

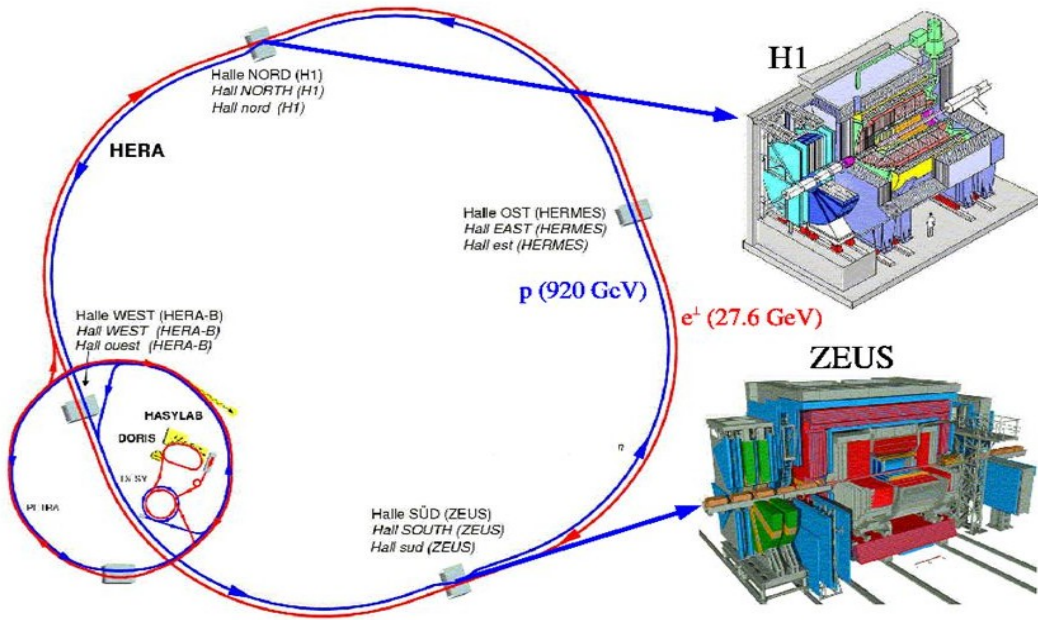
(Simultaneous QCD & Electroweak fits to HERA inclusive DIS data)

K. Wichmann on behalf of H1 & ZEUS collaborations

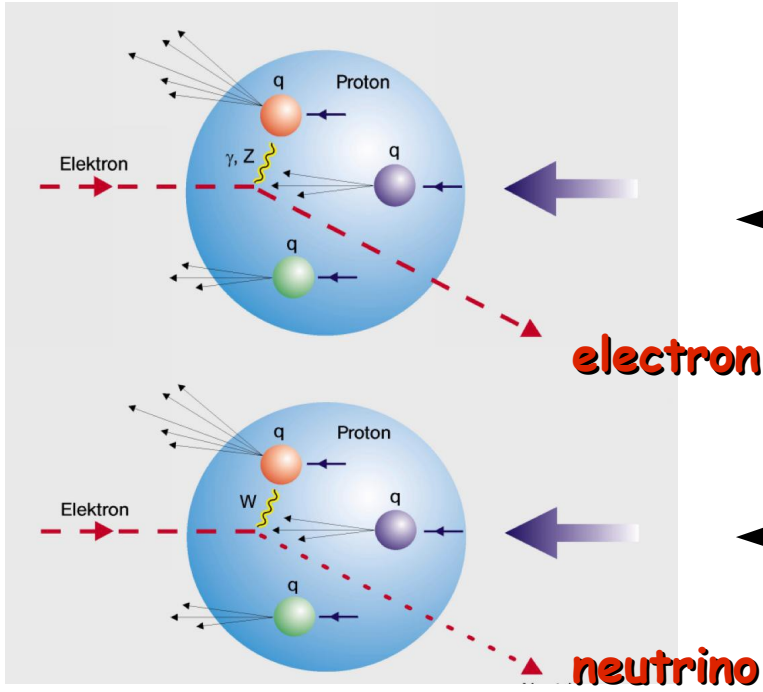
↓ Thanks to Mandy

Everything you would like to know about simultaneous
QCD & Electroweak fits but were afraid to ask...

HERA and DIS



- HERA: ep collider in Hamburg
- Operation: 1992-2007
- Colliding experiments: H1 and ZEUS

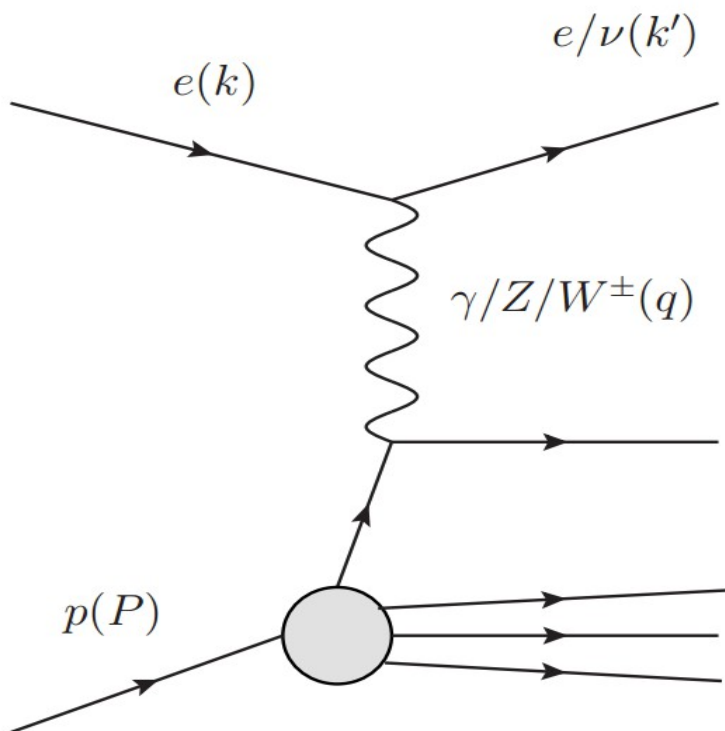


Deep Inelastic Scattering

Neutral Current (NC)
 γ, Z^0 exchange

Charged Current (CC)
 W^\pm exchange

Deep Inelastic Scattering at HERA



- Lepton beams polarised for HERAII
→ crucial for the EW measurements

$$X(P') \quad E_p = 920 (820, 460, 575) \text{ GeV}$$

$$E_e = 27.5 \text{ GeV}$$

$$\sqrt{s} = 318 (300, 225, 252) \text{ GeV}$$

$$Q^2 = -q^2 = -(k - k')^2$$

$$x_{Bj} = \frac{Q^2}{2pq} \quad y = \frac{pq}{pk}$$

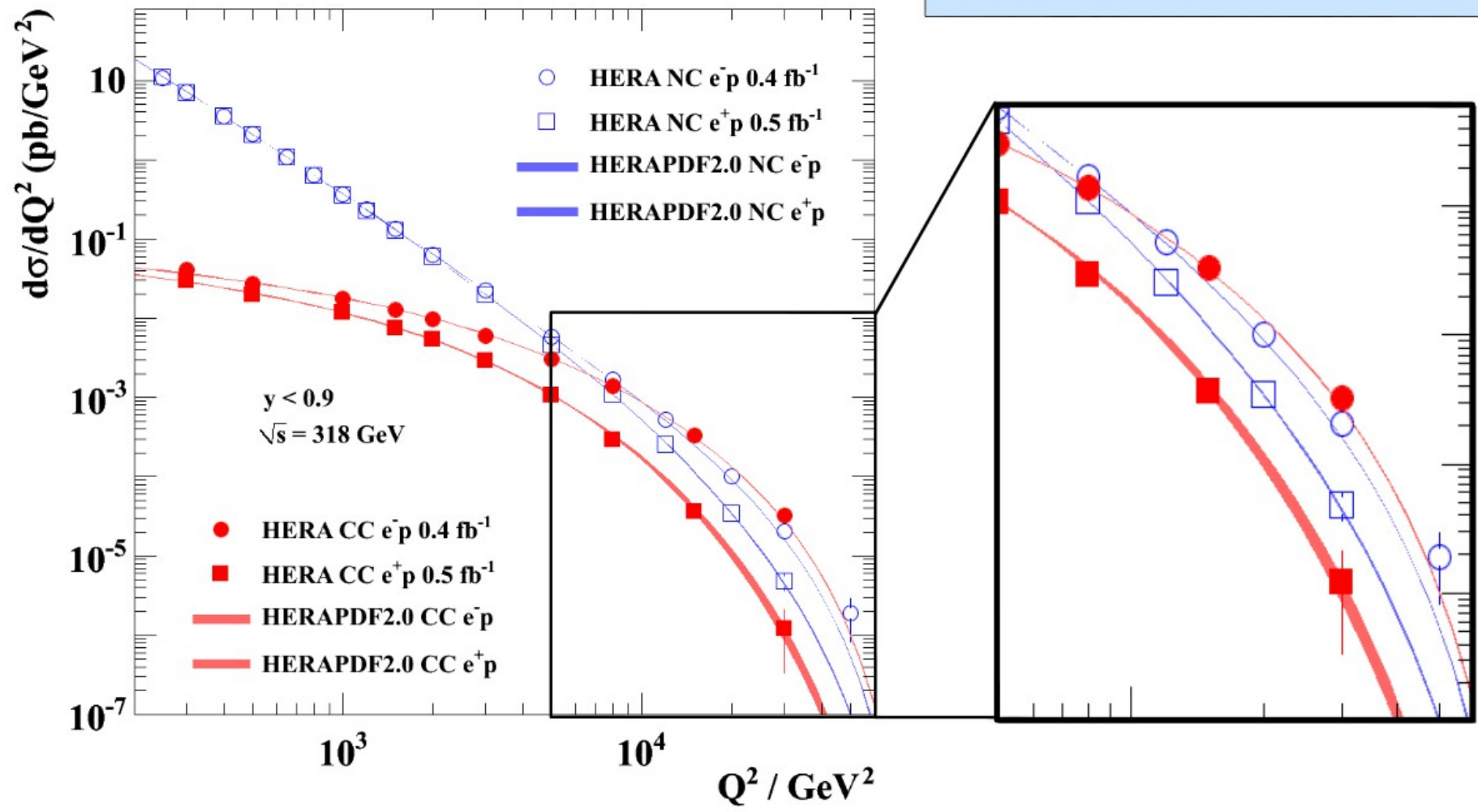
$$s = (p + k)^2 \quad Q^2 = xys$$

Experimental luminosity (H1 & ZEUS):

~ 0.5 fb⁻¹ data from each experiment

Fantastic precision of HERA inclusive final data

H1 and ZEUS



- NC and CC at tree level

$$Y_{\pm} = 1 \pm (1-y)^2$$

$$\frac{d\sigma_{NC}^{\pm}}{dQ^2 dx} = \frac{2\pi\alpha^2}{x} \left[\frac{1}{Q^2} \right]^2 (Y_+ F_2 + Y_- x F_3 + y^2 F_L)$$

$$\frac{d\sigma_{CC}^{\pm}}{dQ^2 dx} = \frac{1 \pm P}{2} \frac{G_F^2}{4\pi x} \left[\frac{m_W^2}{m_W^2 + Q^2} \right]^2 (Y_+ W_2^{\pm} \pm Y_- x W_3^{\pm} - y^2 W_L^{\pm})$$



Polarised DIS

- Generalised structure functions depend on e-beam polarisation

$$P_e = \frac{N_R - N_L}{N_R + N_L}$$

$$\tilde{F}_2^\pm = F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z,$$

$$x\tilde{F}_3^\pm = -(a_e \pm P_e v_e) \chi_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \chi_Z^2 x F_3^Z$$

- Structure functions in QP model

NC

$$[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] x(q + \bar{q}),$$

$$[xF_3^{\gamma Z}, xF_3^Z] = \sum_q [e_q a_q, v_q a_q] 2x(q - \bar{q}),$$

CC

$$\frac{d^2\sigma_{CC}(e^+p)}{dx_{Bj}dQ^2} = (1 + P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(\bar{u} + \bar{c}) + (1 - y)^2 (d + s + b)]$$

$$\frac{d^2\sigma_{CC}(e^-p)}{dx_{Bj}dQ^2} = (1 - P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x [(u + c) + (1 - y)^2 (\bar{d} + \bar{s} + \bar{b})]$$

NC sensitive to $\sin^2\theta_W$ via

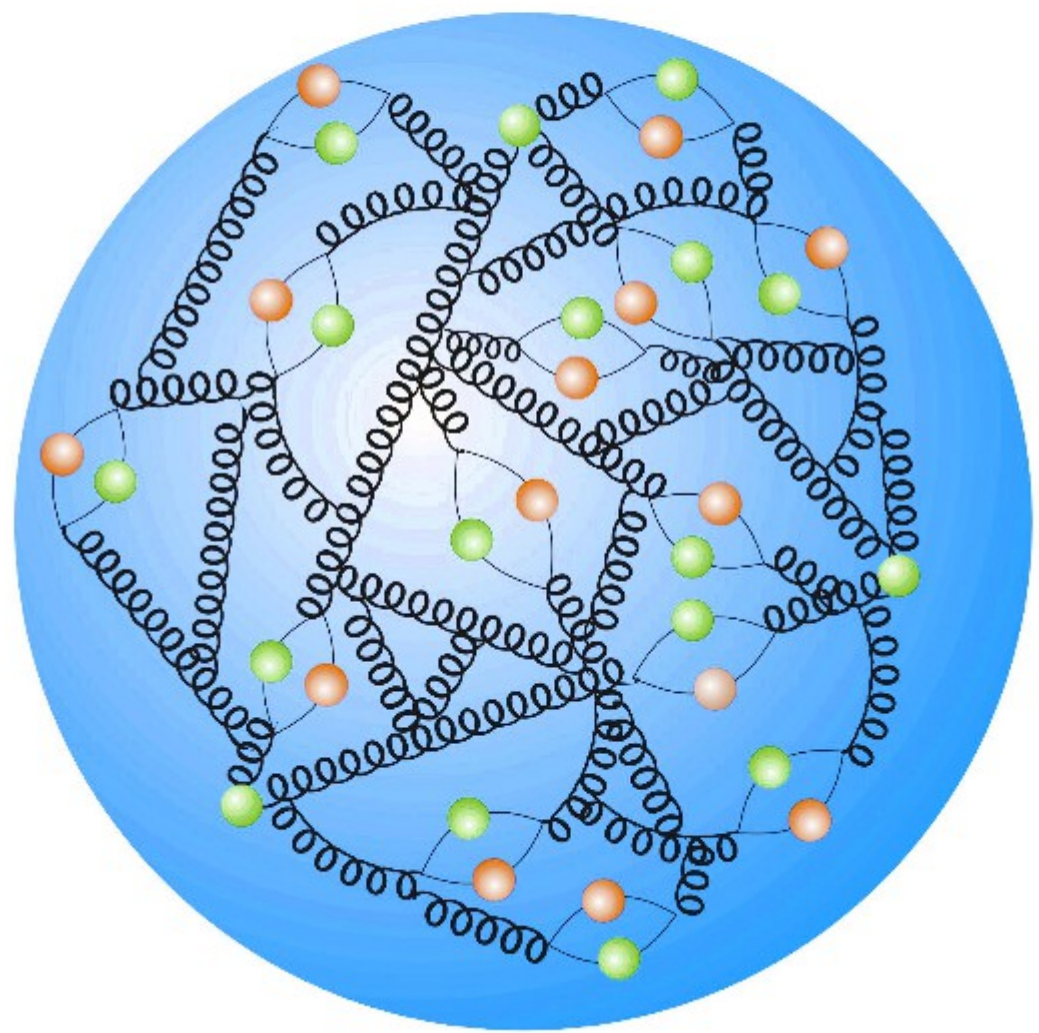
$$\chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2} \frac{1}{1 - \Delta R}$$

