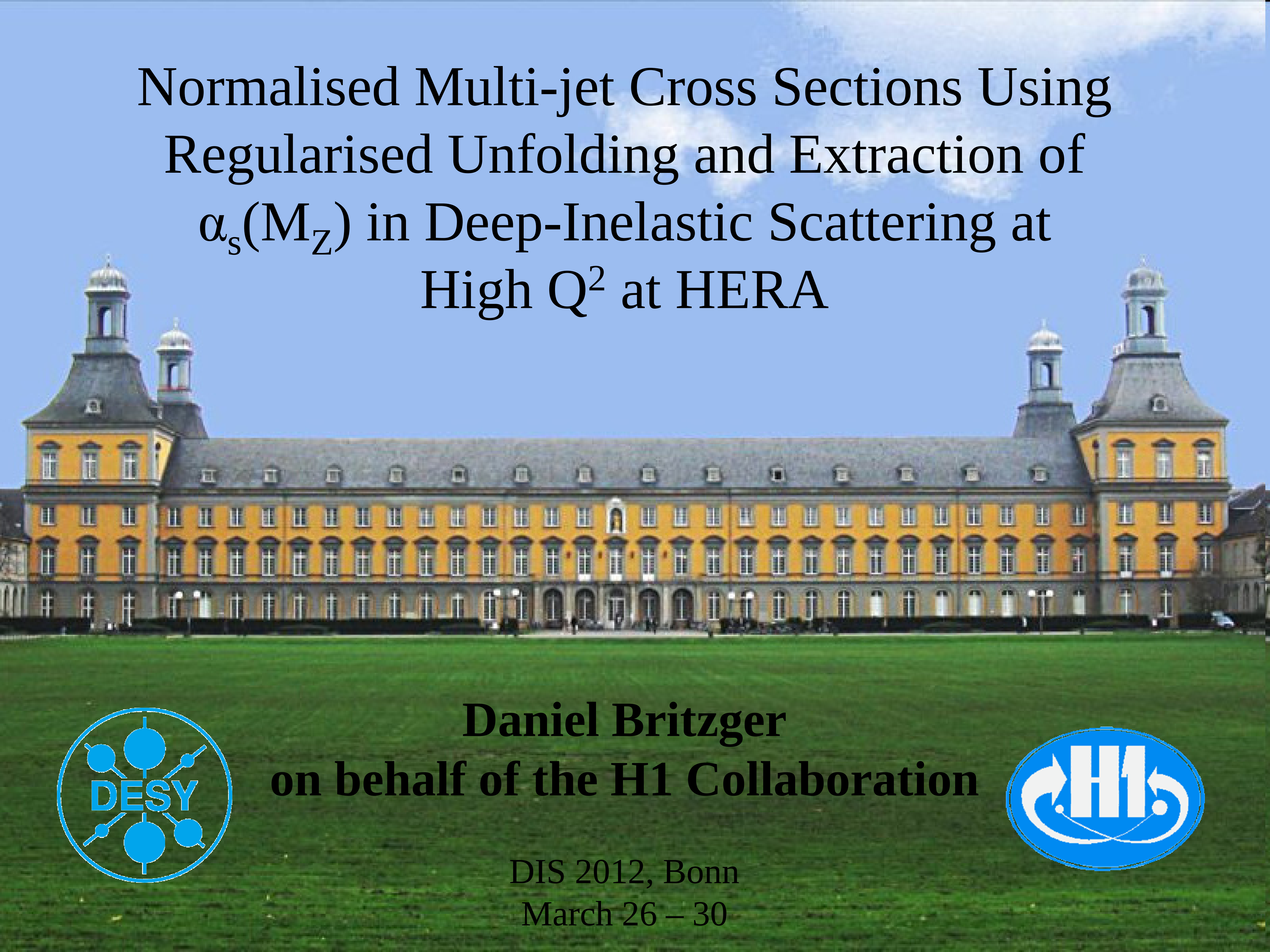


Normalised Multi-jet Cross Sections Using Regularised Unfolding and Extraction of $\alpha_s(M_Z)$ in Deep-Inelastic Scattering at High Q^2 at HERA

