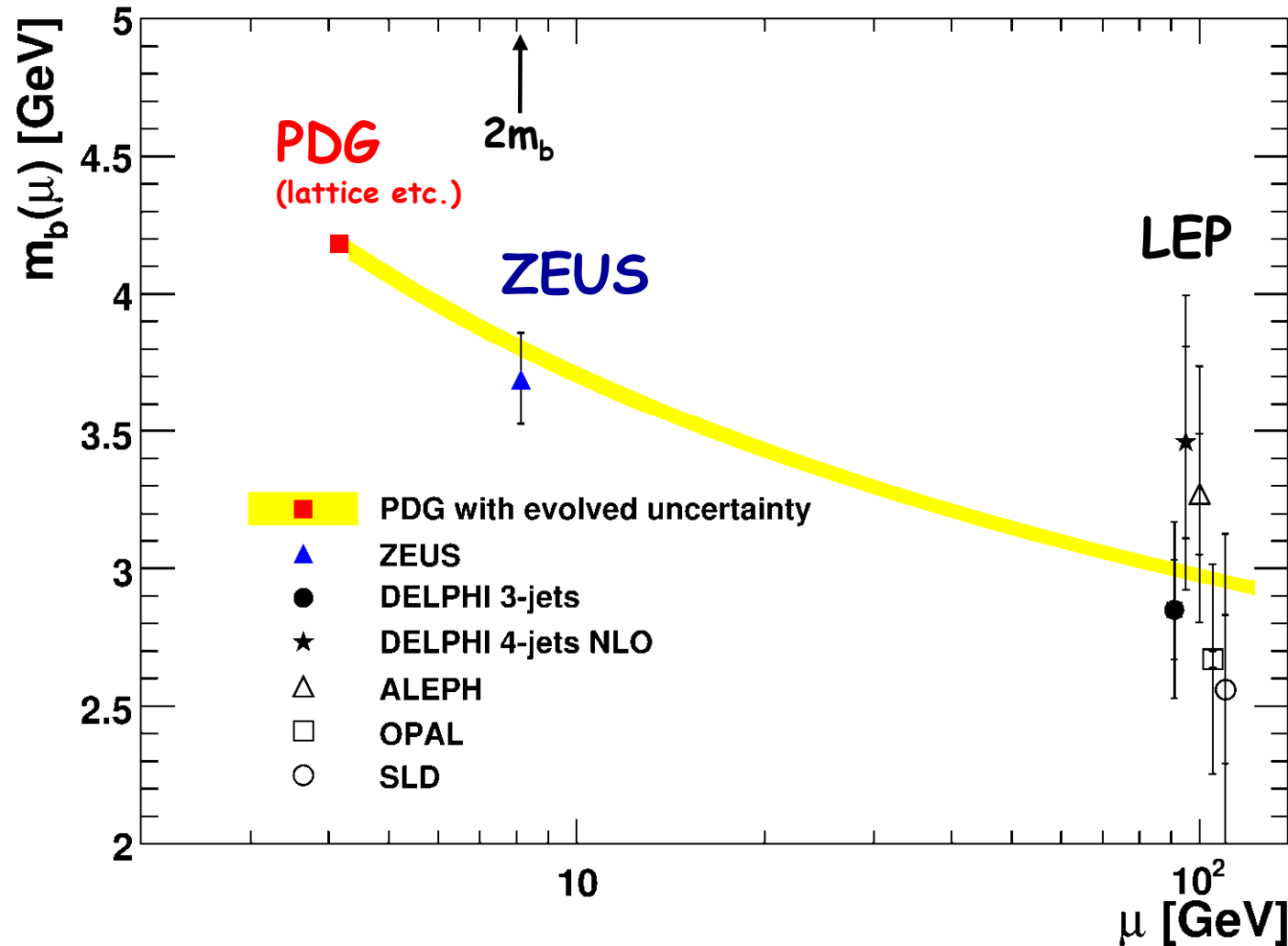
review, arXiv:1506.07519

LEP, Eur. Phys. J. C55 (2008) 525

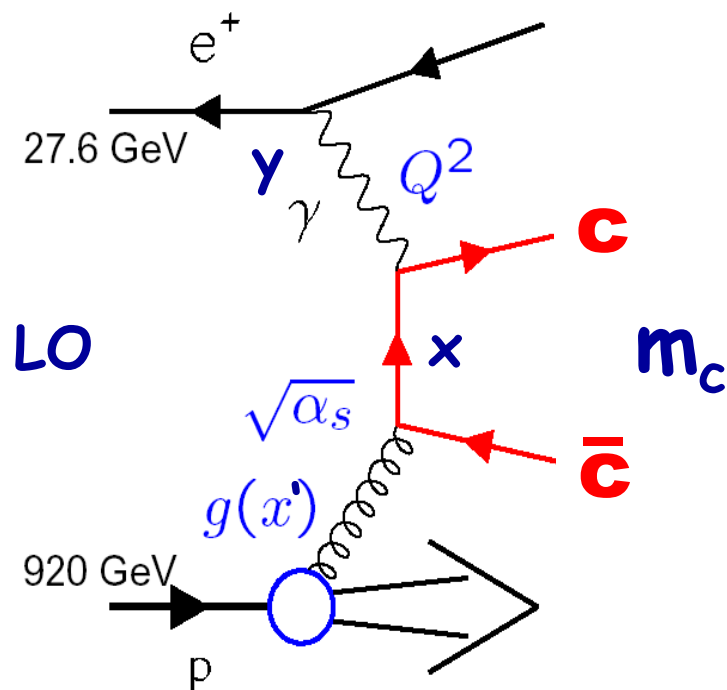
Prog. Part. Nucl. Phys. 84 (2015) 1

translate to  $2m_b$

ZEUS



# Fixed Flavour Number Scheme (FFNS)



+ NLO (+partial NNLO) corrections,

“natural” scale:  
 $\mu^2 = Q^2 + 4m_c^2$

- no charm in proton

- full kinematical treatment of charm mass ☺

(multi-scale problem:  
 $Q^2, p_T, m_c \rightarrow$  logs of ratios)