- Calculation in on-shell scheme

$$G_F = \frac{\pi\alpha_0}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

CC sensitive to $\sin^2\theta_W$

Global QCD fits



Global analysis of parton distributions

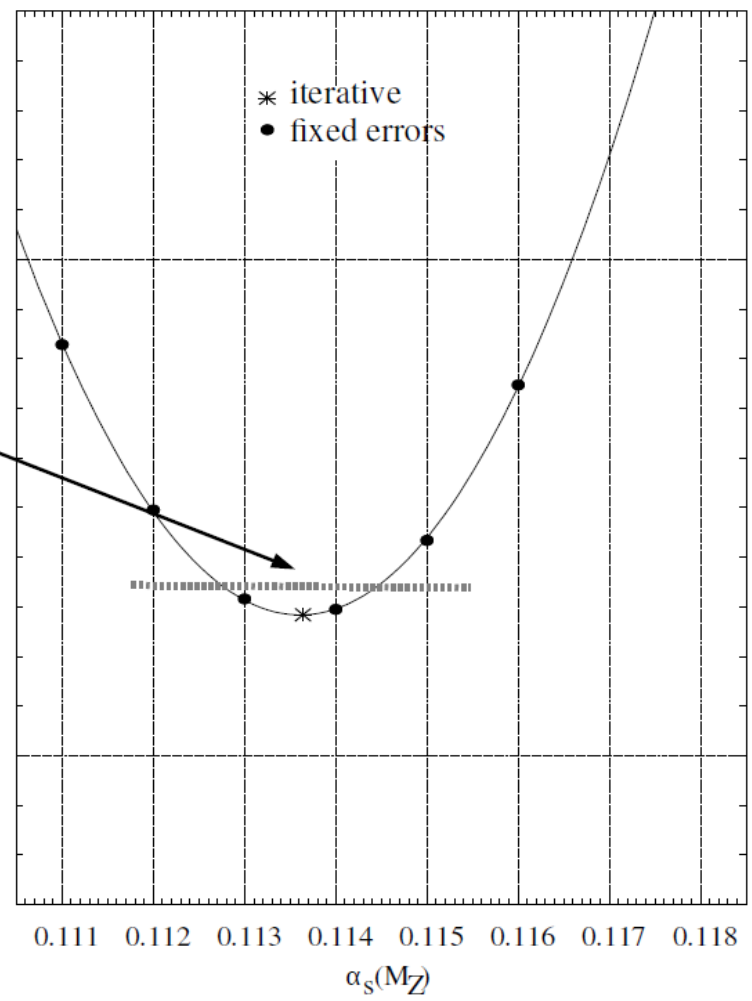
Goal: determination of the *input distributions* (for light quarks and gluons):

Method: Parametrizations $xf(x, Q_0^2) = Nx^a(1-x)^b$ function(x)
and usual *statistical estimation* (fits):

$$\chi^2(p) = \sum_{i=1}^N \left(\frac{\text{data}(i) - \text{theory}(i, p)}{\text{error}(i)} \right)^2$$

Position of minimum gives the value and curvature gives the error (region within a certain “tolerance” $\Delta\chi^2 = 1$) (Monte Carlo methods can also be used)

Usually the chi-square definition is more sophisticated, experimental correlations are also treated, etc.

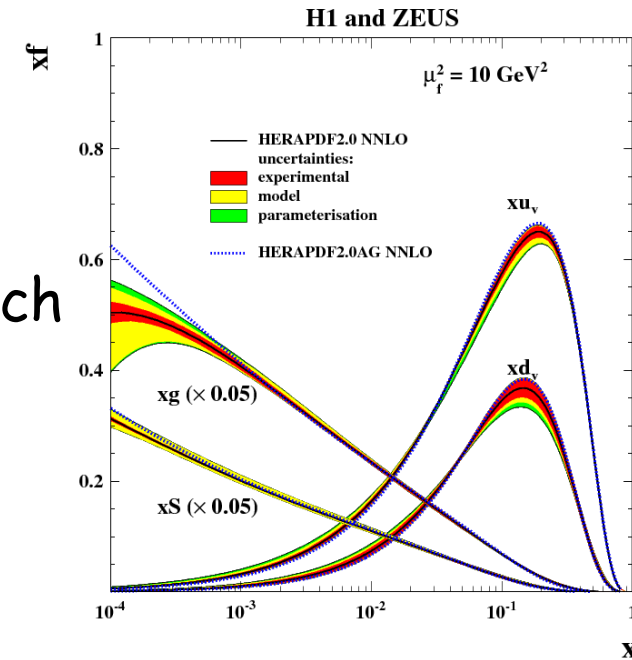


Global QCD fits

- Data: NC & CC, e^+p and e^-p scattering
- Global PDF fits closely follow HERAPDF2.0 approach
- DGLAP evolution using QCDNUM
- 13 parameter fit (HERAPDF2.0 - DUBAR)

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

$$xg(x), xu_v(x), xd_v(x), x\bar{U}(x), x\bar{D}(x)$$



- Starting scale $Q^2_0 = 1.9 \text{ GeV}^2$
- Model and parameterisation uncertainties \rightarrow HERAPDF2.0
- Corrections calculated using EPRC code: ΔR

desy.de/~hspiesb/eprc.html

- No ISR/FSR corrections

Vector and axial-vector couplings



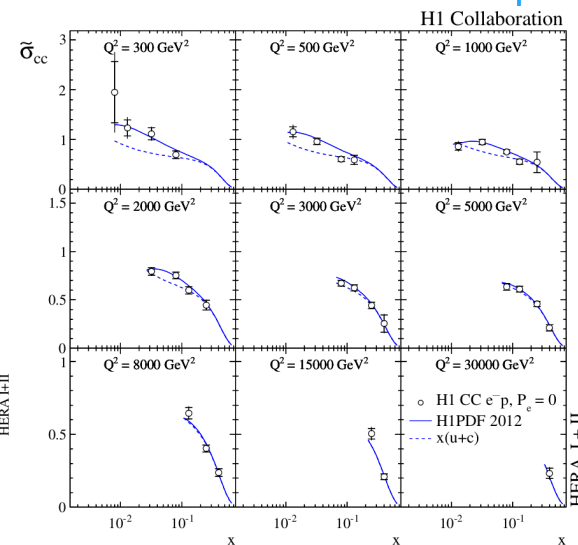
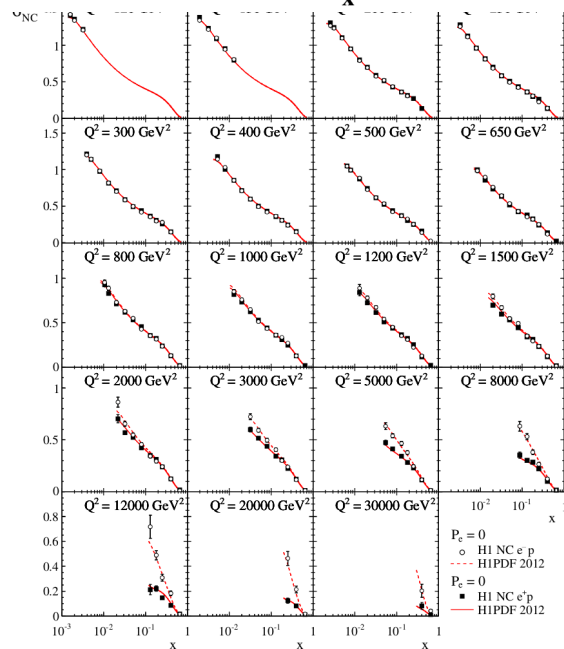
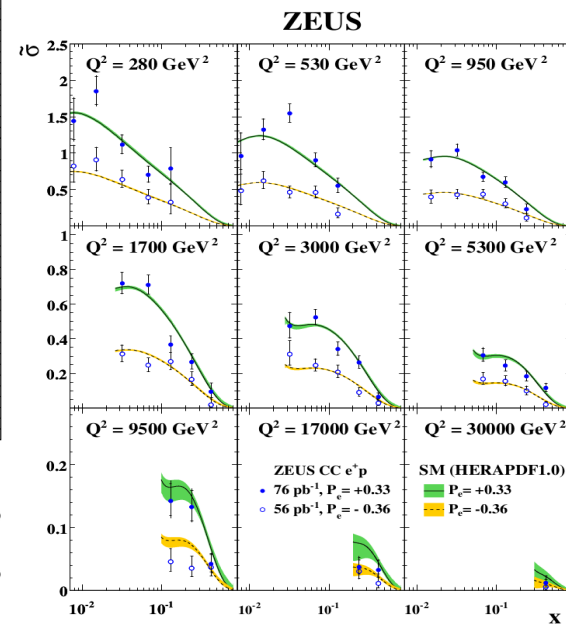
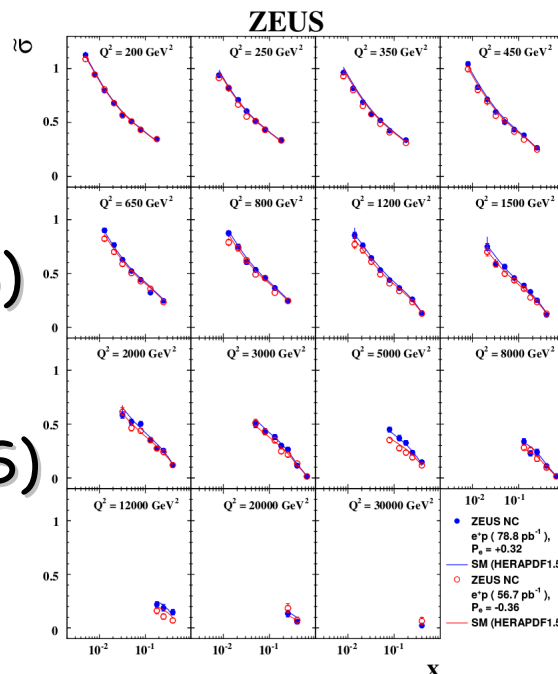
Data used



Uncombined data sets

- All HERAI data (H1 & ZEUS)
 - unpolarised
- Reduced E_p data (H1 & ZEUS)
- HERAII
 - H1 unpolarised data
 - **ZEUS polarised data**

Data from $Q^2 = 3.5 \text{ GeV}^2$

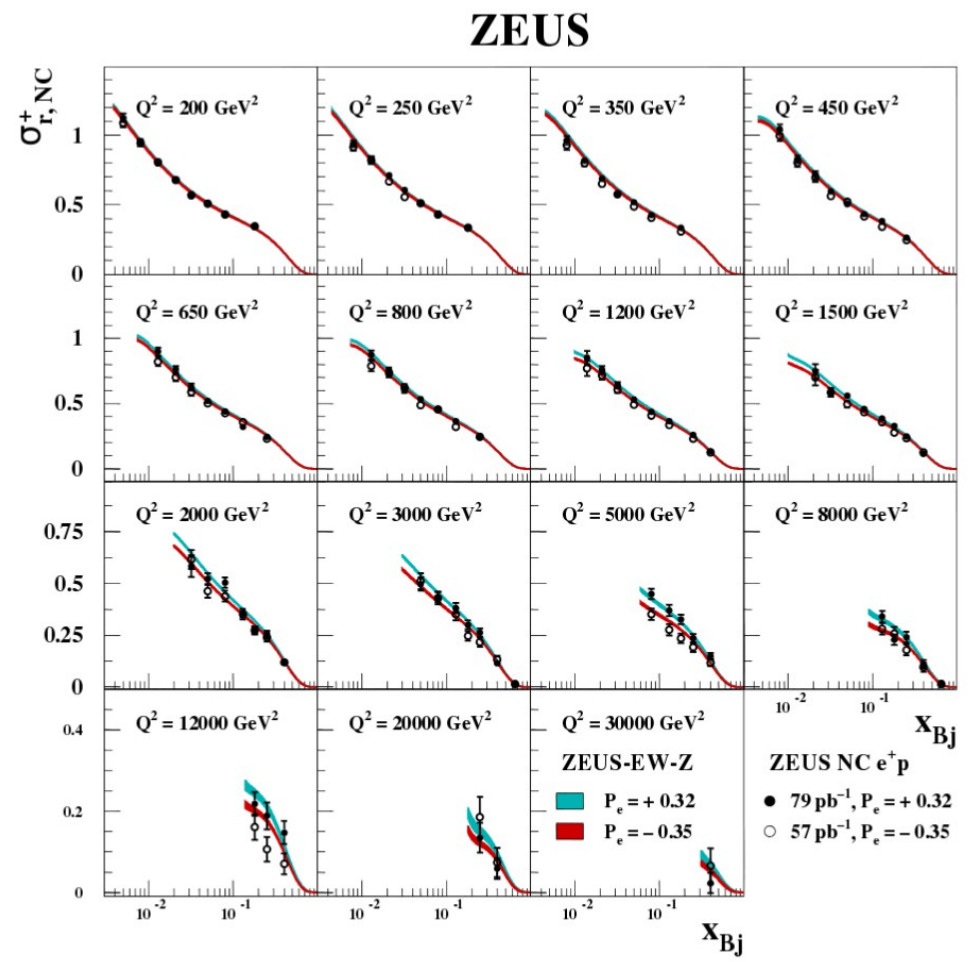
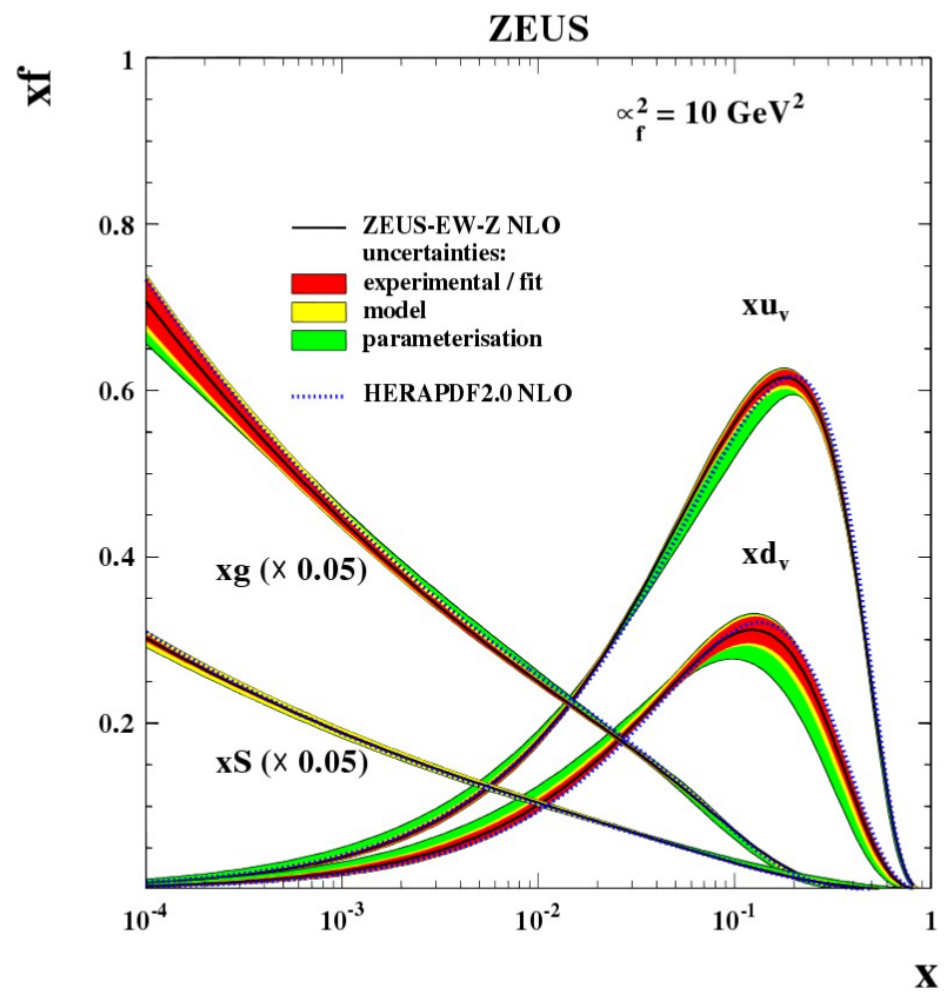