Daniel Britzger

on behalf of the H1 Collaboration

DIS 2012, Bonn

March 26 – 30

Normalised Multi-jet Cross Sections Using Regularised Unfolding and Extraction of $\alpha_s(M_Z)$ in Deep-Inelastic Scattering at High Q^2 at HERA

Jetproduction in DIS

Regularized Unfolding

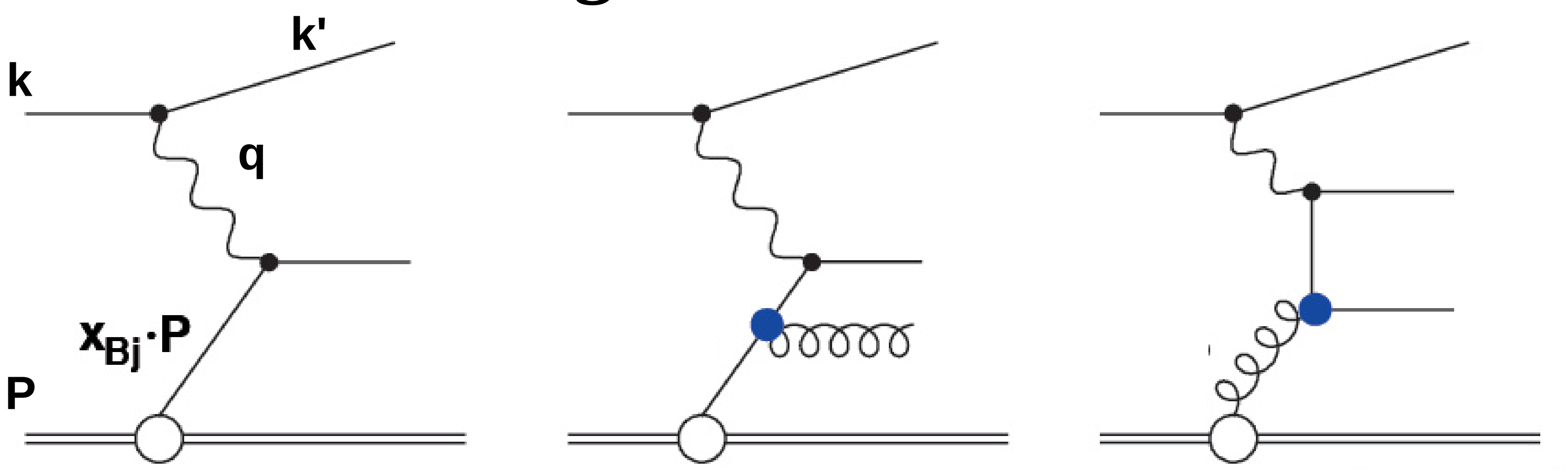
Normalized Multijet Cross Sections

Determination of α_s

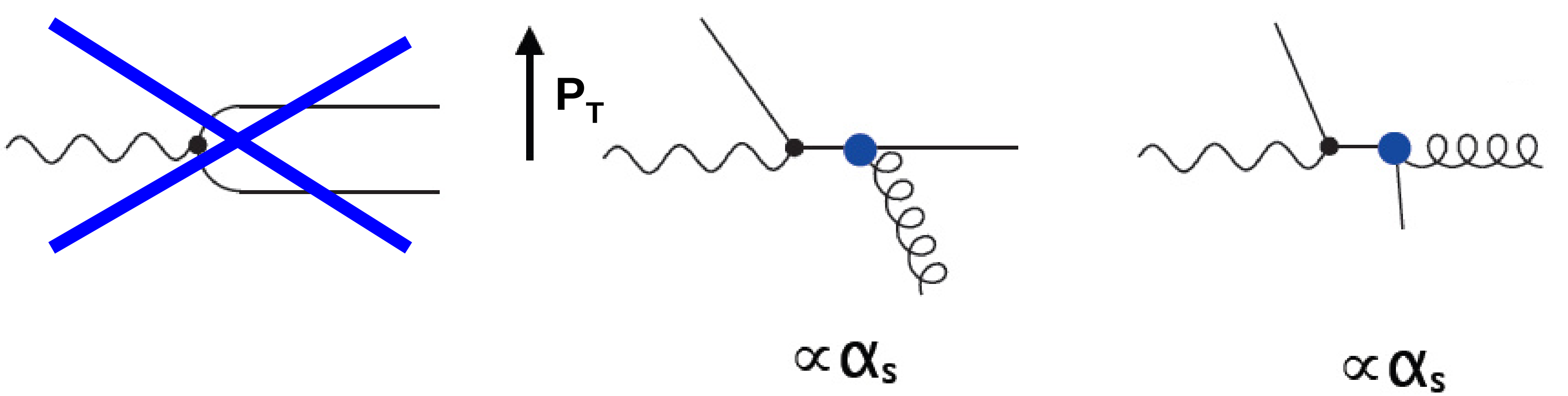
Summary

Jet production in DIS

Jet Production in Leading Order



Analysis is performed in **Breit frame**: $2x_{Bj}P + k = 0$

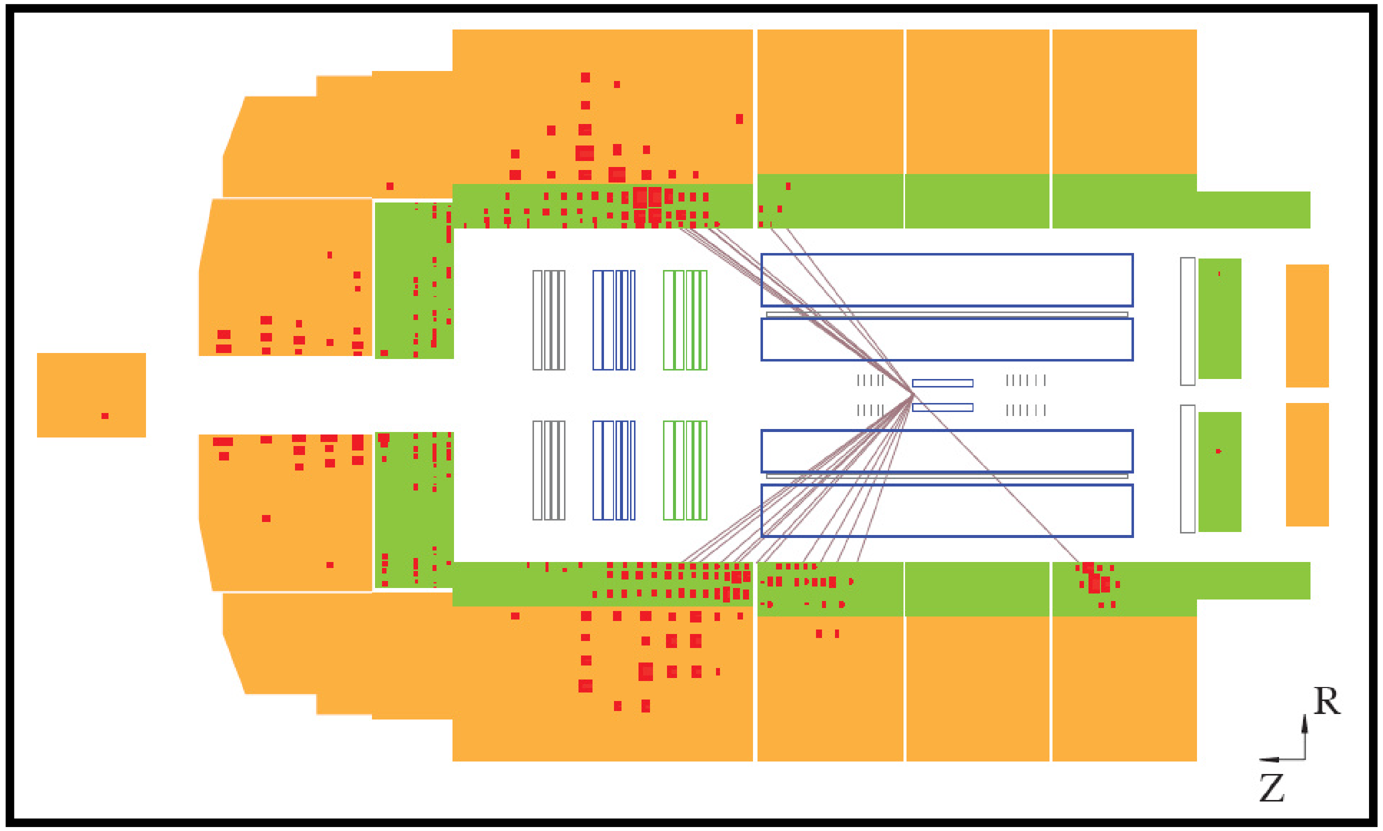


Only hard QCD processes generate considerable P_T in the Breit frame

Leading order: n-jet production ~ α_s^{n-1}

Direct sensitivity to α_s and gluon density

Multijet Measurement



Normalized Jet Measurment

Normalized Jets

$d\sigma_{Jet} / d\sigma_{DIS}$

Four Ingredients

(inclusive) DIS Measurement	$d\sigma/dQ^2$
Inclusive Jet	$d\sigma/dQ^2dP_T$
Dijet	$d\sigma/dQ^2d\langle P_T\rangle$