- no resummation of logs ☹

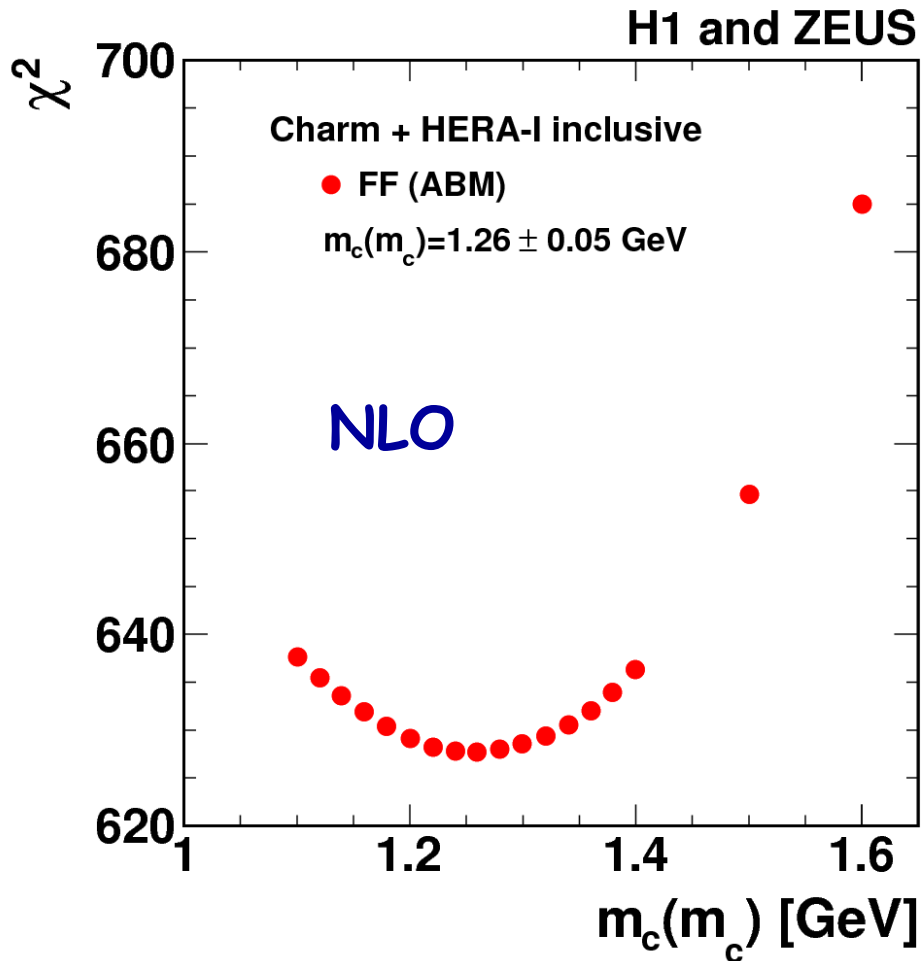
- no extra matching parameters ☺



# Measurement of $\overline{MS}$ charm mass

reminder

EPJ C73 (2013) 2311



simultaneous mass + PDF fit of combined charm data and inclusive HERA I DIS data



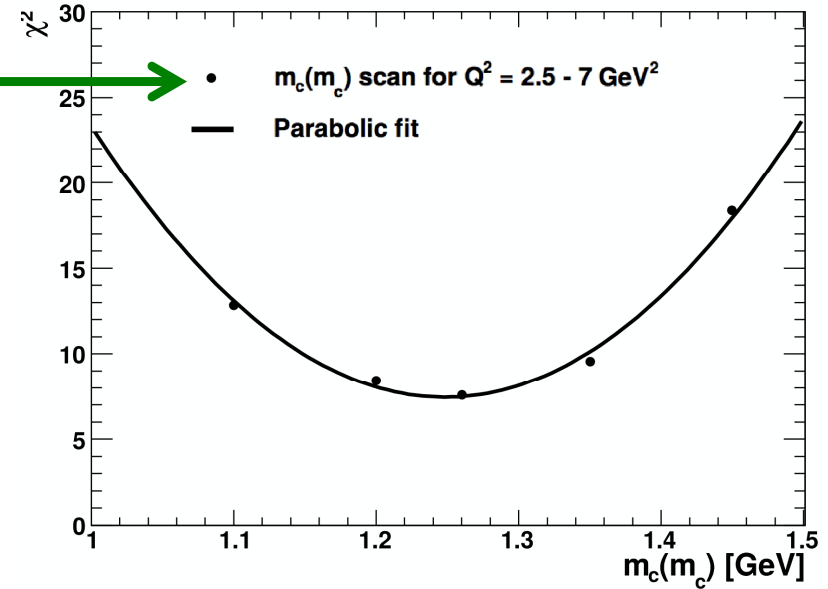
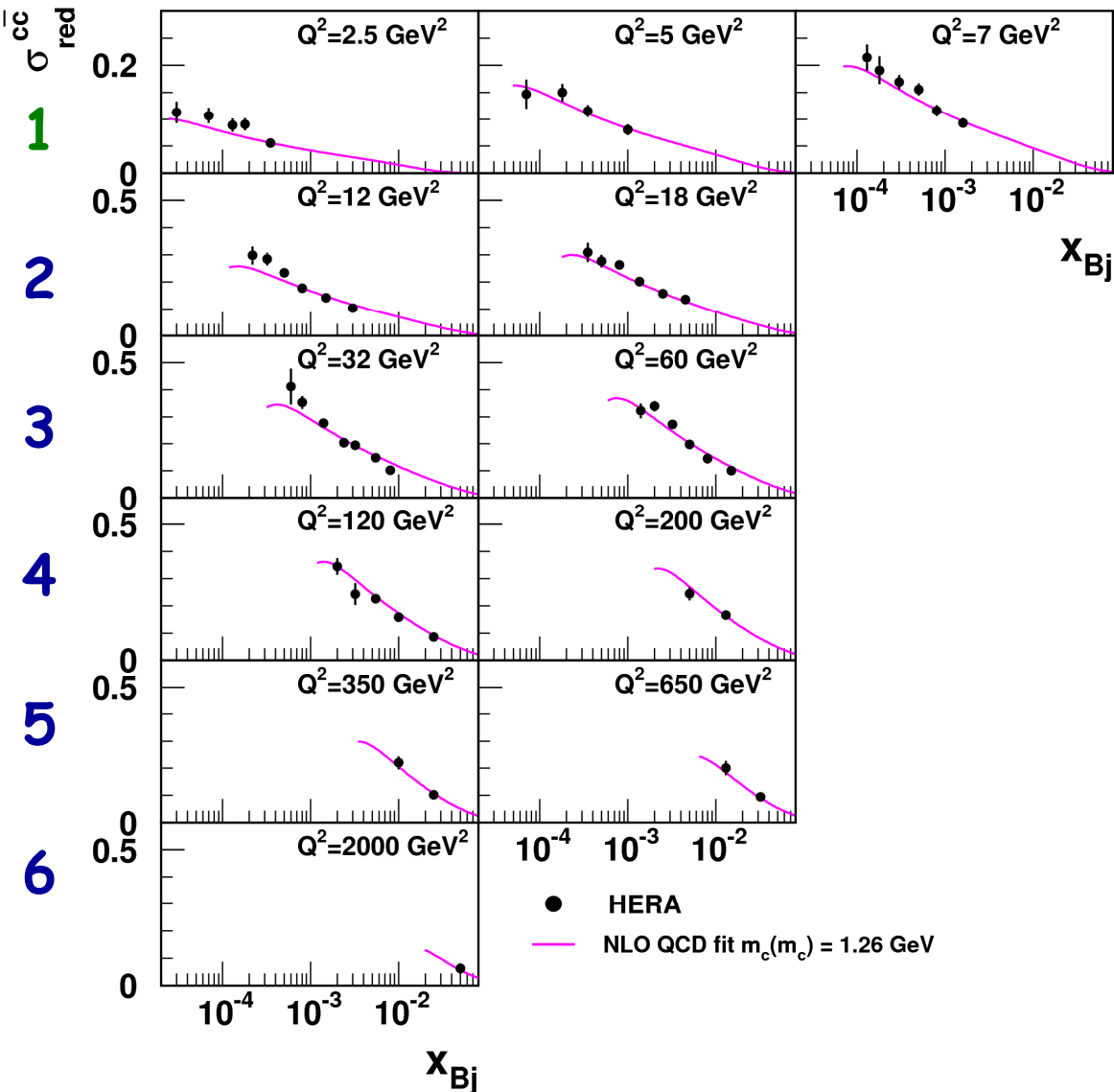
update see talk O. Zenaiev

$$m_c(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\alpha_s} \text{ GeV}$$

PDG:  $1.275 \pm 0.025 \text{ GeV}$  (lattice QCD + time-like processes)

# Measurement of $m_c$ running

A. Gizhko et al., DESY-17-048



**Step 1:**  
extract  $m_c(m_c)$  separately  
for 6 different kinematic  
ranges in  $\mu^2 = Q^2 + 4m_c^2$

(take log average for central scale)



# $m_c$ fit and uncertainties

A. Gizhko et al., DESY-17-048

use appropriate PDF set for each mass  
(from inclusive DIS data only),  
fit charm data

Fit uncertainty

- Was estimated by taking  $\Delta\chi^2 = 1$  (dominant uncertainty)

Parametrisation

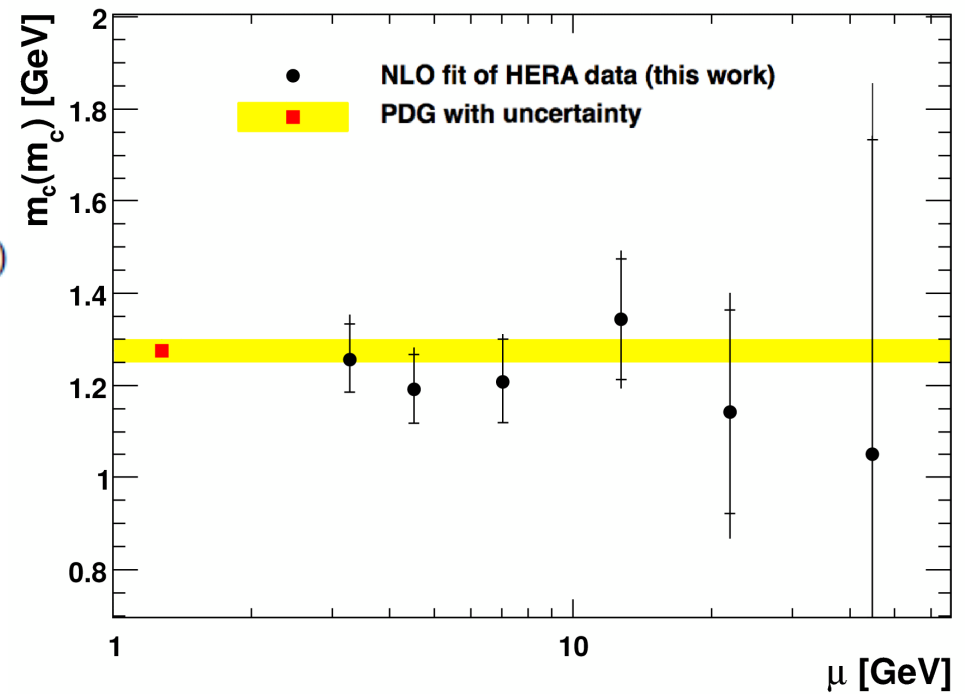
- Adding extra parameter in the PDF parametrisation

Model uncertainty

- Variation of the strangeness suppression factor
- Lower cut on  $Q^2$  for inclusive data
- The evolution starting scale
- The b-quark mass

Theory

- Variation of  $\alpha_s$
- Variation of the factorisation and renormalization scales of heavy quarks by factor 2 → outer error bar



sensitivity to  $m_c(m_c)$  decreases with increasing scale  $\mu^2 = Q^2 + 4m_c^2$

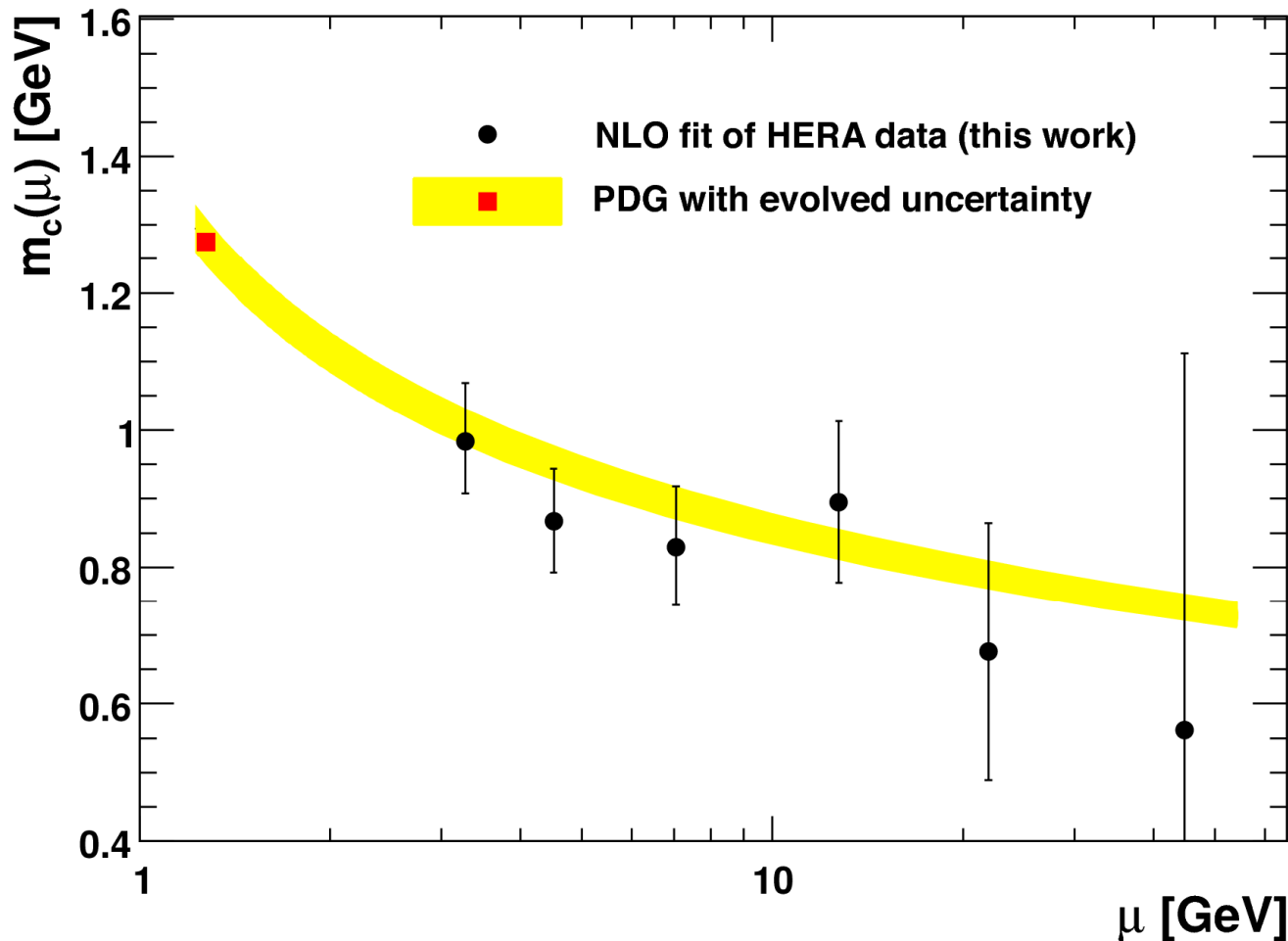
'in reality', have measured  $m_c(\mu)$  at each scale

# The running charm quark mass

A. Gizhko et al., DESY-17-048

Step 2: translate back to  $m_c(\mu)$ , which was actually measured, using LO formula consistent with NLO  $\overline{MS}$  QCD fit

(OpenQCDrad, Alekhin et al.)



running mass  
concept in QCD  
is self-consistent !