- DGLAP evolution @ NLO
- HF scheme - GN VFNS NLO (RT OPT)

Fit results



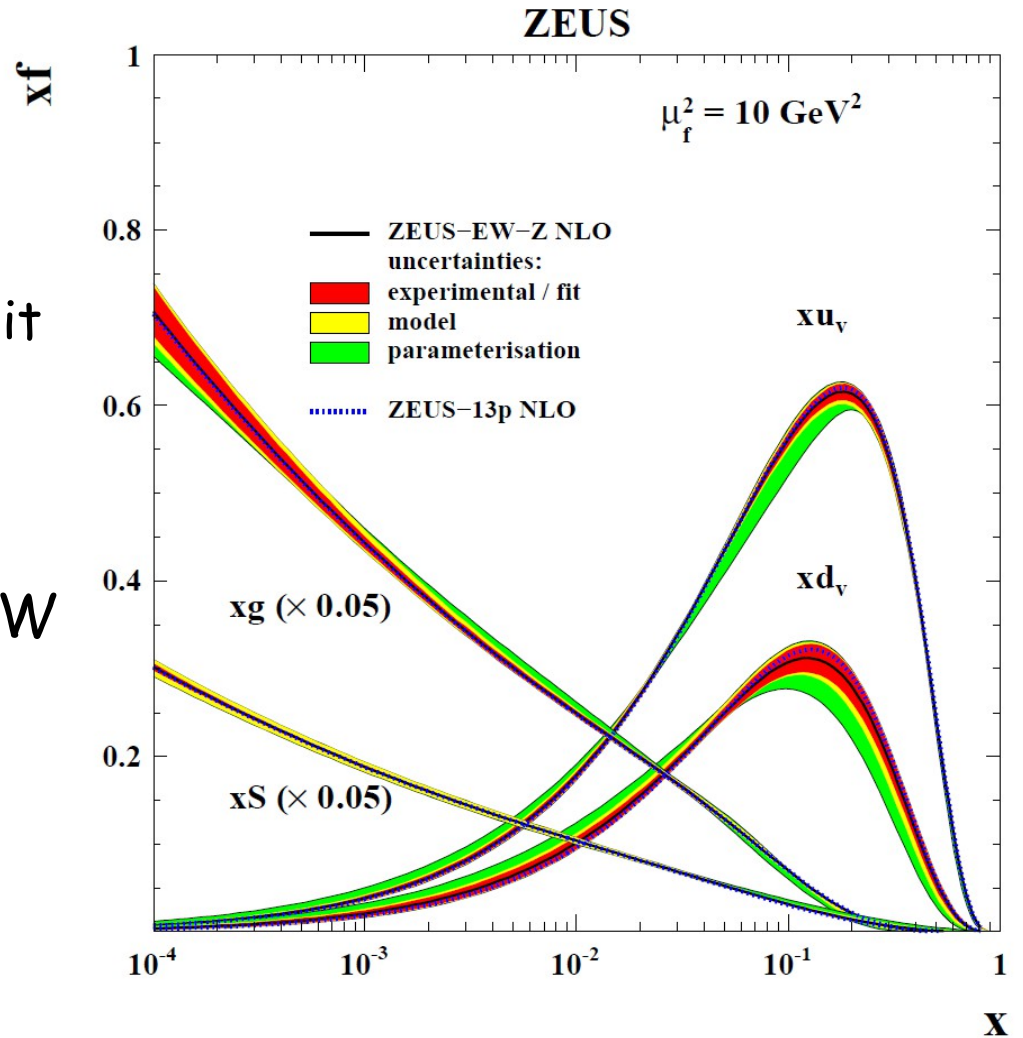
- Simultaneous QCD and EW fit:
 - 13 QCD parameters + 4 EW couplings





QCD & EW parameters uncorrelated

- Reference fit ZEUS-13p:
 - QCD parameters fixed to 13p fit
 - Only 4 EW couplings fitted
- Very similar results
- Correlation between QCD and EW parameters small





QCD & EW parameters uncorrelated

- Detailed studies performed to check stability of EW couplings with respect to various QCD parameters
 - HPDF1: QCD parameters and all constants fixed to HERAPDF2.0
 - HPDF2: QCD parameters fixed to HERAPDF2.0 + on-shell value of $\sin^2\theta_w$
 - 13p - reference fit described before

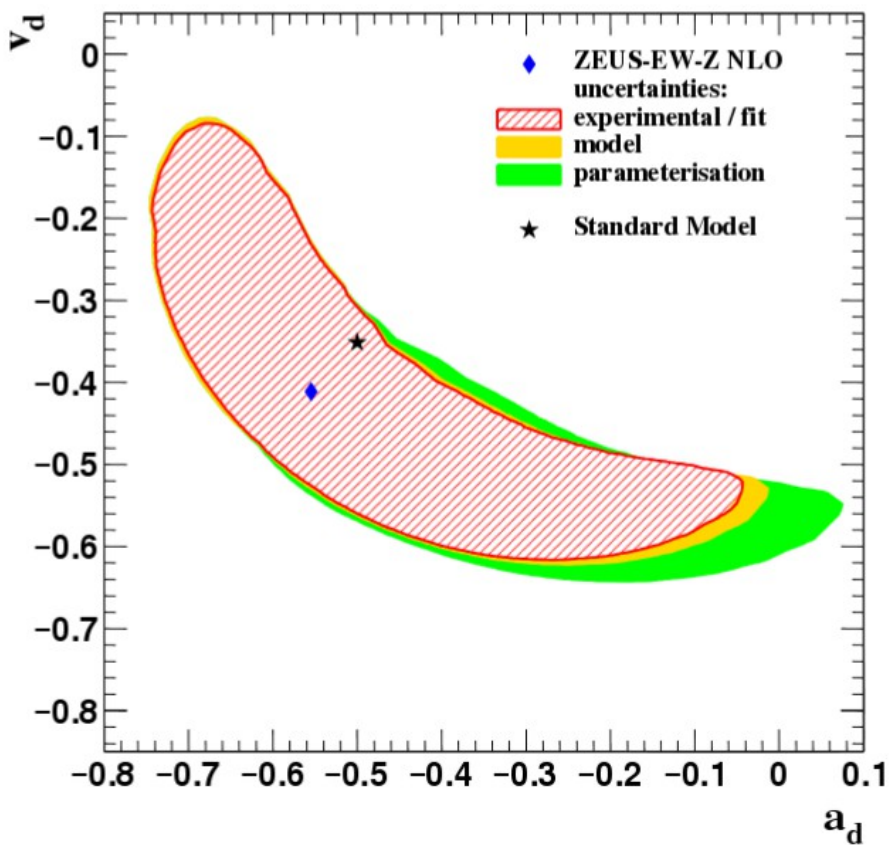
→ Results for couplings very similar

	a_u	exp	tot	a_d	exp	tot	v_u	exp	tot	v_d	exp	tot
EW-Z	+0.50	+0.09 -0.05	+0.12 -0.05	-0.56	+0.34 -0.14	+0.41 -0.15	+0.14	+0.08 -0.08	+0.09 -0.09	-0.41	+0.24 -0.16	+0.25 -0.20
13p	+0.49	+0.07 -0.04		-0.57	+0.30 -0.13		+0.15	+0.08 -0.08		-0.40	+0.22 -0.17	
HPDF1	+0.47	+0.06 -0.03		-0.62	+0.23 -0.11		+0.16	+0.08 -0.08		-0.35	+0.22 -0.19	
HPDF2	+0.49	+0.06 -0.03		-0.63	+0.24 -0.11		+0.15	+0.08 -0.08		-0.36	+0.22 -0.19	
SM	+0.50			-0.50			+0.20			-0.35		

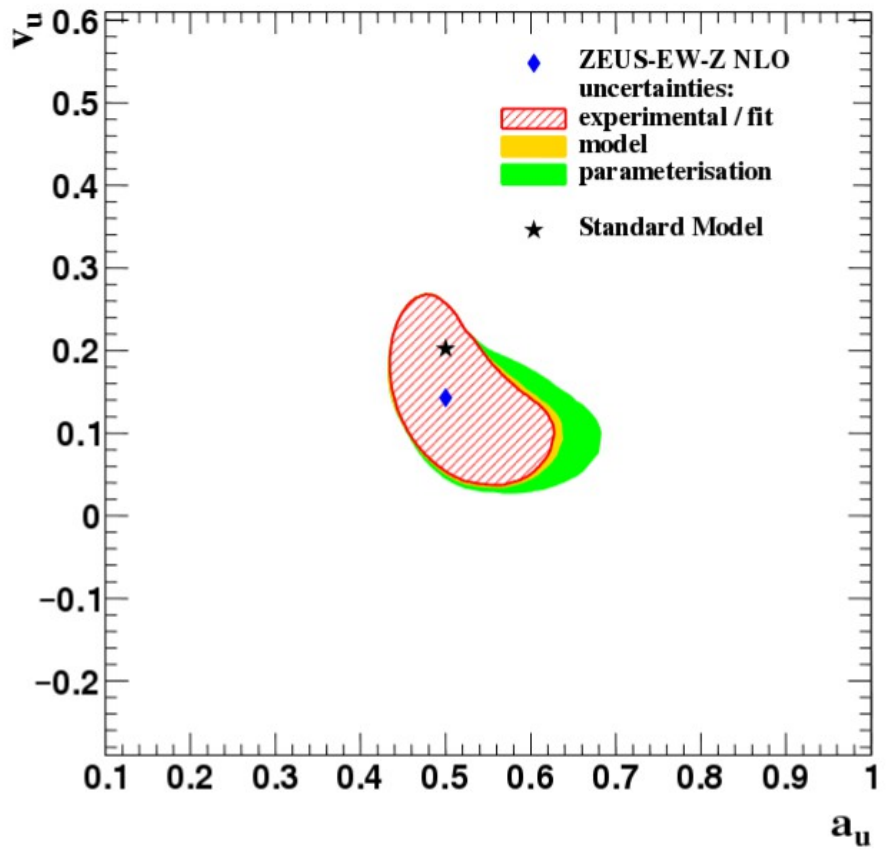
Correlations



ZEUS



ZEUS

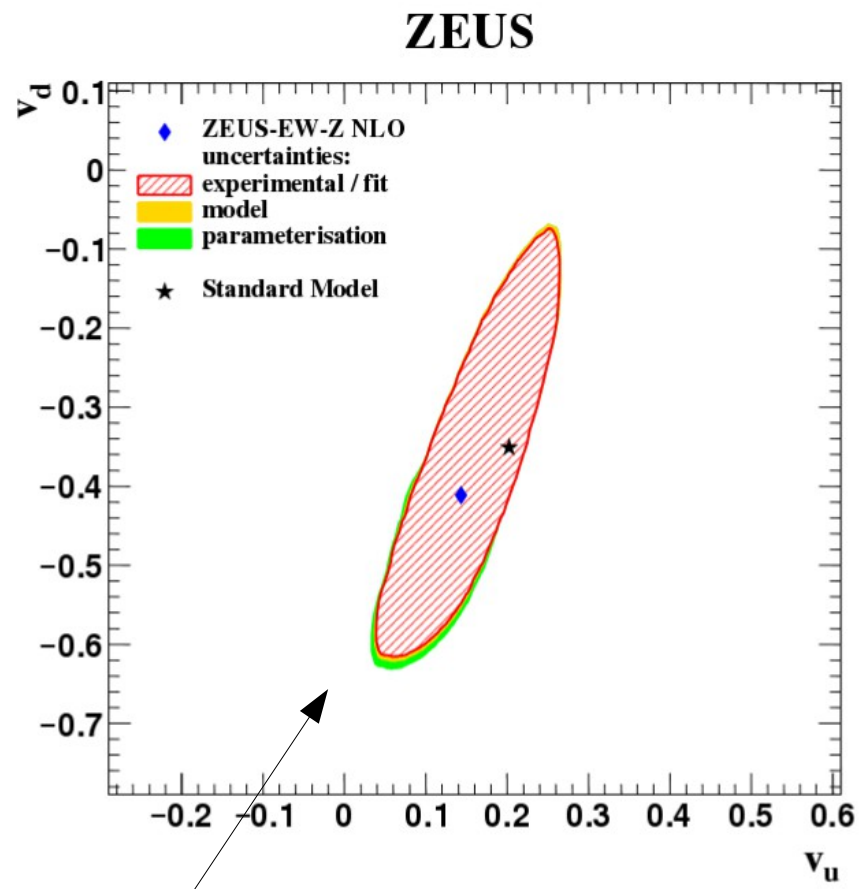
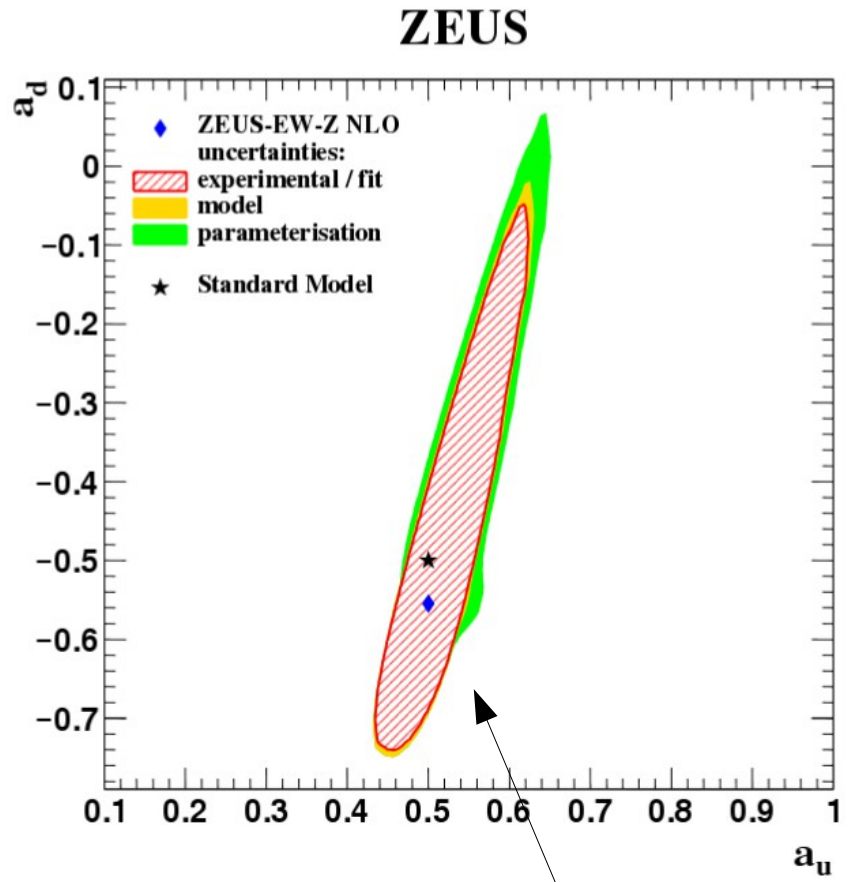


$a_u = 0.50$	$+0.09$ -0.05 (exp/fit)	$+0.04$ -0.02 (mod)	$+0.08$ -0.01 (par)	$= 0.50$	$+0.12$ -0.05 (tot)	0.5	Standard Model
$a_d = -0.56$	$+0.34$ -0.14 (exp/fit)	$+0.11$ -0.05 (mod)	$+0.20$ -0.00 (par)	$= -0.56$	$+0.41$ -0.15 (tot)	-0.5	
$v_u = 0.14$	$+0.08$ -0.08 (exp/fit)	$+0.01$ -0.00 (mod)	$+0.03$ -0.01 (par)	$= 0.14$	$+0.09$ -0.09 (tot)	0.202	
$v_d = -0.41$	$+0.24$ -0.16 (exp/fit)	$+0.04$ -0.07 (mod)	$+0.00$ -0.08 (par)	$= -0.41$	$+0.25$ -0.20 (tot)	-0.351	



Correlations

- Vector and axial-vector couplings in the fit show high correlation



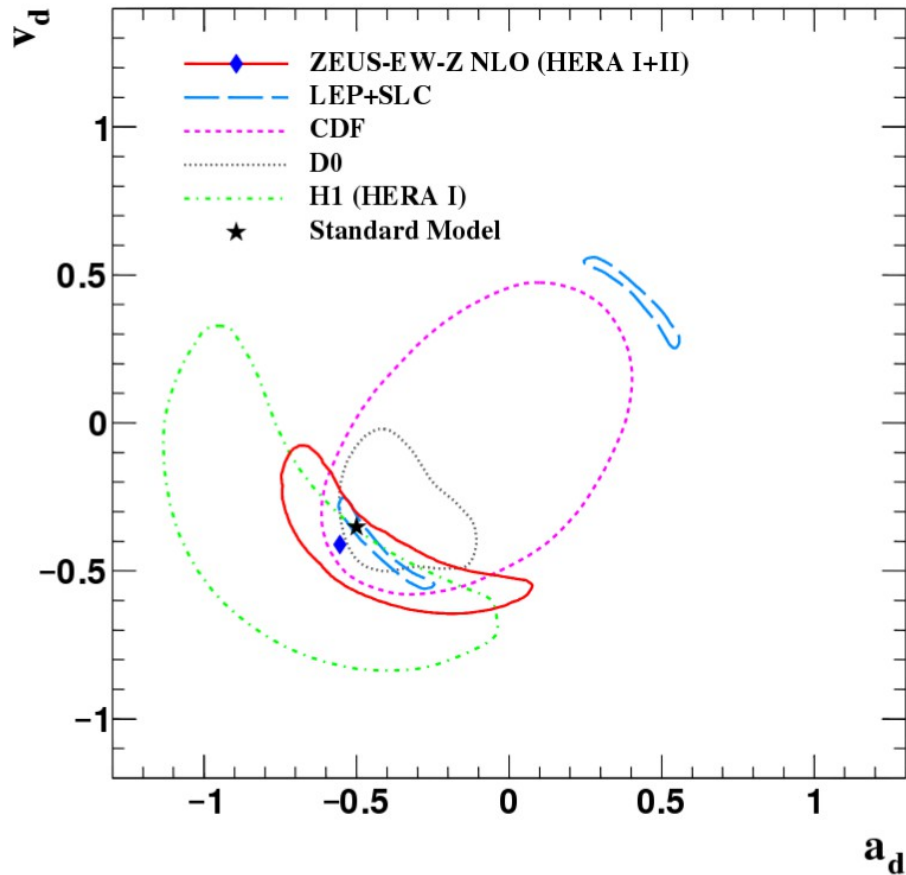
	a_u	a_d	v_u	v_d
a_u	1.000	0.861	-0.555	-0.729
a_d	0.861	1.000	-0.636	-0.880
v_u	-0.555	-0.636	1.000	0.851
v_d	-0.729	-0.880	0.851	1.000

Insignificant correlations of couplings to PDF parameters

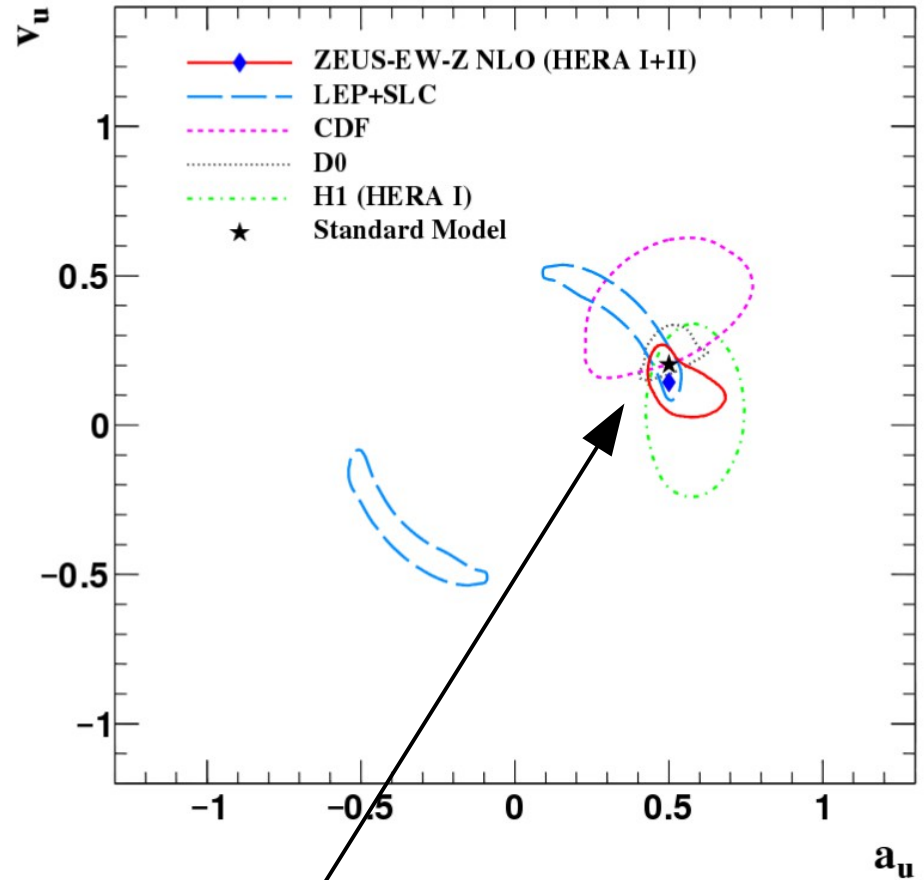


Comparison with other measurements

ZEUS



ZEUS



HERA data remarkably sensitive to **u-type** quark couplings



H1 fit methodology

H1prelim-16-041



K. Wichmann

Low x 2016

- Lots of basics same as in ZEUS measurement
 - Differences/different approaches pointed out
- Calculations performed strictly in on-shell scheme
 - Parameters are: α , m_W , m_Z , (m_t , m_H , ...)
- Polarisation measurements considered as independent measurements in fits
- New C++ code for PDF and more general fits developed: Alpos
- DGLAP evolution @ NNLO

χ^2 Definition

- Uncertainties on cross sections are assumed to be 'log-normal' distributed (relative uncertainties)
- Uncertainties on polarisation measurements are assumed to be 'normal' distributed
- Correlations of syst. uncertainties between different datasets are considered

$$\chi^2 = (\log(d) - \log(t))^T V_R^{-1} (\log(d) - \log(t)) + (d - t)^T V_A^{-1} (d - t)$$

Fit parameters

- 13 PDF parameters
- 4 polarisation values
- 4 Light-quark couplings (or other SM parameters)
- More general also 'nuisance parameters' of syst. uncertainties



Data used

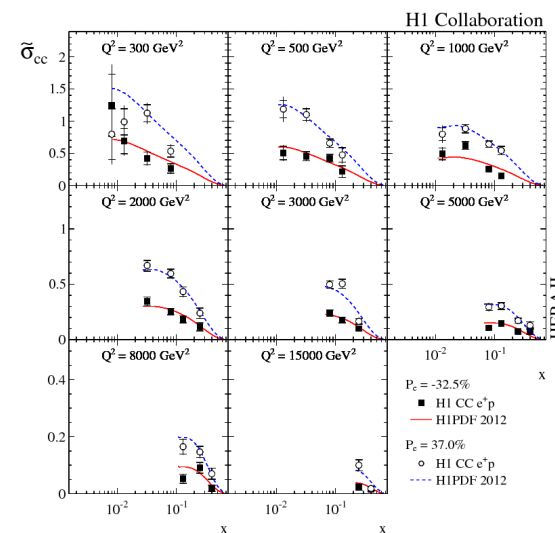
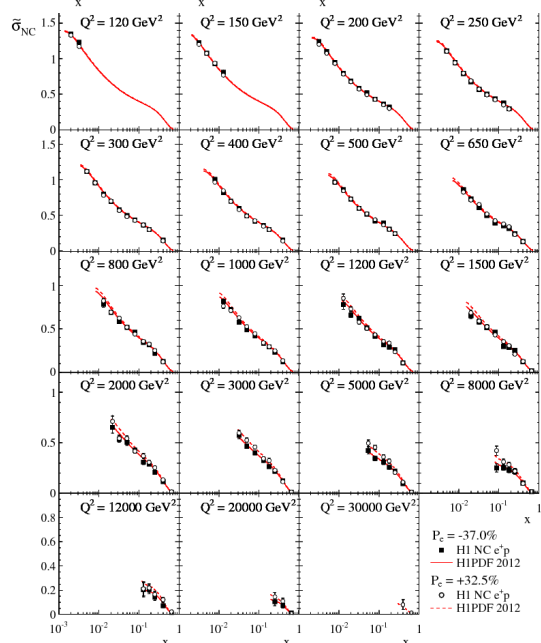
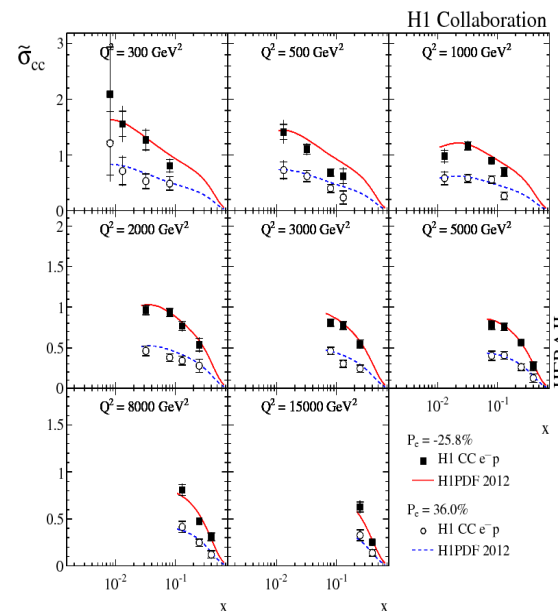
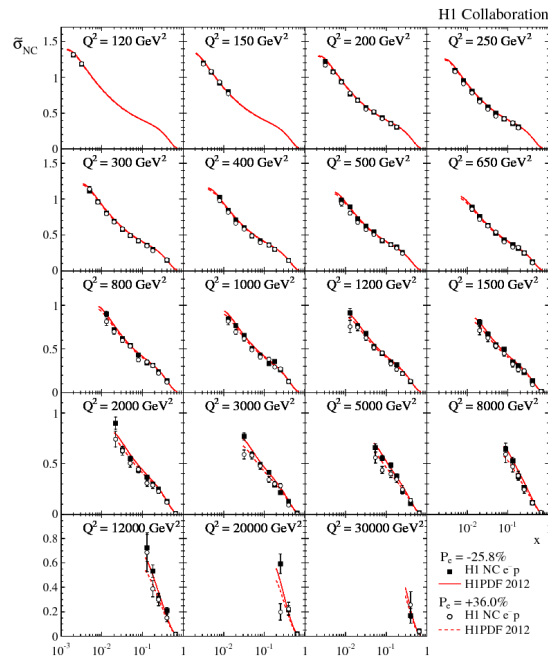