# Numerical details

Subset	$N_{\text{dat}}$	$Q^2$ range [GeV <sup>2</sup> ]	$\mu$ [GeV]	$m_c(m_c)$		$m_c(\mu)$	
				[GeV] fit	scale	[GeV] fit	
1	15	2.5–7	3.3	1.256 <sup>+0.078</sup> <sub>−0.070</sub>	<sup>+0.054</sup> <sub>−0.000</sub>	0.984 ± 0.061	
2	12	12–18	4.5	1.192 <sup>+0.075</sup> <sub>−0.073</sub>	<sup>+0.043</sup> <sub>−0.000</sub>	0.867 ± 0.055	
3	13	32–60	7.0	1.208 <sup>+0.092</sup> <sub>−0.088</sub>	<sup>+0.045</sup> <sub>−0.000</sub>	0.830 ± 0.063	
4	7	120–200	12.7	1.344 <sup>+0.130</sup> <sub>−0.131</sub>	<sup>+0.073</sup> <sub>−0.074</sub>	0.895 ± 0.087	
5	4	350–650	21.9	1.143 <sup>+0.222</sup> <sub>−0.221</sub>	<sup>+0.133</sup> <sub>−0.163</sub>	0.676 ± 0.132	
6	1	2000	44.8	1.050 <sup>+0.684</sup> <sub>−0.760</sub>	<sup>+0.400</sup> <sub>−0.149</sub>	0.562 ± 0.412	

Table 1: Values of  $m_c(m_c)$  at different scales  $\mu$ , determined from six different subsets, and corresponding values of  $m_c(\mu)$ . The first uncertainty (fit) corresponds to the uncertainty  $\delta_{\text{fit}}^{\text{exp}}$  added in quadrature with the symmetrised systematic uncertainties  $\delta_1 - \delta_6$ . The second uncertainty (scale) of  $m_c(m_c)$  corresponds to the scale variation uncertainty  $\delta_7$ . No scale uncertainty is quoted for  $m_c(\mu)$  (see text). The range of  $Q^2$  values contributing to the six data subsets shown in Fig. 1 is given. Also given is the corresponding logarithmic average scale  $\mu$  for each subset according to Eq. (2), and the number  $N_{\text{dat}}$  of charm data points contributing to each measurement.

# Breakdown of uncertainties on $m_c$

A. Gizhko et al., DESY-17-048

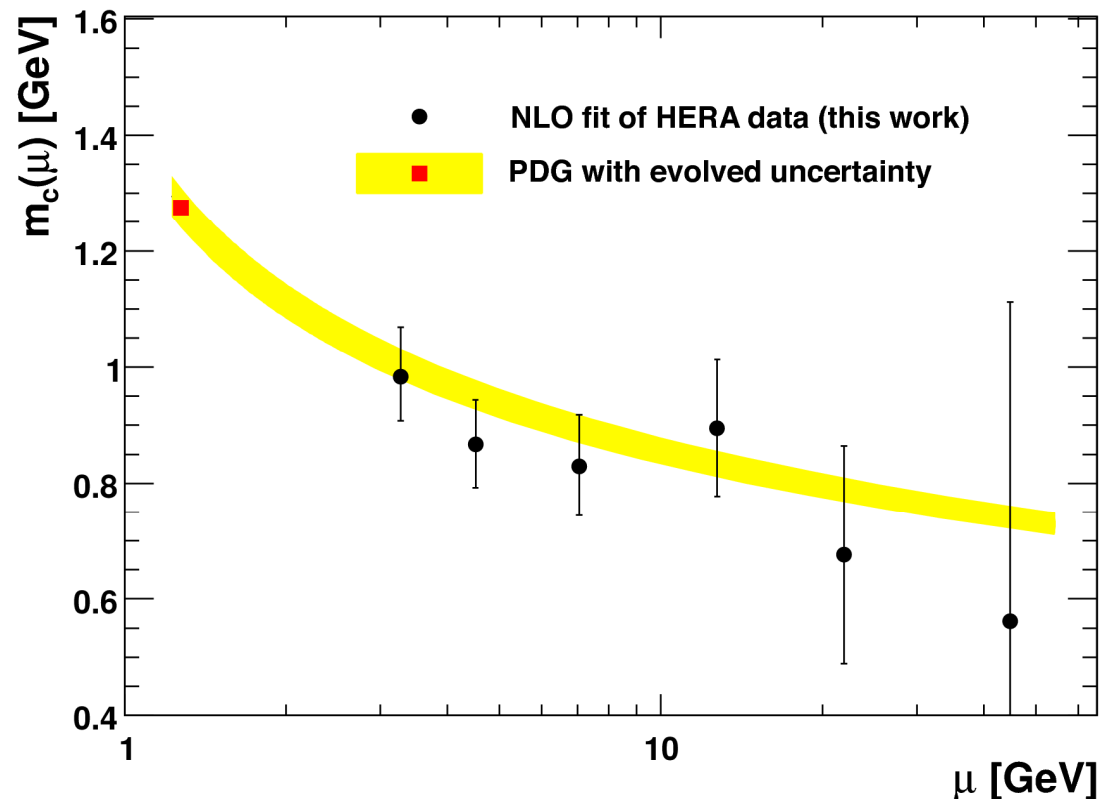
Subset	$\delta_{\text{fit}}^{\text{exp}}$	$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_5$	$\delta_6$	$\delta_7$
		$(m_b)$	$(\alpha_s)$	$(f_s)$	$(Q_0)$	$(Q_{\text{min}}^2)$	(param)	(scale)
	[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]
1	$\pm 5.4$	+0.1 -0.4	-1.2 +2.6	-0.4 +0.2	+0.5	+1.4	+0.5	+3.1 +4.3
2	$\pm 6.0$	+0.2 -0.5	-0.9 +0.7	-0.5 +0.2	+0.3	+1.0	+0.9	+2.4 +3.6
3	$\pm 7.2$	+0.3 -0.7	-0.4 +0.3	-0.8 +0.3	+1.7	+0.3	+1.8	+0.1 +3.7
4	$\pm 9.6$	+0.5 -0.8	+0.7 -0.6	-0.8 +0.5	+0.5	-1.2	+0.1	-5.5 +5.4
5	$\pm 19.2$	+0.5 -1.2	+1.6 -1.8	-1.2 +0.5	-0.5	+2.1	-1.7	-14.3 +11.6
6	$\pm 63.8$	-7.4 -2.9	+5.9 -5.7	-3.0 -7.6	+6.5	-33.3	+9.5	+38.1 -14.2

Table 2: Summary of the systematic uncertainties in the  $m_c(m_c)$  determinations. The definitions of the uncertainty sources, the meaning of the symbols in the first and second row and related details are given in the text. In cases where opposite variations of a variable yield uncertainties with the same sign, only the larger one is considered for the uncertainty combination in Table 1. Except for  $\delta_7$ , these uncertainties also apply to  $m_c(\mu)$ .

# Conclusions, part I

A. Gizhko et al., DESY-17-048

- Subdividing HERA DIS charm data into 6 kinematic intervals, running of charm-quark mass in  $\overline{MS}$  scheme has been determined for the first time (conceptually similar to running of  $\alpha_s$  from jets)
- Interplay/treatment of correlations between mass and PDF fits nontrivial, details see DESY-17-048
- Charm-quark mass running consistent with QCD



# Higgs Yukawa couplings from $m_Q$

PoS CHARM2016 (2017) 012

relate  $m_t, m_b, m_c$  to associated Higgs Yukawa couplings

LO EW (+NLO QCD) formula:

$$y_Q = \sqrt{2}m_Q/v$$

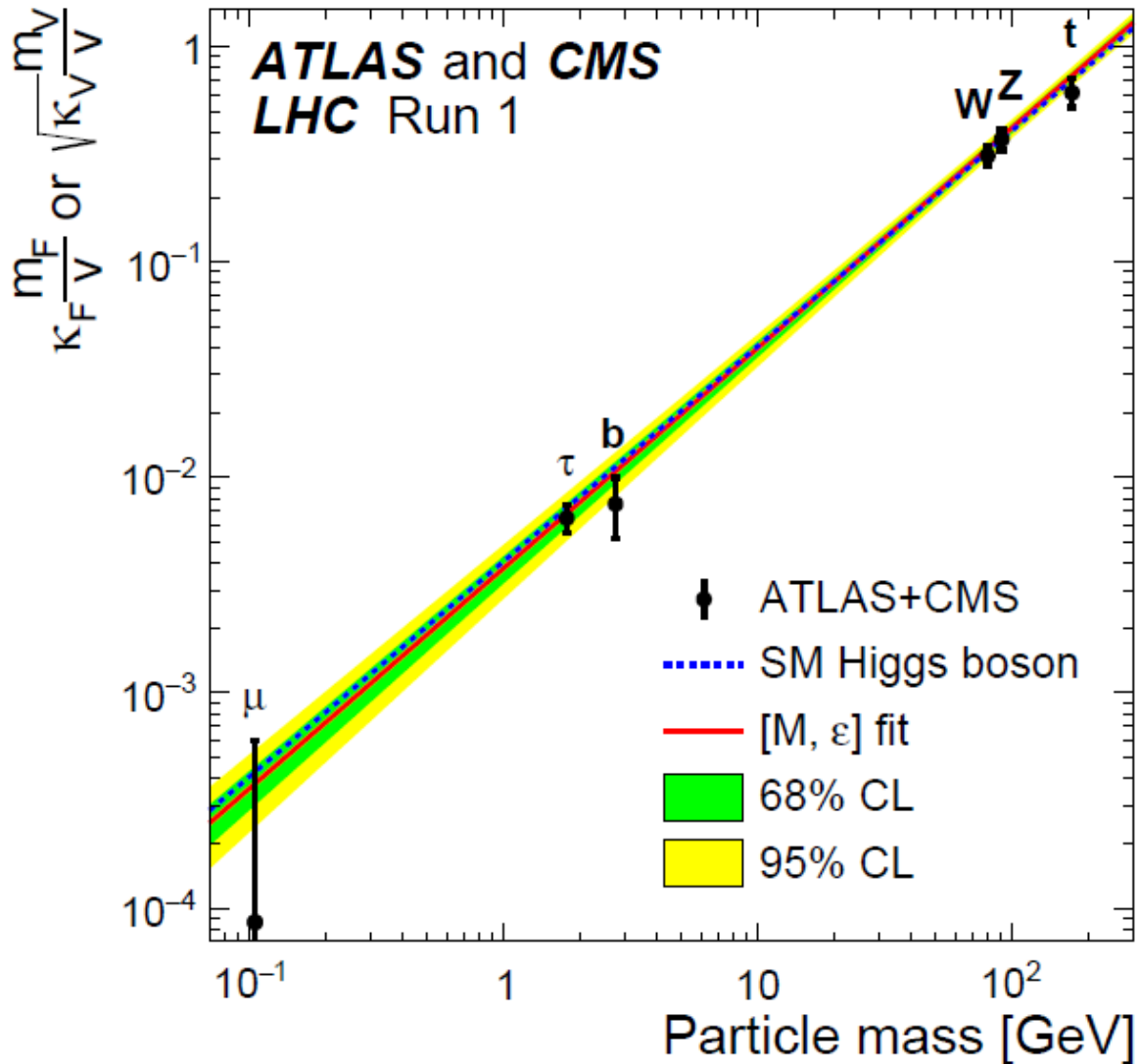
use Higgs/EW scheme in which this relation is exact !



source: viXra blog

# Direct measurements of Higgs Yukawa couplings

ATLAS and CMS, JHEP08 (2016) 045



# Running of $\alpha_s$ and quark Yukawa couplings

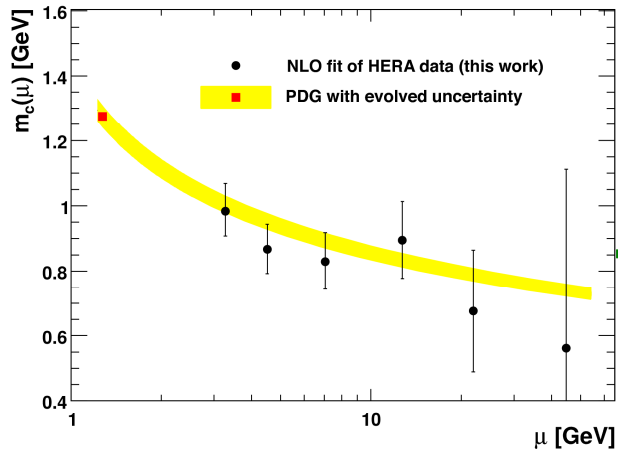
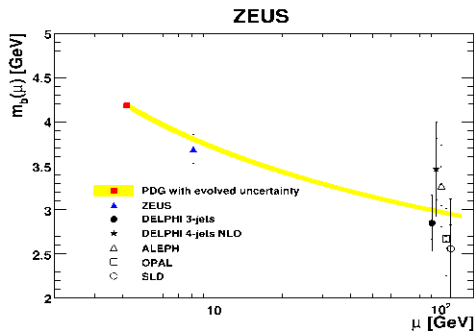
PoS CHARM2016 (2017) 012

relate  $m_t$ ,  $m_b$ ,  $m_c$  to associated Higgs Yukawa couplings

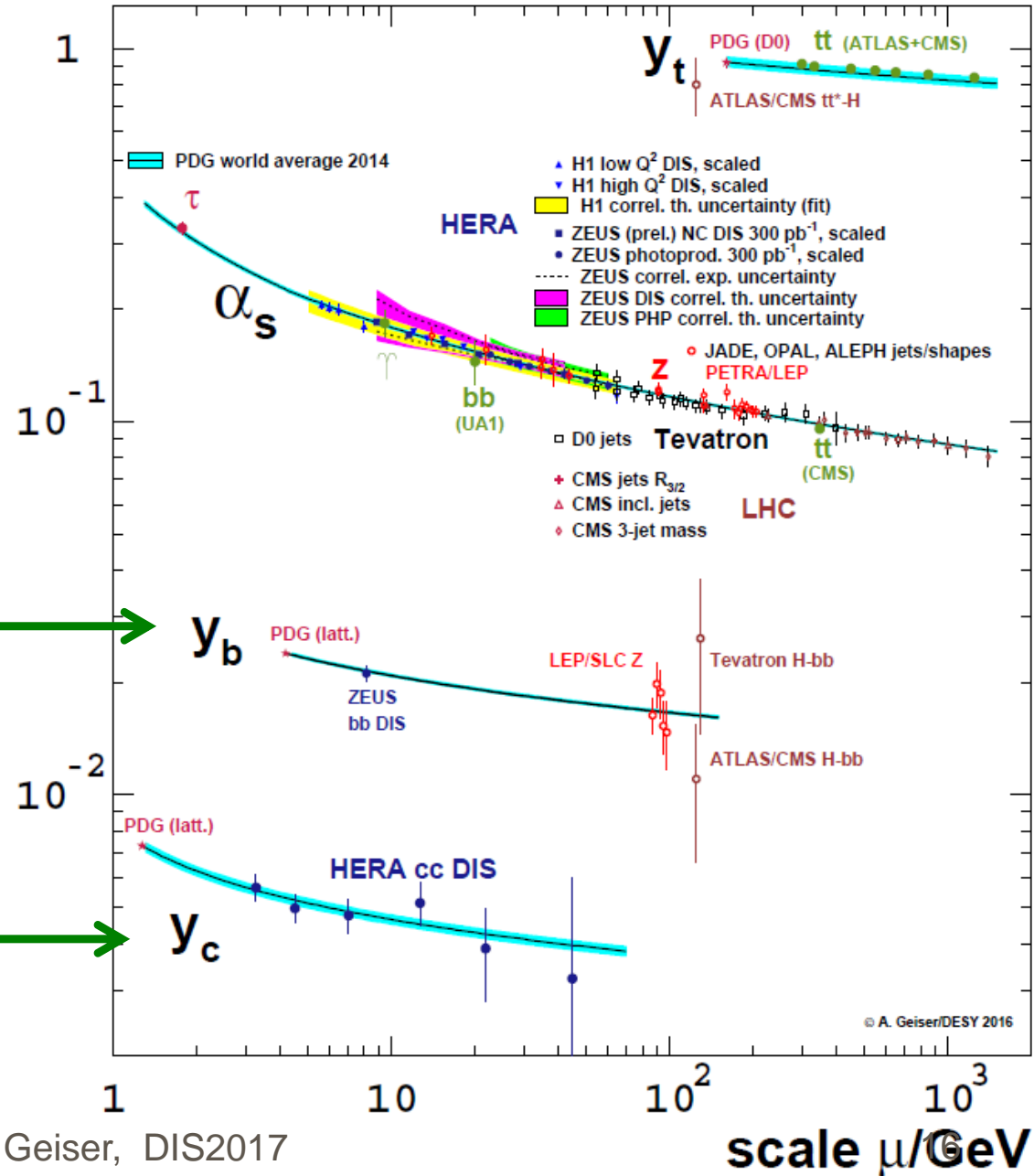
LO EW (+NLO QCD) formula:

$$y_Q = \sqrt{2}m_Q/v$$

for top see backup



running coupling





# Conclusion

## Part II

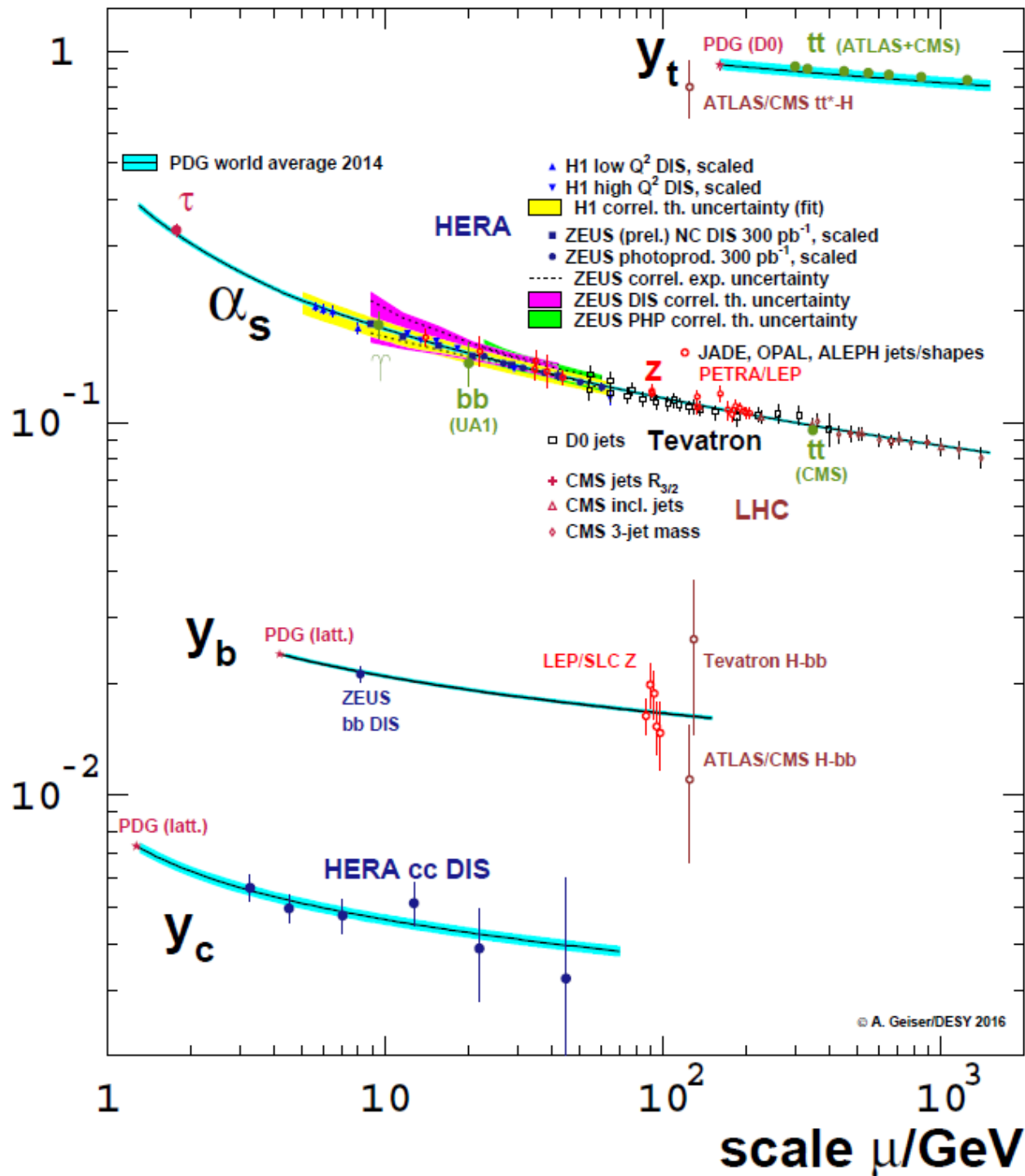
experimental  
representation of  
running Yukawa couplings  
obtained  
for the first time

heavy quark  
physics is also  
QCD + Higgs physics

so far, Higgs couplings  
and their running  
as obtained from quark  
masses are consistent  
with directly measured  
Higgs couplings