Uncombined data sets

- H1 HERAI data
 - unpolarised
- Reduced E_p H1 data
- HERAII

- H1 polarised data

Data from $Q^2 = 12 \text{ GeV}^2$



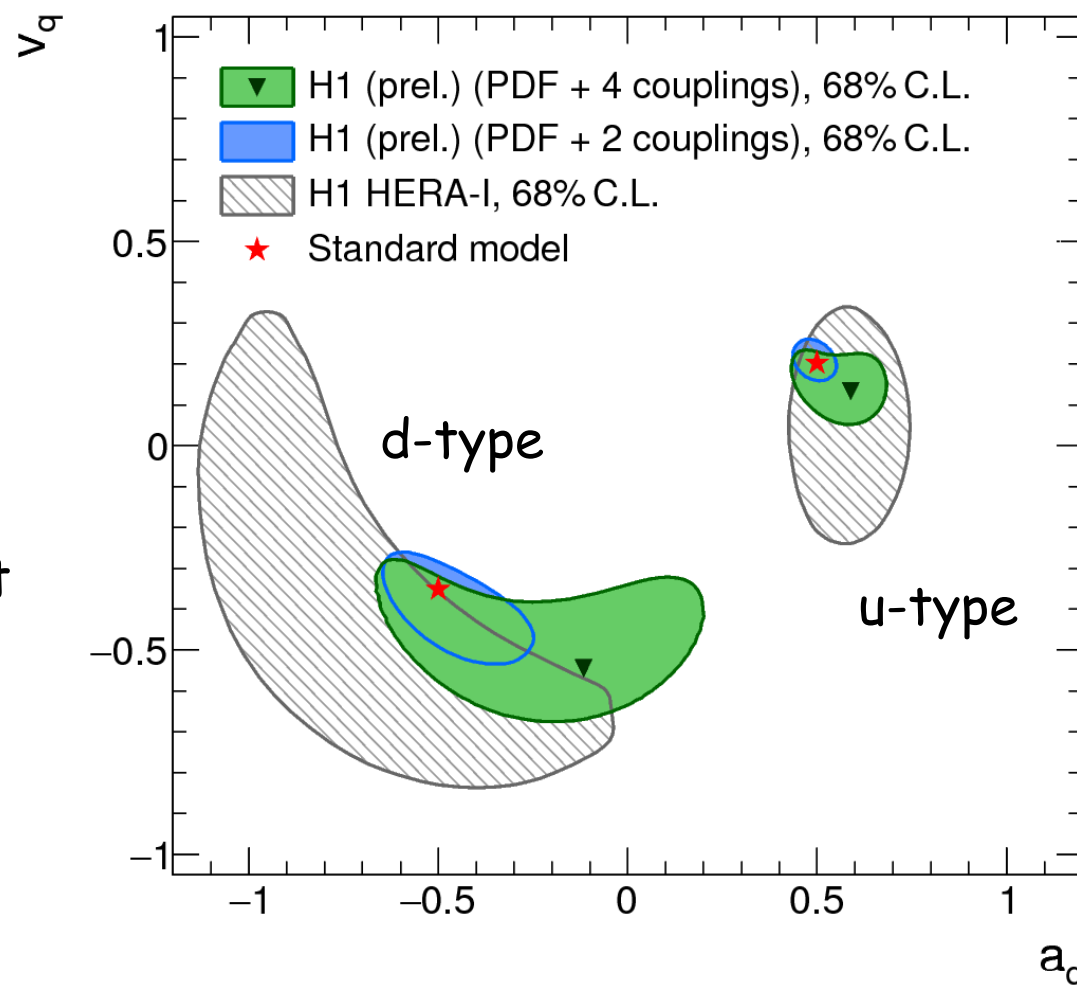
- HF scheme: ZM-VFNS as implemented in QCDNUM

Fit: PDF + 4 couplings

- $\chi^2 / \text{ndf} = 1370.5 / (1388 - 21)$
- u-type couplings constrained better than d-type
- sensitivity from valence quarks
- Results compatible with SM
- PDF uncertainties small
- Considerably improved sensitivity using final H1 HERA-II data
- Polarisation in HERA-II important for vector couplings

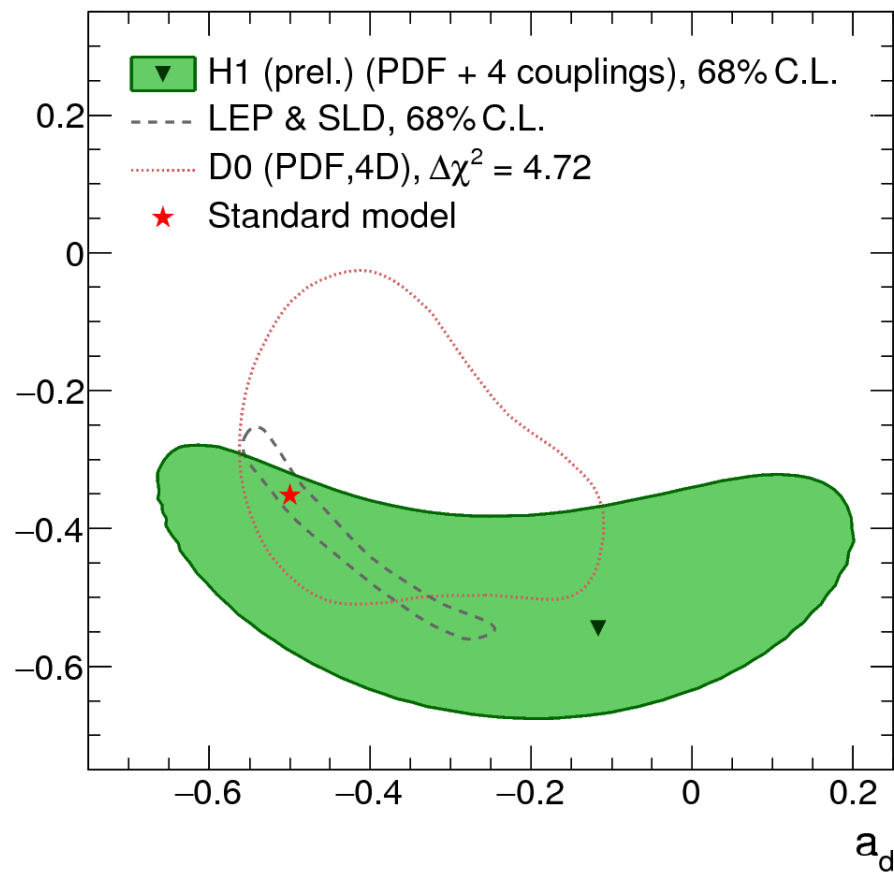
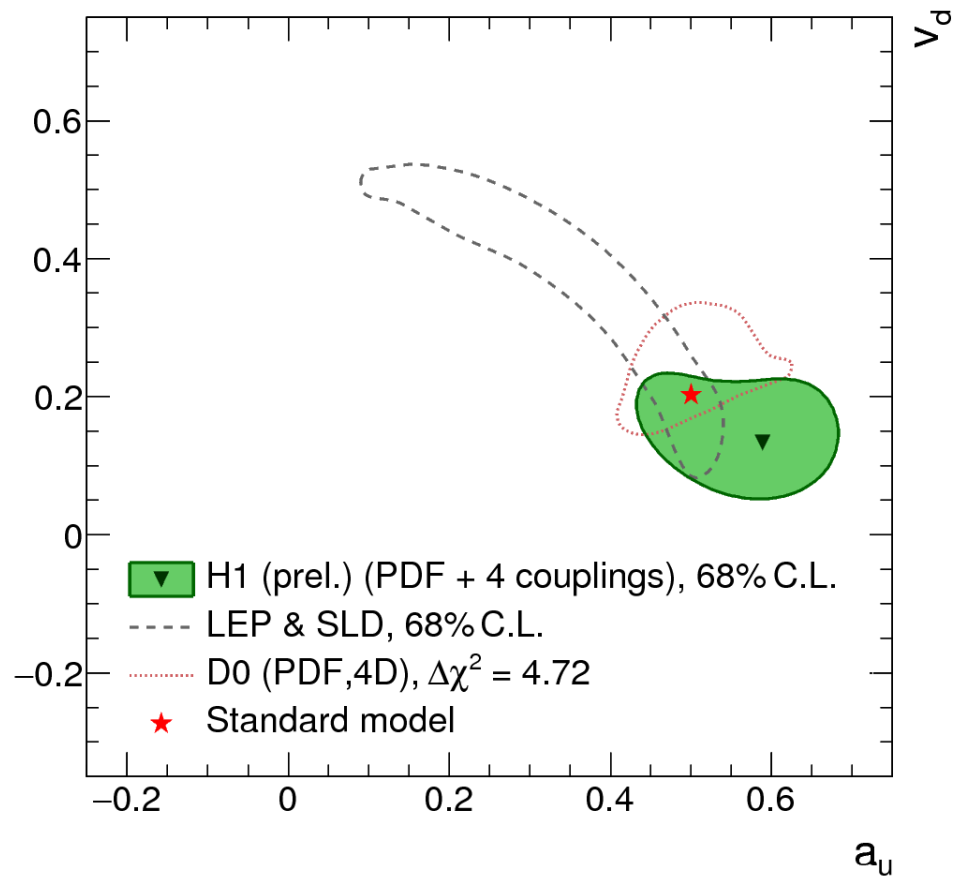
Fit: PDF + 2 couplings

- Reduced correlations and uncertainties
- Correlations between $a_u - a_d$ and $v_u - v_d$ large