05. 04. 17

running coupling



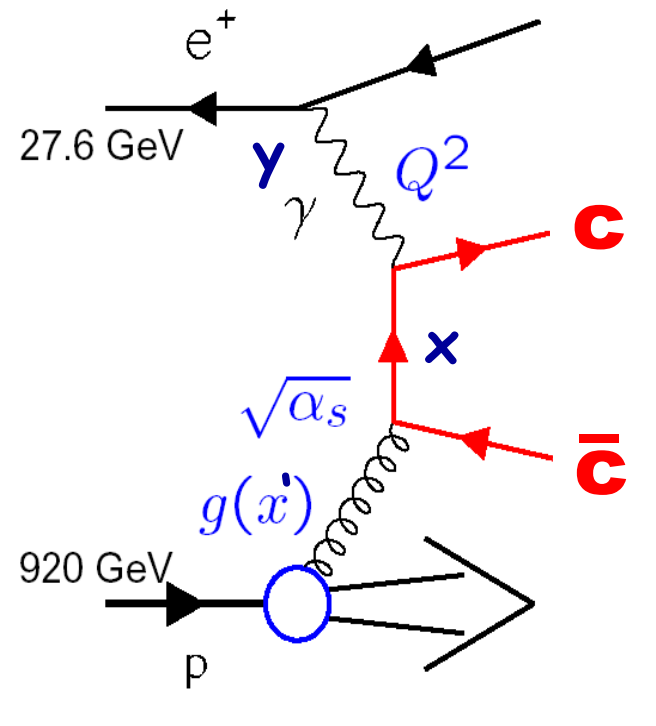
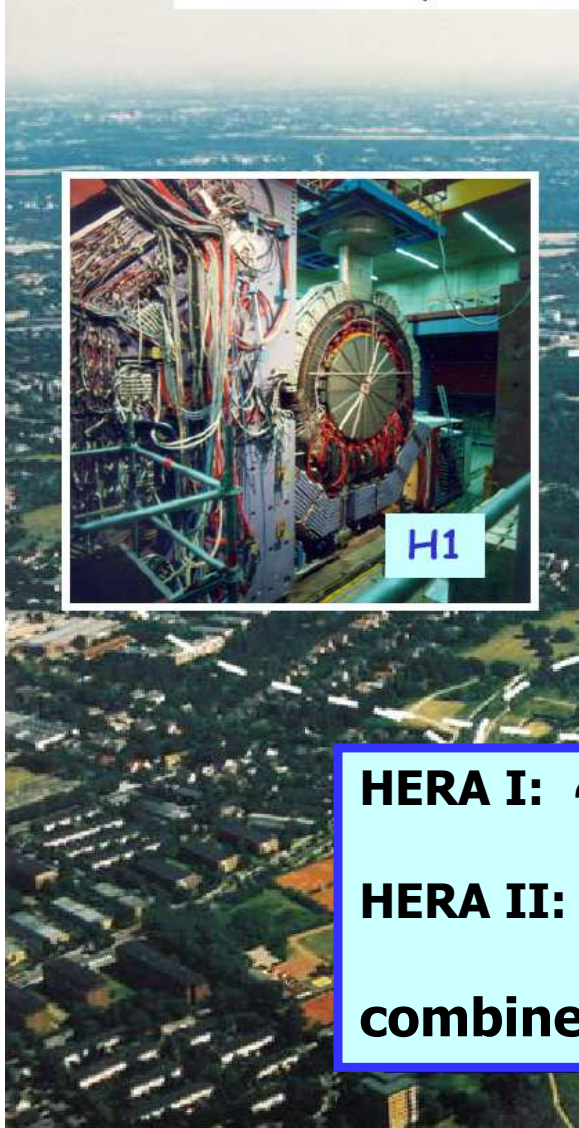
A. Geiser, DIS2017

17



# Backup

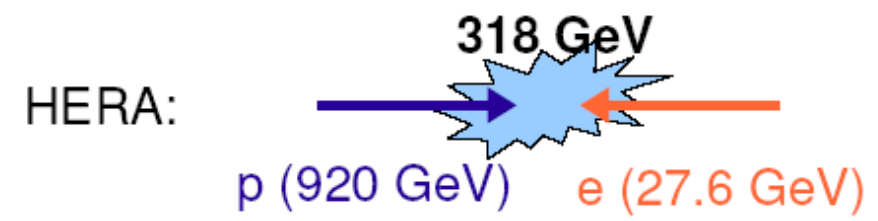
# The HERA ep collider and experiments



up to 30%  
of cross section

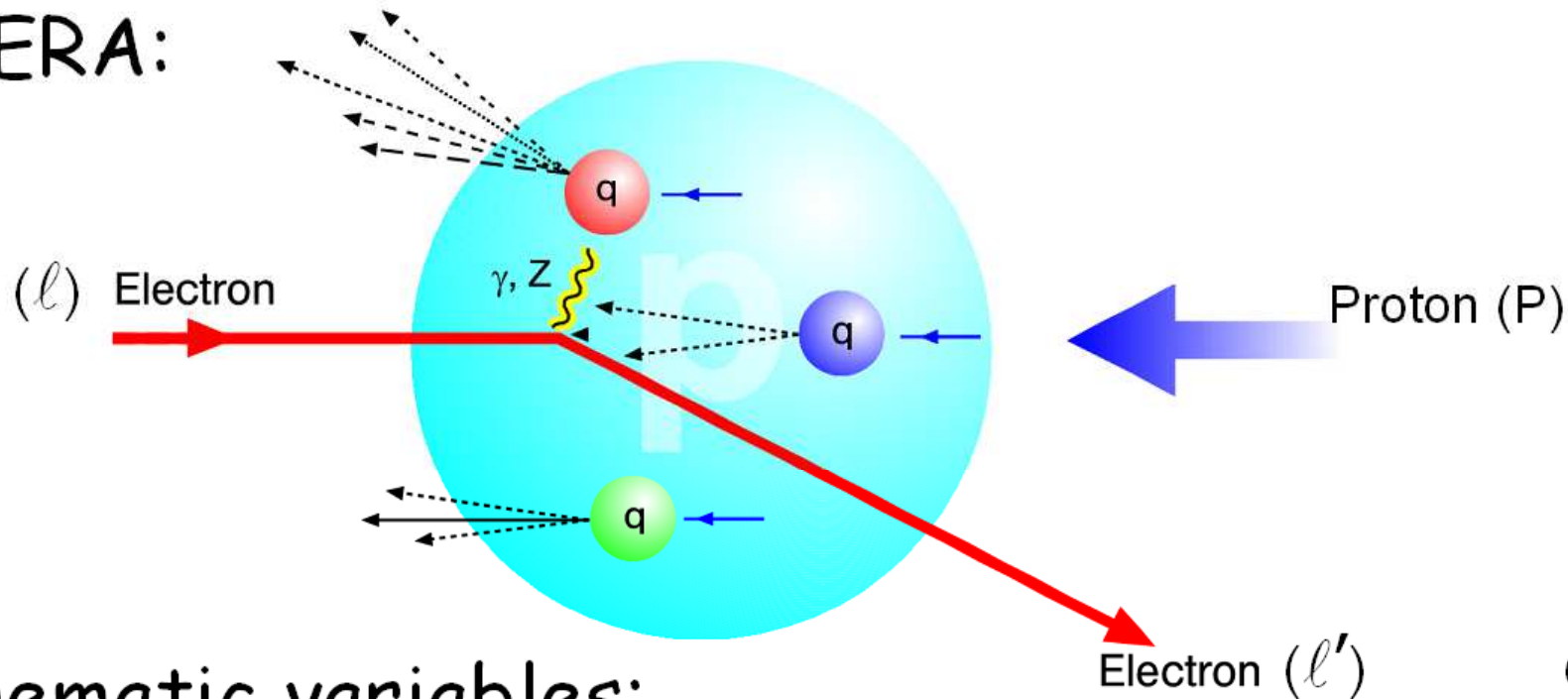


**HERA I:  $\sim 130 \text{ pb}^{-1}$  (physics)**  
**HERA II:  $\sim 380 \text{ pb}^{-1}$  (physics)**  
**combined:  $\sim 2 \times 0.5 \text{ fb}^{-1}$**



# Deep Inelastic ep Scattering at HERA

HERA:



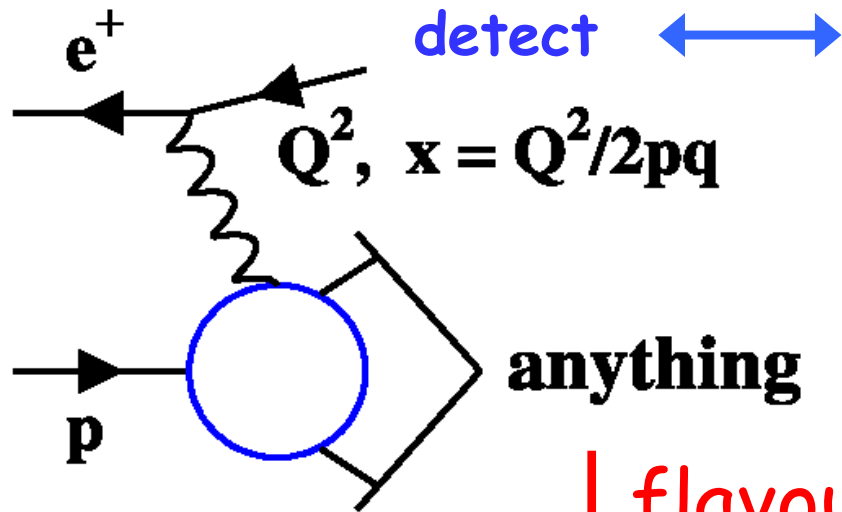
kinematic variables:

$Q^2 = -q^2$	photon (or $Z$ ) virtuality, squared momentum transfer
$x = \frac{Q^2}{2Pq}$	Bjorken scaling variable, for $Q^2 \gg (2m_q)^2$ : momentum fraction of $p$ constituent
$y = \frac{qP}{lP}$	inelasticity, $\gamma$ momentum fraction (of $e$ )

$Q^2 \lesssim 1 \text{ GeV}^2$ :  
photoproduction

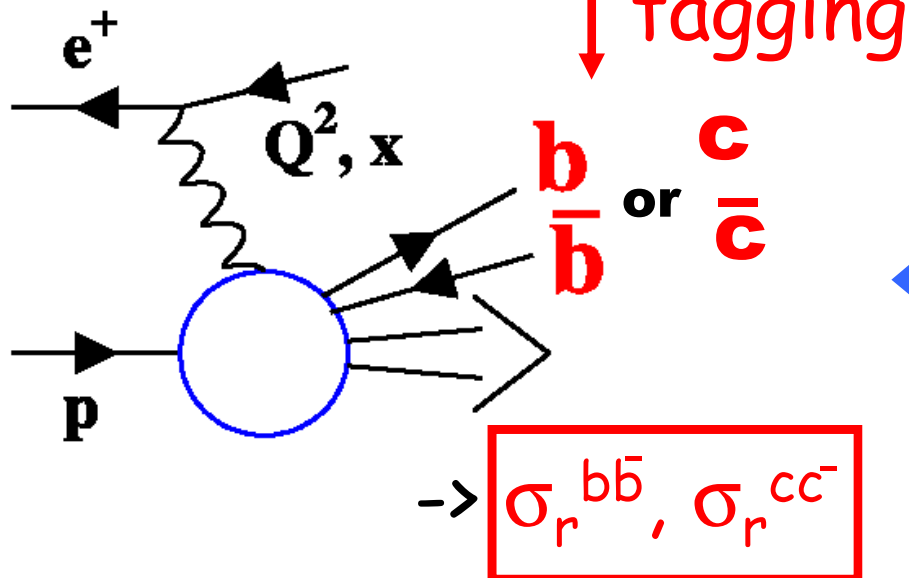
$Q^2 \gtrsim 1 \text{ GeV}^2$ :  
DIS

# Heavy flavour contributions to $F_2$

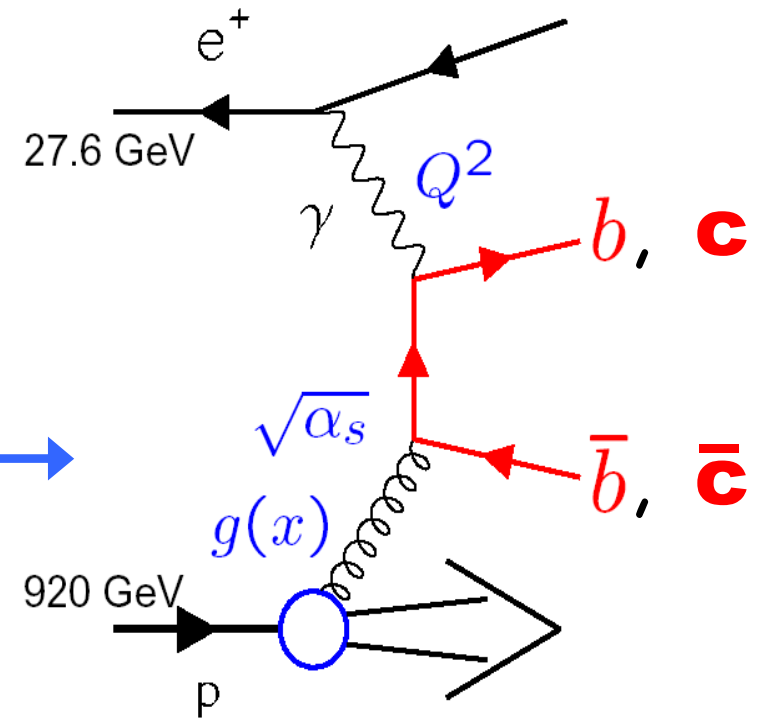


Measure cross section

$$\frac{d^2\sigma}{dx dQ^2} \approx \frac{2\pi\alpha^2}{Q^4 x} \left\{ \left[ 1 + (1-y)^2 \right] \sigma_r(x, Q^2) \right\}$$



QCD  $\longleftrightarrow$



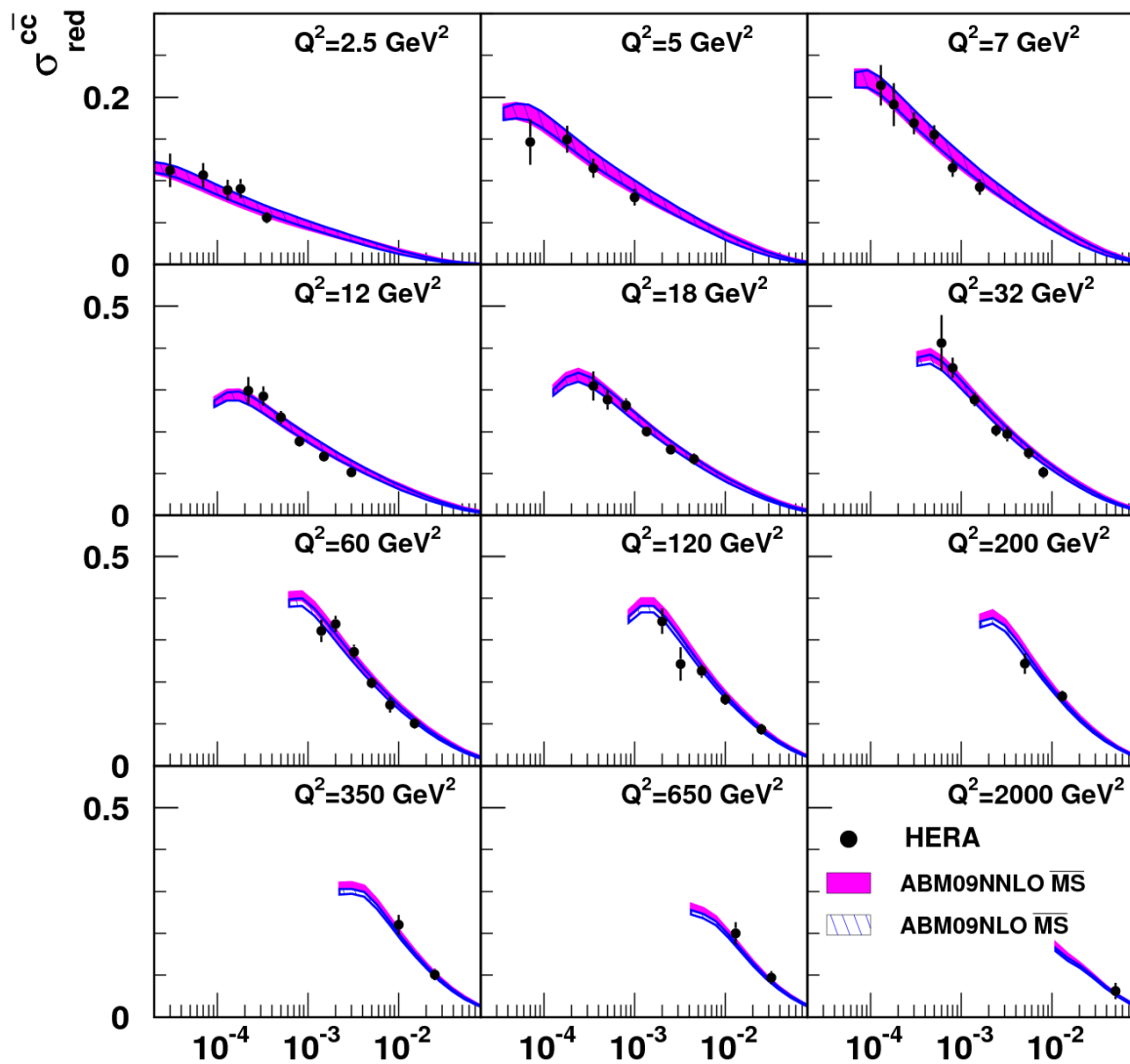


# comparison to ABM FFNS



EPJ C73 (2013) 2311

## H1 and ZEUS



very good description  
of data  
in full kinematic range

unambiguous treatment  
of  $m_c$  in all terms of  
calculation

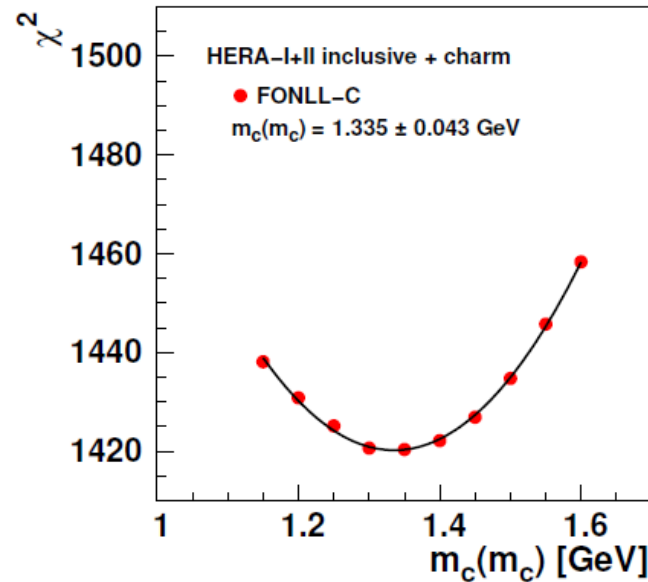
here:  $\overline{MS}$  running mass

(similar predictions for  
pole mass)

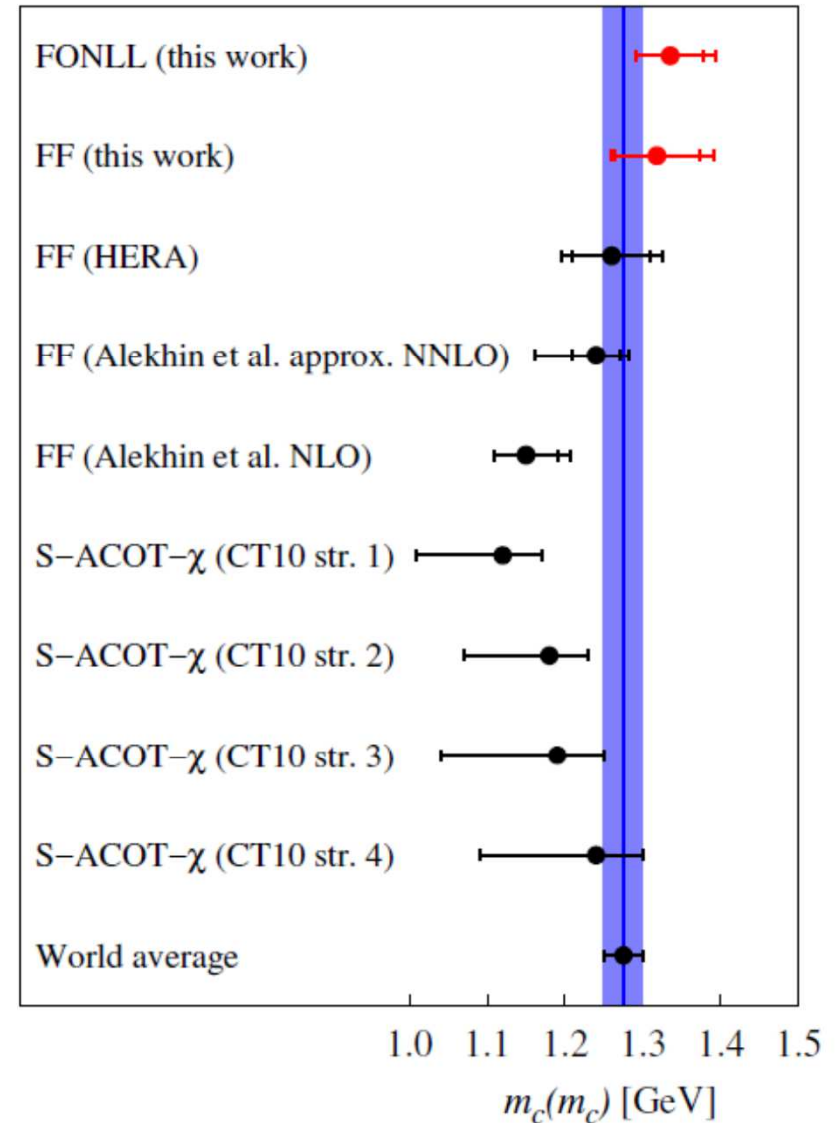
x

# $m_c(m_c)$ from FONLL fit of HERA data

V. Bertone et al., arXiv 1605.01946, JHEP 1608 (2016) 050



scheme	$m_c(m_c)$ [GeV]
FONLL (this work)	$1.335 \pm 0.043(\text{exp})_{-0.000}^{+0.019}(\text{param})_{-0.008}^{+0.011}(\text{mod})_{-0.008}^{+0.033}(\text{th})$
FFN (this work)	$1.318 \pm 0.054(\text{exp})_{-0.010}^{+0.011}(\text{param})_{-0.019}^{+0.015}(\text{mod})_{-0.004}^{+0.045}(\text{th})$
FFN (HERA) [9]	$1.26 \pm 0.05(\text{exp}) \pm 0.03(\text{mod}) \pm 0.02(\text{param}) \pm 0.02(\alpha_s)$
FFN (Alekhin <i>et al.</i> ) [24]	$1.24 \pm 0.03(\text{exp})_{-0.02}^{+0.03}(\text{scale})_{-0.07}^{+0.00}(\text{th})$ (approx. NNLO) $1.15 \pm 0.04(\text{exp})_{-0.00}^{+0.04}(\text{scale})$ (NLO)
S-ACOT- $\chi$ (CT10) [29]	$1.12_{-0.11}^{+0.05}$ (strategy 1) $1.18_{-0.11}^{+0.05}$ (strategy 2) $1.19_{-0.15}^{+0.06}$ (strategy 3) $1.24_{-0.15}^{+0.06}$ (strategy 4)
World average [53]	$1.275 \pm 0.025$