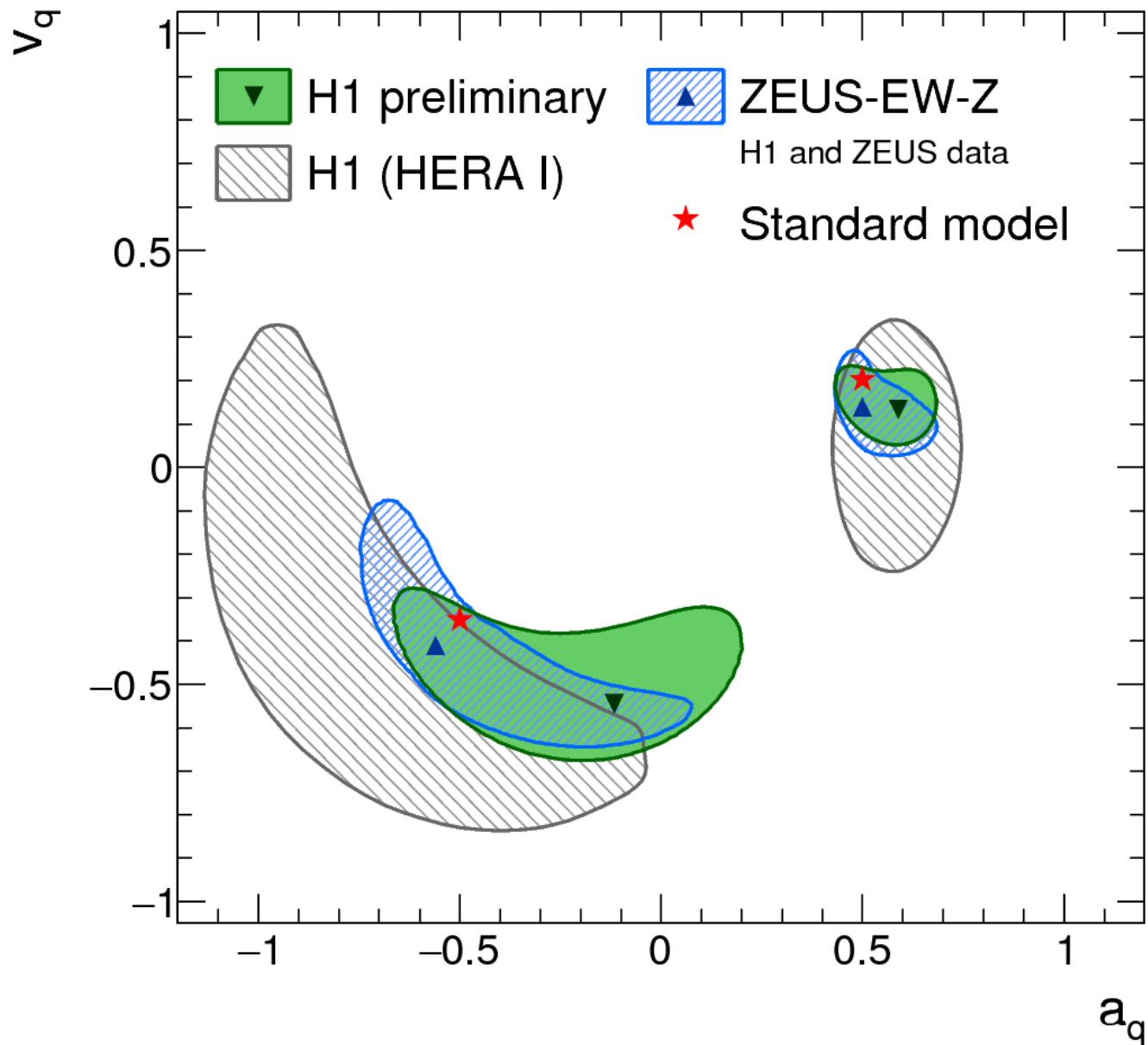
Comparison with other measurements

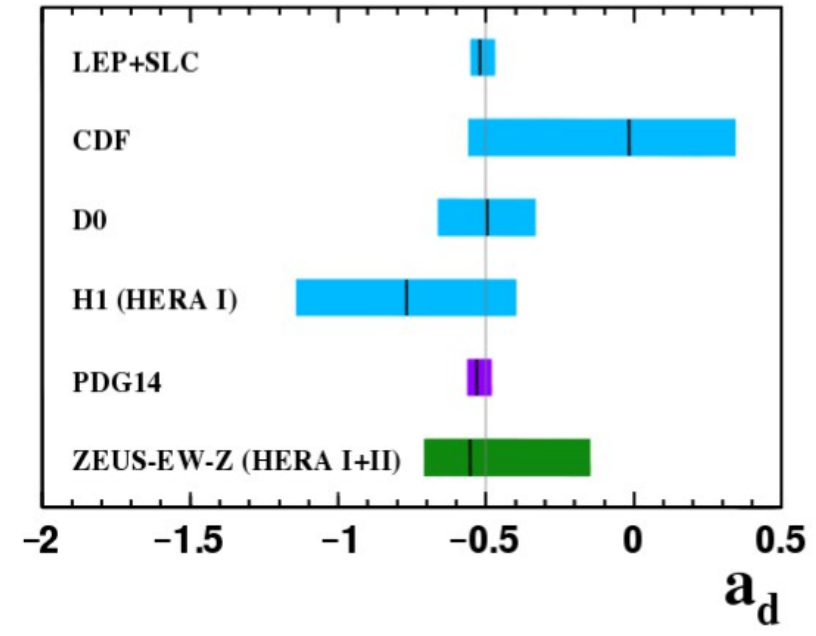
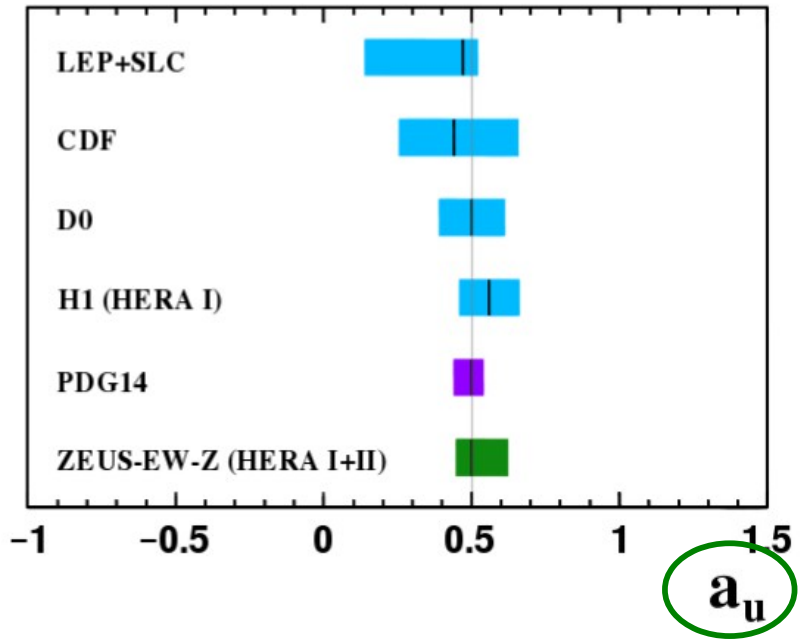
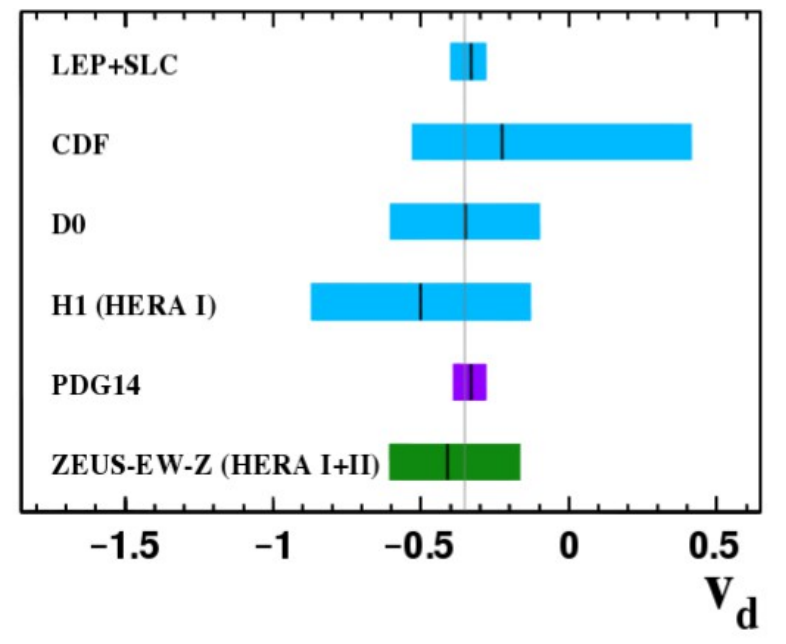
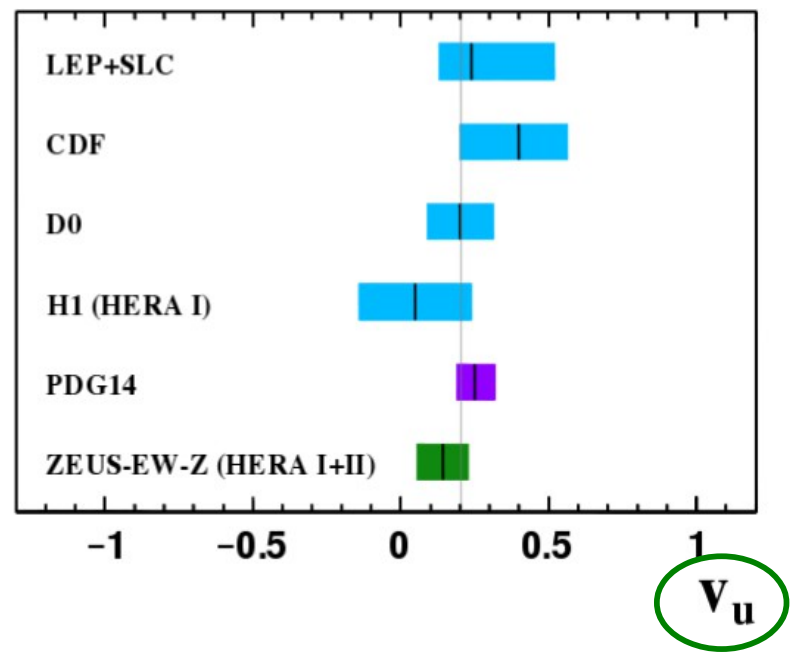


- Comparable precision of complementary processes



H1 and ZEUS Results Combined



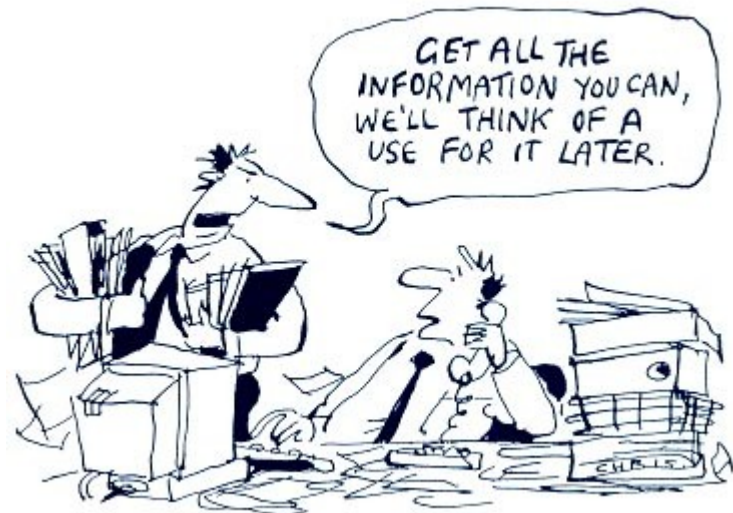


High sensitivity of HERA data to u-type quark couplings

Probing Standard Model

Standard Model is now overconstrained

- Important to study consistency in many complementary processes
- HERA: Space-like momentum transfers
- Only purely virtual exchange of bosons

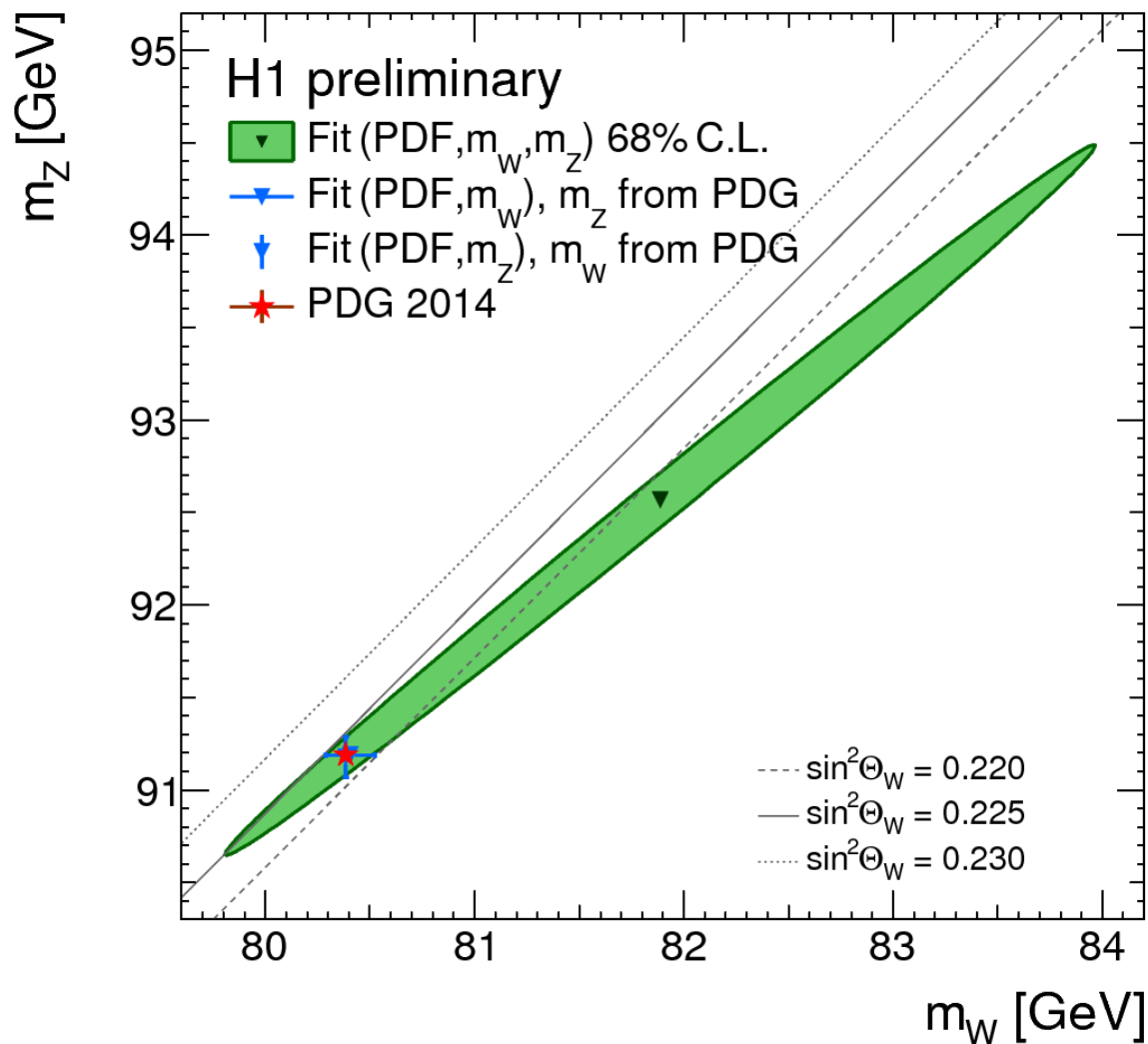


$(m_W - m_Z) + \text{PDF fits}$

- Assume α is known
- on-shell masses m_W and m_Z are only free EW parameters
- Agreement with SM
- Large correlation between m_W and m_Z

Mass of W boson

- Take other masses (m_Z) as external input to calculations



$$m_W = 80.407 \pm 0.118 \text{ (exp, pdf-fit)} \pm 0.005 \text{ (} m_Z, m_t, m_H \text{)} \text{ GeV}$$

$$M_W^{PDG 14} = 80.385 \pm 0.015 \text{ GeV}$$

Different view on SM parameters

- Fermi coupling constant G_F

$$G_F = \frac{\pi \alpha}{\sqrt{2} m_W^2 \sin^2 \theta_W} (1 + \Delta r)$$

- Weak mixing angle

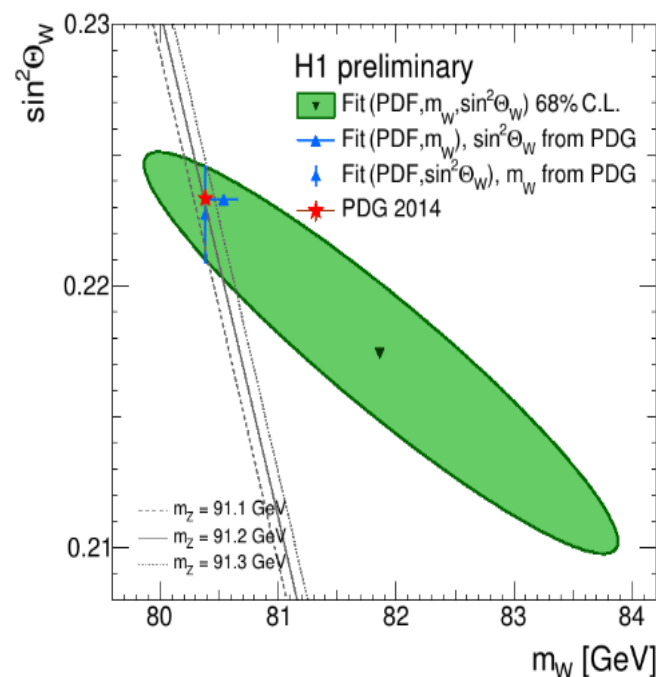
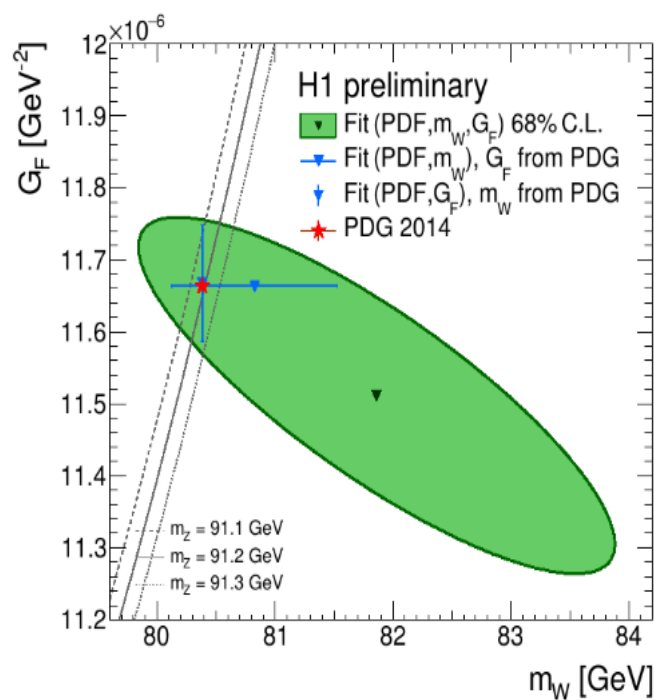
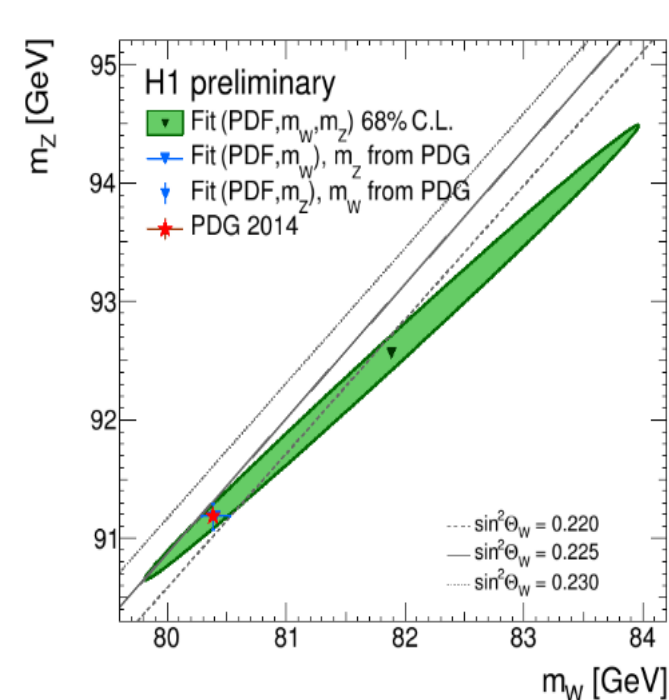
$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