# top quark mass running

very preliminary procedure (with caveats, "cheated" a bit):

- use (conceptually constant) LO MC mass measured as function of scale-dependent quantity (e.g.  $m_{\text{++}}^-$ )
- check self-consistency of cross section measurements with data used for mass determination
- 'convert' LO MC mass to NLO pole mass by comparing MC and pole mass extractions from same data
- convert pole mass to  $\overline{\text{MS}}$  mass using 3-loop QCD
- use 1-loop evolution for actual running (NLO QCD)

(in the future, like for  $m_c$  and  $m_b$ , extract NLO (or NNLO) running mass directly from data, e.g. cross section, in each kinematic bin)

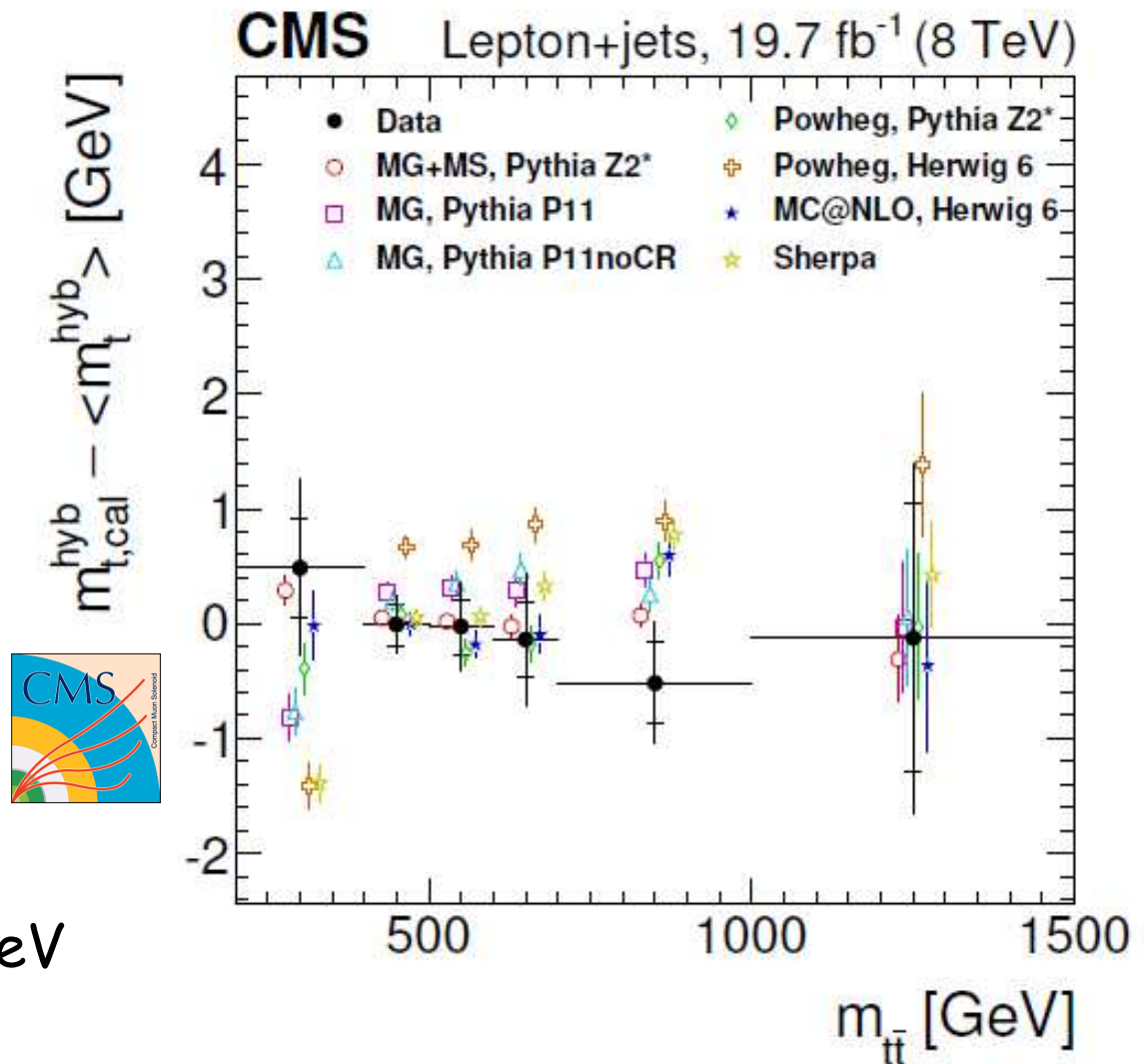


# top quark mass as function of $m_{t\bar{t}}$

CMS-TOP-14-022, Phys. Rev. D93 (2016) 072004

"MC mass"

deviation  
from average of  
 $172.35 \pm 0.16_{\text{stat}} \pm 0.48_{\text{sys}} \text{ GeV}$



# differential top cross section shape consistent with NLO

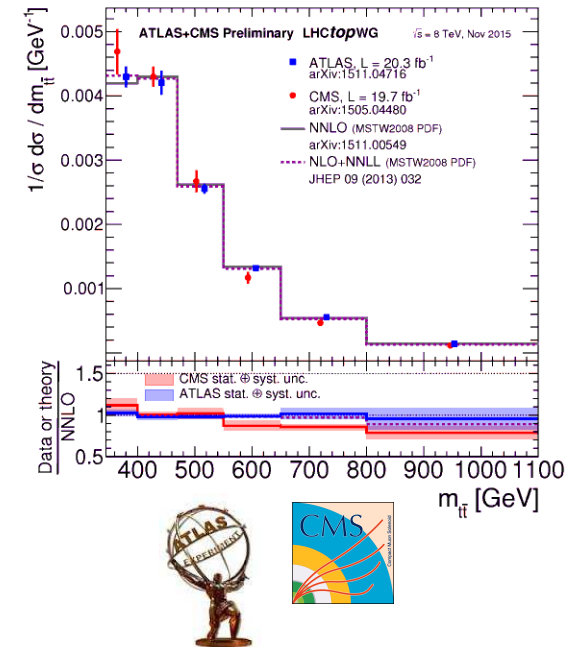
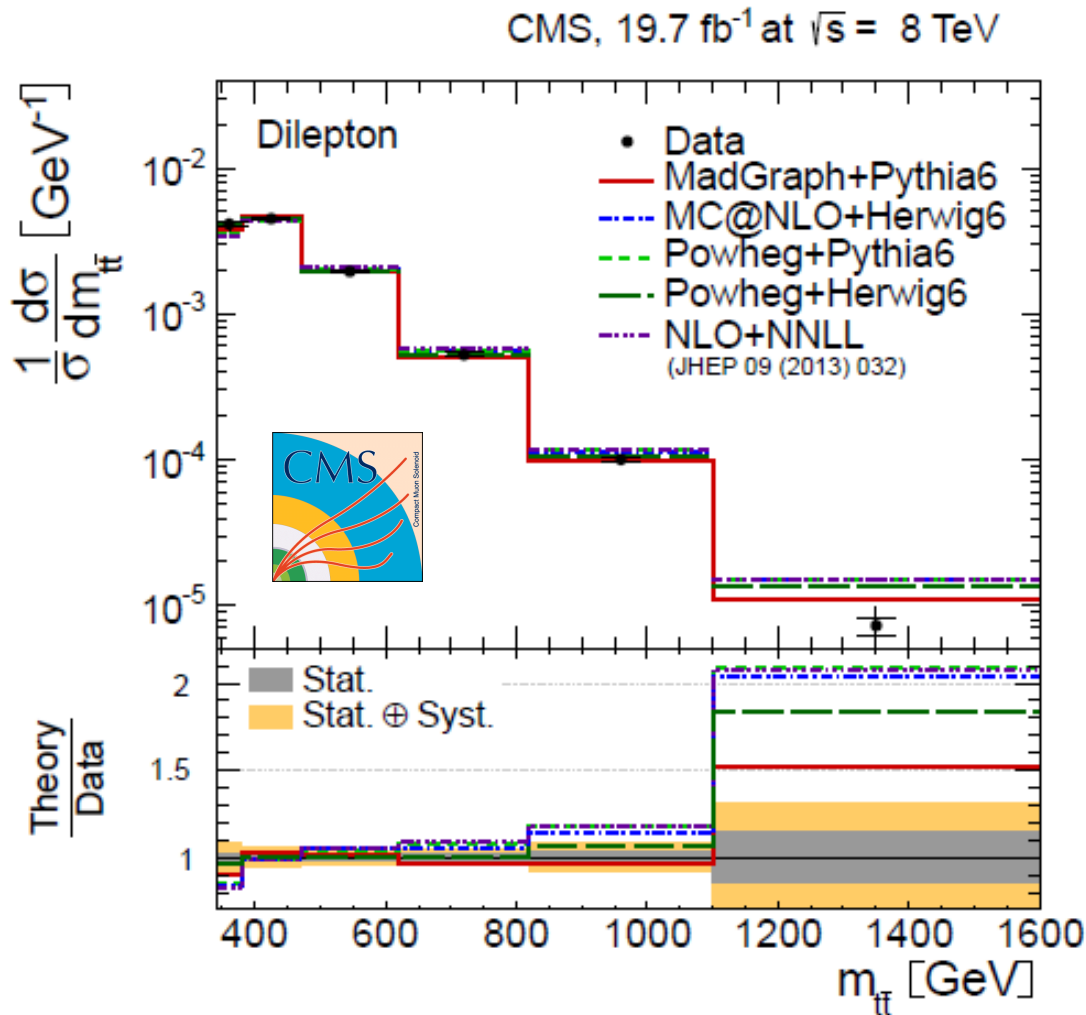
CMS-TOP-12-028, Eur. Phys. J. C75 (2015) 542