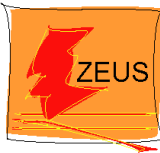
Perform calculations consistently in on-shell scheme (α, m_Z, m_W)

- Calculate m_Z (iteratively) from G_F or $\sin^2 \theta_W$

Results from fits together with PDF and m_W

- H1 values consistent with precise values from PDG
- Correlation to m_W are different for m_Z , $\sin^2 \theta_W$ and G_F





Simultaneous extraction of $\sin^2\theta_W$ and M_W

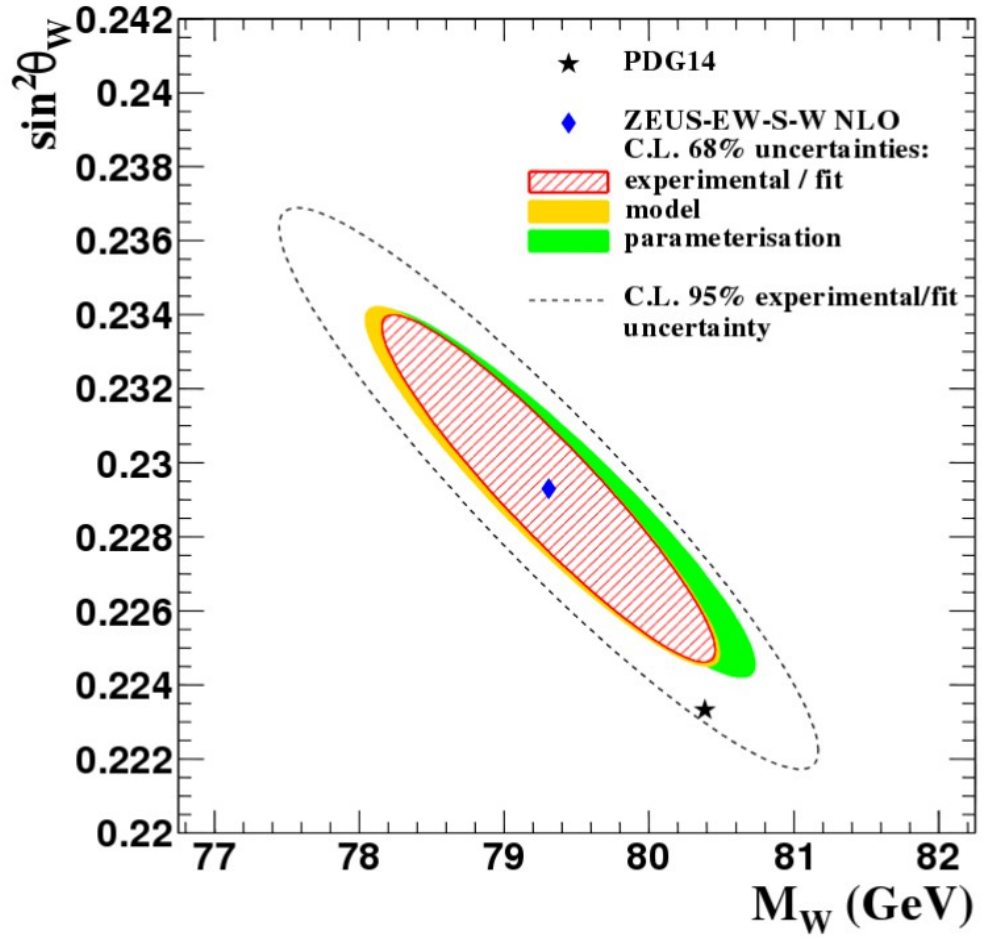
- Similar measurement by ZEUS

$$M_W = 79.30 \pm 0.76_{(expl/fit)} \begin{matrix} +0.38 \\ -0.08(mod) \end{matrix} \begin{matrix} +0.48 \\ -0.10(par) \end{matrix} GeV = 79.30^{+0.98}_{-0.77(tot)} GeV$$

$$\sin^2\theta_W = 0.2293 \pm 0.0031_{(expl/fit)} \begin{matrix} +0.0005 \\ -0.0001(mod) \end{matrix} \begin{matrix} +0.0003 \\ -0.0001(par) \end{matrix} = 0.2293^{+0.0032}_{-0.0031(tot)}$$

- All extracted quantities agree with world average values

ZEUS



$$M_W^{PDG14} = 80.385 \pm 0.015 GeV$$

$$\sin^2\theta_W^{PDG14 On-shell} = 0.22333 \pm 0.00011$$

$$corr(M_W, \sin^2\theta_W) = -0.930$$



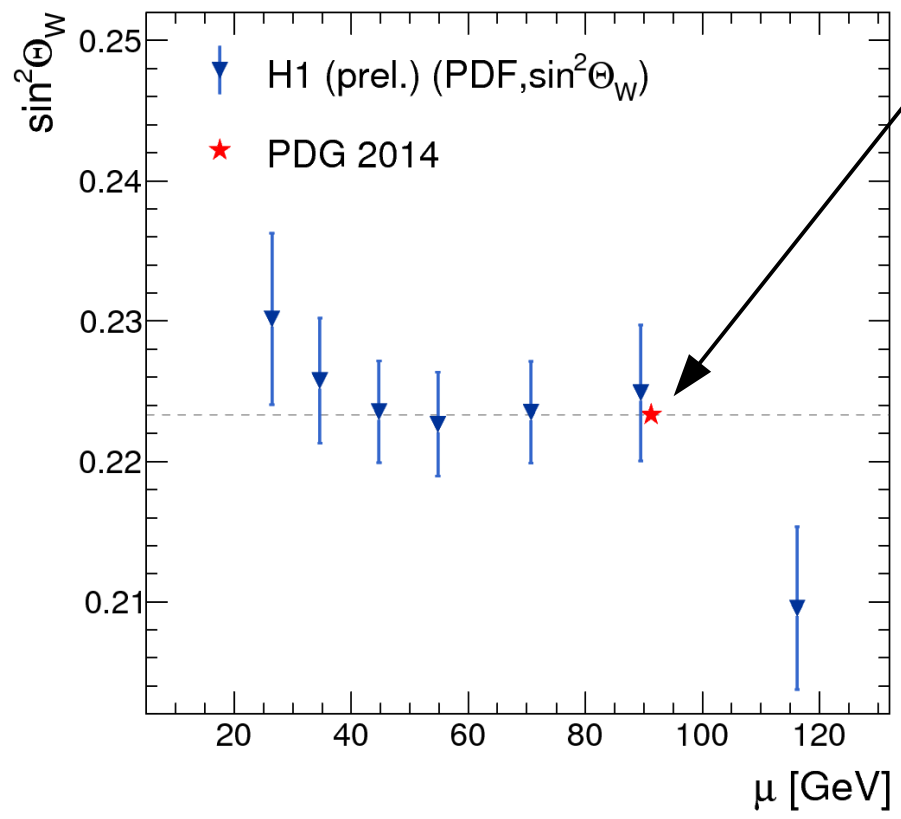
On-shell $\sin^2\theta_W$

- $\sin^2\theta_W$ determined simultaneously with PDF parameters (ZEUS-EW-S)

$$\sin^2\theta_W = 0.2252 \pm 0.0011_{(exp/fit)} \begin{matrix} +0.0003 \\ -0.0001(mod) \end{matrix} \begin{matrix} 0.0007 \\ -0.0001(par) \end{matrix} = \mathbf{0.2252^{+0.0013}_{-0.0011}(tot)}$$

- Consistent with PDG14

$$\sin^2\theta_W^{PDG14\ On-shell} = 0.22333 \pm 0.00011$$

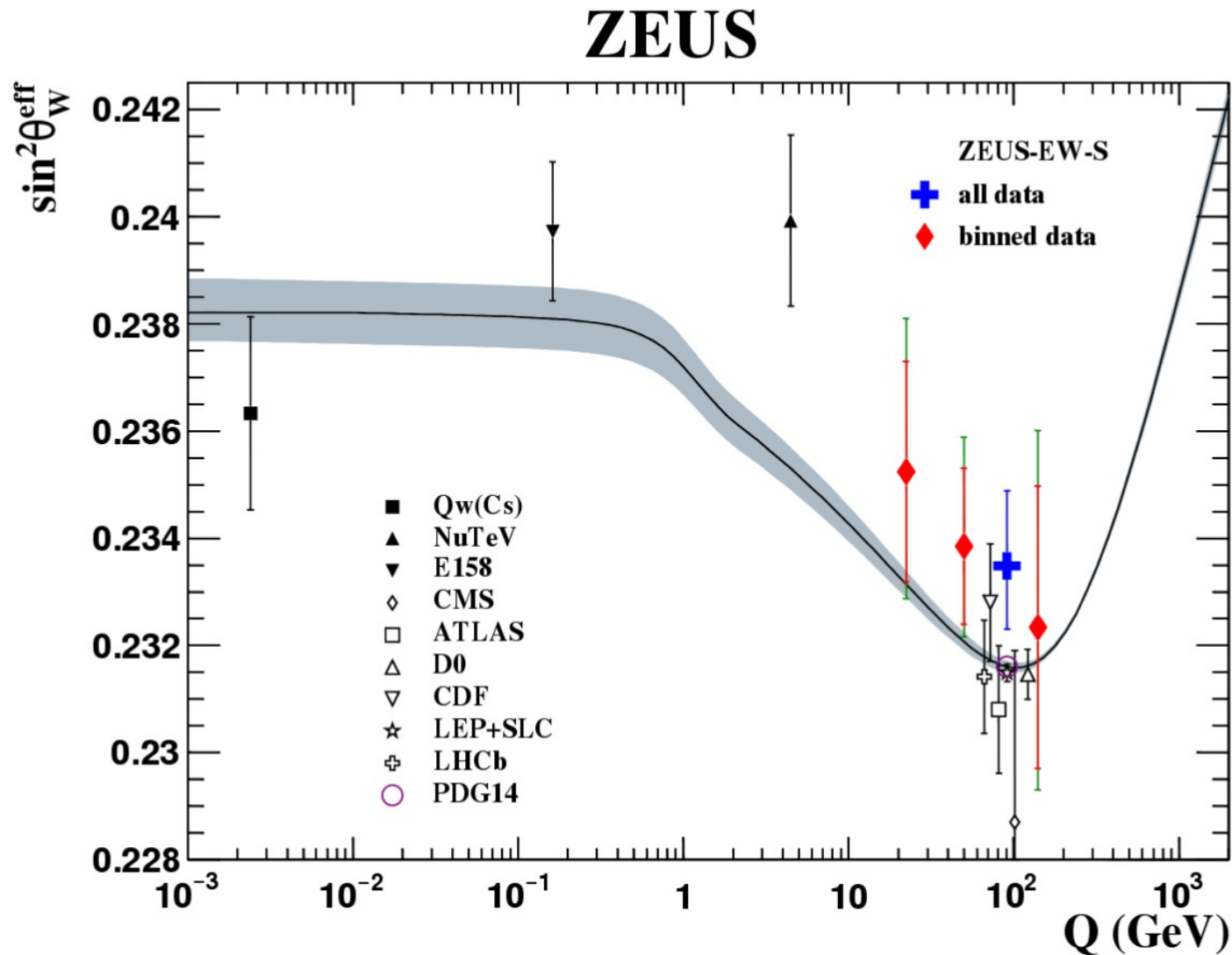


- $\sin^2\theta_W$ determined in Q^2 bins
- Unique measurement of weak mixing angle at different scales
- Agreement with PDG14
- Can be translated to \overline{MS} scheme



Effective $\sin^2\theta_W$

- On-shell measurements were translated to $\sin^2\theta_W^{eff}$
- First observation of effective $\sin^2\theta_W$ running from single machine

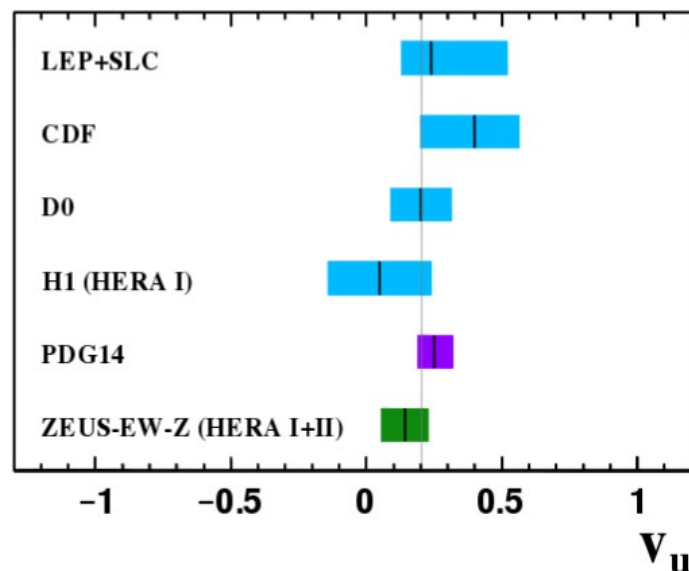




Summary



- HERA polarised inclusive data sensitive to electroweak parameters
 - Simultaneous PDF and EW fits
- Axial and vector-axial couplings to quarks agree with world average
- Measurements of **u-type** quark couplings among the most accurate



- Standard Model tests performed
 - Good consistency for M_Z , M_W , G_F and weak mixing angle
 - value of $\sin^2\theta_W$ competitive with measurements from neutrino sector
- $\sin^2\theta_W$ on-shell and effective determined for different scales
- Mass of W boson was determined at space-like momentum transfer

Back-up slides

On $\sin^2\theta_W(+X)$ fits to DIS data

DIS inclusive cross sections depend on $\sin^2\theta_W$ through:

- **Z propagator** in NC cross sections;
- **Vector couplings** of Z to quarks;

$$\tilde{F}_2^\pm = F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z$$

$$x\tilde{F}_3^\pm = -(a_e \pm P_e v_e) \chi_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \chi_Z^2 x F_3^Z$$

$$\chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2} \frac{1}{1 - \Delta R}$$

- **W propagator** (G_F);

$$\frac{d^2\sigma_{CC}(e^+p)}{dx_{Bj}dQ^2} = (1 + P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x[(\bar{u} + \bar{c}) + (1 - y)^2(d + s + b)]$$

$$\frac{d^2\sigma_{CC}(e^-p)}{dx_{Bj}dQ^2} = (1 - P_e) \frac{G_F^2 M_W^4}{2\pi x_{Bj} (Q^2 + M_W^2)^2} x[(u + c) + (1 - y)^2(\bar{d} + \bar{s} + \bar{b})]$$

$$G_F = \frac{\pi\alpha}{\sqrt{2} \sin^2 \theta_W M_W^2} \frac{1}{1 - \Delta R}$$

ΔR is an EW correction.

[arXiv:hep-ph/9902277](https://arxiv.org/abs/hep-ph/9902277)

Re-expressing G_F through $\sin^2\theta_W$ and M_W allows to use both CC and NC for $\sin^2\theta_W$ determination.