LHCtopWG

NLO theory  
uses  
pole mass  
scheme

use CMS to  
be consistent  
with previous  
slide

similar results  
for lepton+jets  
channel only



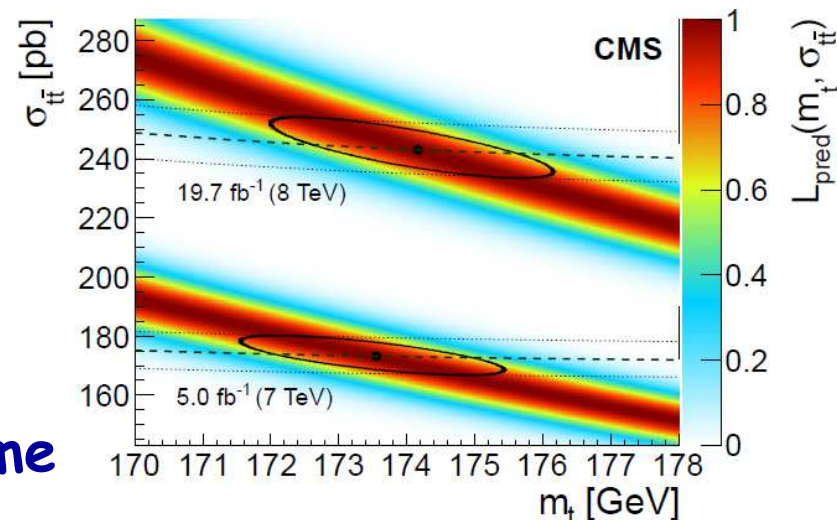
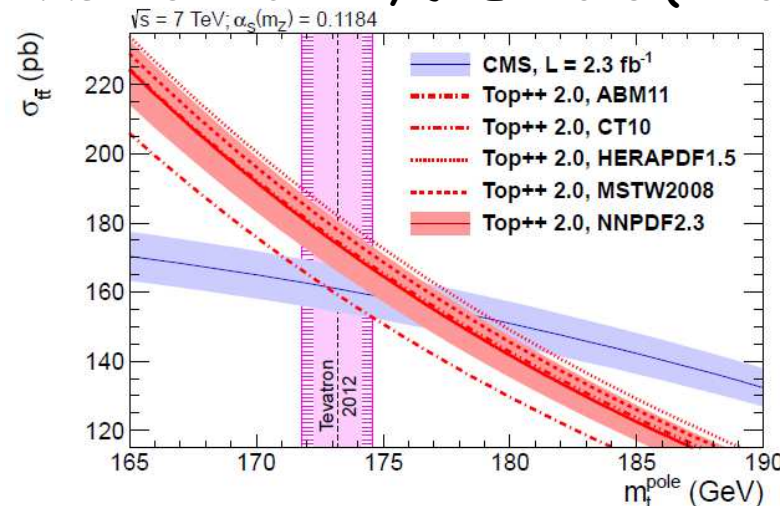
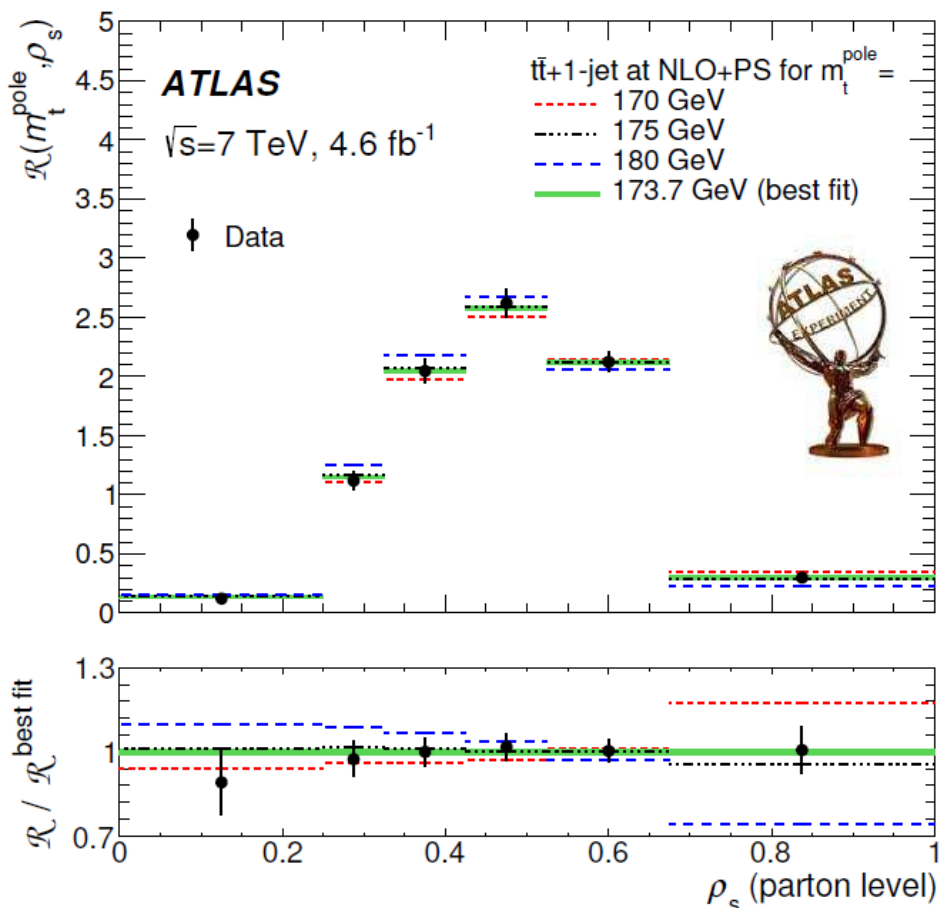
-> measurements and LO+PS/NLO theory are self-consistent and consistent with ATLAS and NNLO

# convert top MC mass to pole mass

ATLAS, JHEP10 (2015) 121,

CMS-TOP-12-022, Phys. Lett. B728 (2014) 526

CMS-TOP-13-004, JHEP 1608 (2016) 029



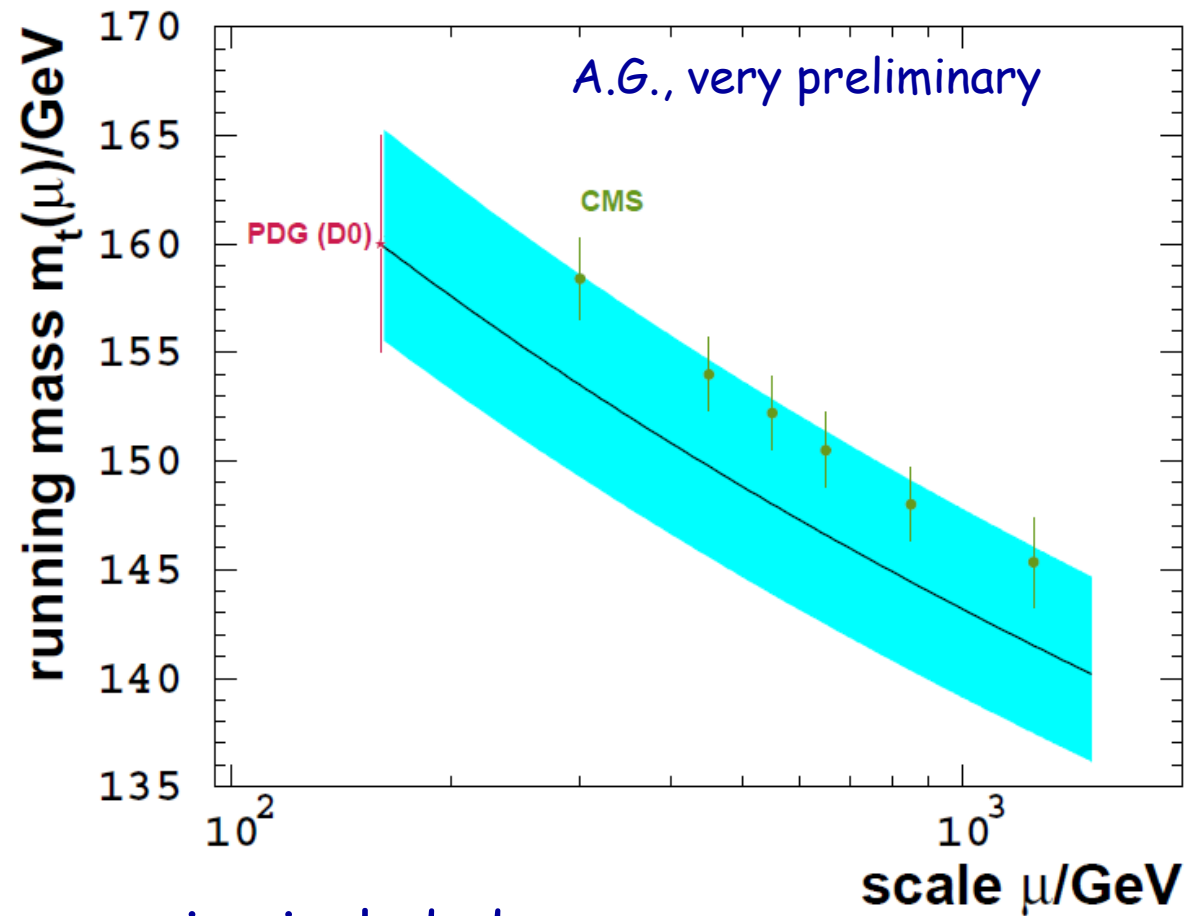
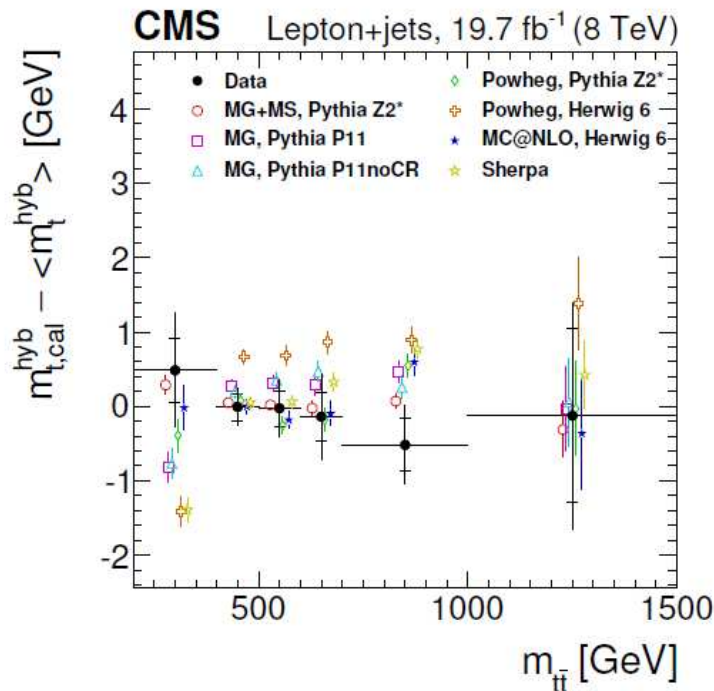
“MC” and pole masses almost the same

ATLAS:  $m_t(\text{pole}) = 173.7 \pm 1.5_{\text{stat}} \pm 1.4_{\text{syst}} + 1.0 - 0.5_{\text{th}} \text{ GeV}$

CMS:  $m_t(\text{pole}) = 173.8 + 1.7 - 1.8_{\text{total}} \text{ GeV} \leftrightarrow m_t(\text{MC}_{\text{CMS}, l+\text{jets}}) = 172.35 \pm 0.16_{\text{stat}} \pm 0.48_{\text{syst}} \text{ GeV}$

PDG:  $m_t(\text{pole}) = 176.7 \pm 4.0 - 3.4 \text{ GeV}$ , “ $m_t(\text{MC})$ ” =  $173.21 \pm 0.51_{\text{stat}} \pm 0.71_{\text{syst}} \text{ GeV}$

# convert pole masses to running mass



caveat:

not all uncertainties from conversion included

(needs theoretically better defined procedure!)

-> take with grain of salt, **for illustration purposes**

# Discussion

of future conceptual improvements

- avoid MC mass and pole mass intermediate steps for top  
-> extract  $m_+(\mu)$  directly from data, as already done for b,c  
(e.g. from absolute  $m_{++}$  cross sections in CMS-TOP-16-008)  
need NLO QCD theory for LHC using running mass
- extend LO EW + NLO QCD approach  
(running of Higgs couplings is purely QCD-induced!)  
to NLO EW + NNLO QCD + interference  
highly non-trivial but eventually necessary  
(Standard Model is not QCD only)