- Current analysis exploits all three dependences for $\sin^2\theta_W$ extraction.
- $\sin^2\theta_W$ values extracted in current analysis correspond to **On-shell scheme**.

Quark couplings to Z

Now consider fits to electroweak NC couplings as well as PDF parameters

The total cross-section : $\sigma = \sigma^0 + P \sigma^P$

The unpolarised cross-section is given by $\sigma^0 = Y_+ F_2^0 + Y_- xF_3^0$

$$F_2^0 = \sum_i A_i^0(Q^2) [xq_i(x, Q^2) + xq_i(\bar{x}, Q^2)]$$

$$xF_3^0 = \sum_i B_i^0(Q^2) [xq_i(x, Q^2) - xq_i(\bar{x}, Q^2)]$$

$$A_i^0(Q^2) = e_i^2 - 2 e_i \mathbf{v}_i \mathbf{v}_e P_Z + (\mathbf{v}_e^2 + \mathbf{a}_e^2)(\mathbf{v}_i^2 + \mathbf{a}_i^2) P_Z^2$$

$$B_i^0(Q^2) = -2 e_i \mathbf{a}_i \mathbf{a}_e P_Z + 4 \mathbf{a}_i \mathbf{a}_e \mathbf{v}_i \mathbf{v}_e P_Z^2$$

$$P_Z = \frac{1}{\sin^2 2\theta} \frac{Q^2}{(M_Z^2 + Q^2)}$$

The polarised cross-section is given by $\sigma^P = Y_+ F_2^P + Y_- xF_3^P$

$$F_2^P = \sum_i A_i^P(Q^2) [xq_i(x, Q^2) + xq_i(\bar{x}, Q^2)]$$

$$xF_3^P = \sum_i B_i^P(Q^2) [xq_i(x, Q^2) - xq_i(\bar{x}, Q^2)]$$

$$A_i^P(Q^2) = 2 e_i \mathbf{v}_i \mathbf{a}_e P_Z - 2 \mathbf{v}_e \mathbf{a}_e (\mathbf{v}_i^2 + \mathbf{a}_i^2) P_Z^2$$

$$B_i^P(Q^2) = 2 e_i \mathbf{a}_i \mathbf{v}_e P_Z - 2 \mathbf{a}_i \mathbf{v}_i (\mathbf{v}_e^2 + \mathbf{a}_e^2) P_Z^2$$

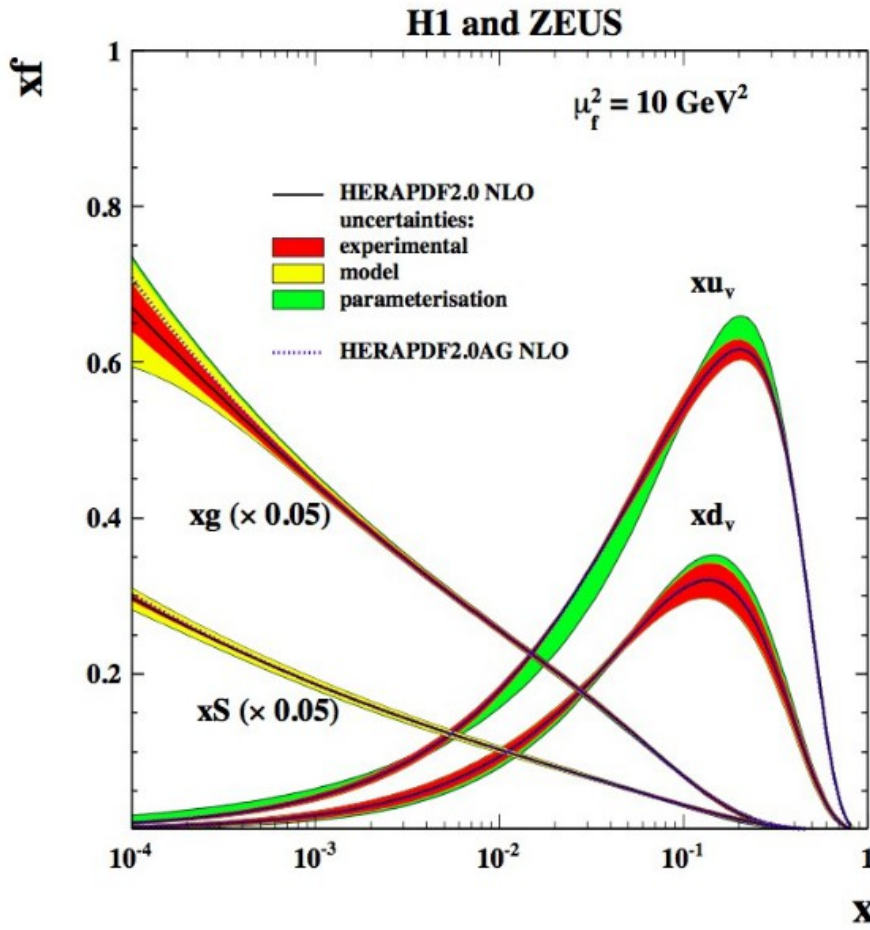
$P_Z \gg P_Z^2$ (γZ interference is dominant)
 \mathbf{v}_e is very small (~ 0.04).



unpolarized $xF_3 \rightarrow \mathbf{a}_i$,
 polarized $F_2 \rightarrow \mathbf{v}_i$

From slides by Amanda Cooper-Sarkar

Color decomposition of uncertainties



Experimental uncertainties:

- Hessian method
- Conventional $\Delta\chi^2 = 1 \Rightarrow 68\% \text{ CL}$

Variation	Standard Value	Lower Limit	Upper Limit
Q_{\min}^2 [GeV ²]	3.5	2.5	5.0
Q_{\min}^2 [GeV ²] HiQ2	10.0	7.5	12.5
M_c (NLO) [GeV]	1.47	1.41	1.53
M_c (NNLO) [GeV]	1.43	1.37	1.49
M_b [GeV]	4.5	4.25	4.75
f_s	0.4	0.3	0.5
μ_{f_0} [GeV]	1.9	1.6	2.2

Adding D and E parameters to each PDF

Parametrisation uncertainties

- largest deviation

Model uncertainties

- all variations added in quadrature

Fit methodology I

Determine light-quark couplings

- Use iterative minimisation procedure ('fit') of cross section predictions to data

Unfortunate correlation

- PDFs have considerable uncertainties
- These PDFs are essentially determined from H1 structure function data
-> Large correlations
- Consider PDF uncertainty by simultaneous fit of PDFs and light quark couplings

Consistency of fit-parameters in SM formalism

- Perform calculations strictly in on-shell scheme
Parameters are: α , m_Z , m_W , (m_t , m_H , ...)

Polarisation measurement

- Measurements of the beam polarisations are measurements on their own
-> Consider these measurements as independent measurements in fit

1-loop EW corrections

- May be considered in terms of 'EW form factors'
- Are ignored in the present analysis, but will be included in the future

Fit methodology II

New C++-based fitting code for PDF and more general fits developed (Alpos)

- DGLAP evolution of PDFs in NNLO QCD (QCDNUM with ZMVFNS)
- PDFs are parameterised at starting scale $Q_0^2 = 1.9\text{GeV}^2$ (similar to HERAPDF2.0)

xg	xg	$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}$	<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="width: 15px; height: 15px; background-color: #ccc; border: 1px solid #000;"></div> fixed or constrained by sum-rules </div> <div style="display: flex; justify-content: space-between; align-items: center; margin-top: 5px;"> <div style="width: 15px; height: 15px; background-color: #007bff; border: 1px solid #000;"></div> parameters set equal but free </div>
xu_v	$xU = xu + xc$	$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2)$	
xd_v	$xD = xd + xs$	$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$	
$x\bar{U}$	$x\bar{U} = x\bar{u} + x\bar{c}$	$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x)$	
$x\bar{D}$	$x\bar{D} = x\bar{d} + x\bar{s}$	$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$	

- Use only data with $Q^2 \geq 12 \text{ GeV}^2$

χ^2 Definition

- Uncertainties on cross sections are assumed to be 'log-normal' distributed (relative uncertainties)
- Uncertainties on polarisation measurements are assumed to be 'normal' distributed
- Correlations of syst. uncertainties between different datasets are considered

$$\chi^2 = (\log(d) - \log(t))^T V_R^{-1} (\log(d) - \log(t)) + (d - t)^T V_A^{-1} (d - t)$$

Fit parameters

- 13 PDF parameters
- 4 polarisation values
- 4 Light-quark couplings (or other SM parameters)
- More general also 'nuisance parameters' of syst. uncertainties

Polarised deep-inelastic ep scattering

Neutral and charged current at tree level

$$\frac{d\sigma_{NC}^{\pm}}{dQ^2 dx} = \frac{2\pi\alpha^2}{x} \left[\frac{1}{Q^2} \right]^2 (Y_+ F_2 + Y_- x F_3 + y^2 F_L)$$

$$\frac{d\sigma_{CC}^{\pm}}{dQ^2 dx} = \frac{1 \pm P}{2} \frac{G_F^2}{4\pi x} \left[\frac{m_W^2}{m_W^2 + Q^2} \right]^2 (Y_+ W_2^{\pm} \pm Y_- x W_3^{\pm} - y^2 W_L^{\pm})$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

Calculations in on-shell scheme

$$G_F = \frac{2\pi\alpha}{2\sqrt{2}m_W^2} \left(1 - \frac{m_W^2}{m_Z^2} \right)^{-1} (1 + \Delta r)$$

Corrections to G_F

$$\Delta r = \Delta r(\alpha, m_W, m_Z, m_t, m_H, \dots)$$

Parameters to calculations

Parameters to cross section calculation: $\alpha, m_Z, m_W (m_t, m_H, \dots)$

More general, also couplings: v_e, a_e, v_u, a_u and v_d, a_d

Generalised structure functions

$$F_2 = F_2^y + \kappa_Z (-v_e \mp P a_e) F_2^{yZ} + \kappa_Z^2 (v_e^2 + a_e^2 \pm P v_e a_e) F_2^Z$$

$$x F_3 = +\kappa_Z (\pm a_e + P v_e) F_3^{yZ} + \kappa_Z^2 (\mp 2 v_e a_e - P (v_e^2 + a_e^2)) x F_3^Z$$

Z⁰-exchange

$$\kappa_Z(Q^2) = \frac{Q^2}{Q^2 + m_Z^2} \frac{G_F m_Z^2}{2\sqrt{2}\pi\alpha}$$

Structure functions in QPM

$$[F_2, F_2^{yZ}, F_2^Z] = x \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] \{q + \bar{q}\}$$

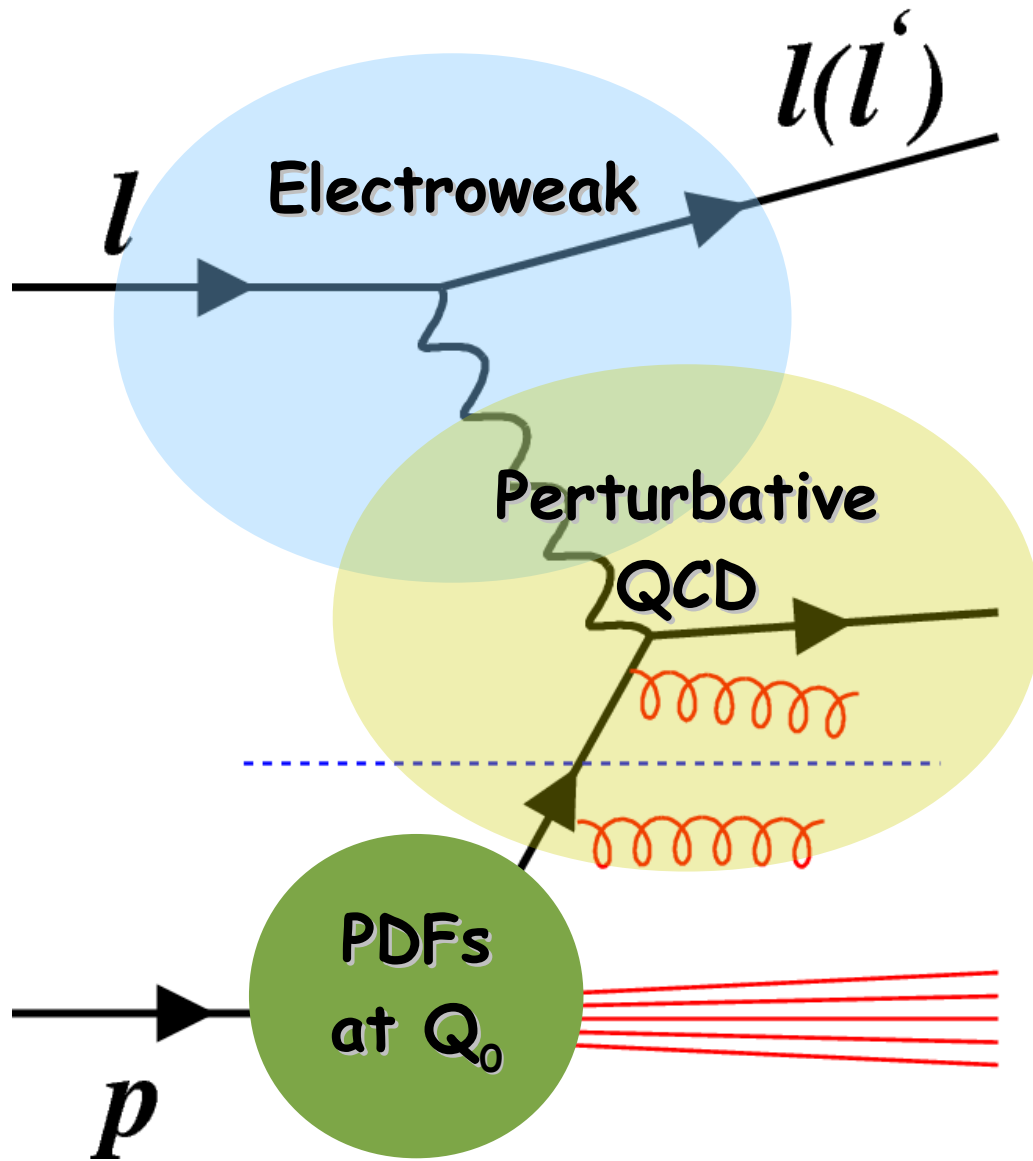
$$[xF_3^{yZ}, xF_3^Z] = x \sum_q [2e_q a_q, 2v_q a_q] \{q - \bar{q}\}$$

Weak couplings to Z-boson

$$v_f = I_{f,L}^{(3)} - 2e_f \sin^2 \theta_W \quad (f = e, u, d, \dots)$$

$$a_f = I_{f,L}^{(3)}$$

Deep Inelastic Scattering @ HERA



- Fix pQCD & PDFs
! Test Electroweak
- Fix Electroweak
! Test pQCD & PDFs

- Fix Electroweak & pQCD
! Determine PDFs