Trijet	$d\sigma/dQ^2d\langle P_T\rangle$

Phase Space

Neutral current phase space

- $150 < Q^2 < 15000$ GeV^2
- $0.2 < y < 0.7$

Jet phase space

- $-1.0 < \eta_{Lab} < 2.5$

Inclusive Jet

- $7.0 < P_T < 50$ GeV

Dijet and Trijet

- $5 < P_T < 50$ GeV
- $7 < \langle P_T\rangle < 50$ GeV
- $M_{12} > 16$ GeV

Kinematics

DIS Event = scattered electron

$Q^2 \quad = -q^2 = (k - k')^2$

$y \quad = q\cdot P / k\cdot P$ (Inelasticity)

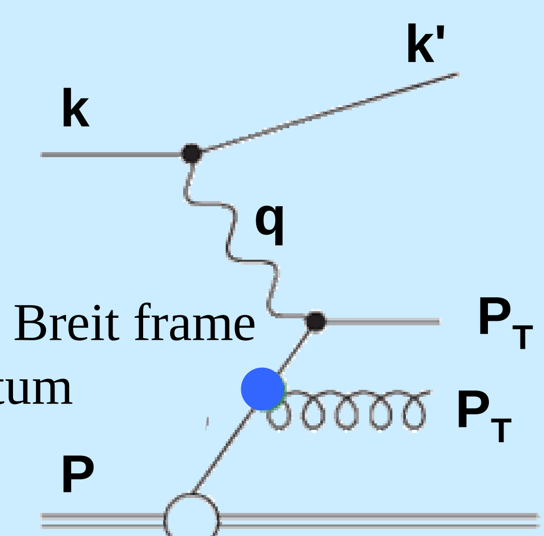
Jet kinematics

p_T = jet transverse momentum in Breit frame

$\langle p_T\rangle$ = Average transverse momentum of 2 (resp. 3) jets

η = pseudorapidity of Jet

M_{12} = invariant mass of two leading jets



Correction for Detector Effects

Aim

- Cross section on particle level
- Correction for detector effects

Detector Response

- Steeply falling P_T and Q^2 spectra
- Boost to Breit frame
- Finite resolution

Bin-by-bin correction

- Detector correction based on bin-wise correction factors
- Some error propagation from migrations

Migrations are getting relevant

-> not respected by bin-by-bin method

Full/Correct error propagation is difficult

Regularized Unfolding using TUnfold

Regularized Unfolding

(Migration) Matrix A is describing detector response

$$\vec{m} = A \cdot \vec{x}$$

m: measured distribution (Detector level)

x: 'true' distribution (Particle level)

Find particle level x by analytic minimization of χ^2 as function of x

$$\chi^2(x) = \tfrac{1}{2}(m - Ax)^T V^{-1}(m - Ax) + \tau^2 \cdot L$$

TUnfold v17 (S.Schmitt)

Regularisation parameter τ suppresses large fluctuations/errors in comparision to direct matrix inversion

Unfolding of four measurements at once

NC-DIS, Inclusive Jet, Dijet und Trijet

-> same dataset, same events

-> Covariance matrix V contains correlations

Schematic Definition of Migration Matrix

Inclusive Jet

Jet algorithm runs on

Detector level

Particle level

Matching with geometrical measure

$$r \equiv \sqrt{(\varphi_1 - \varphi_2)^2 + (\eta_1 - \eta_2)^2} < 0.9$$

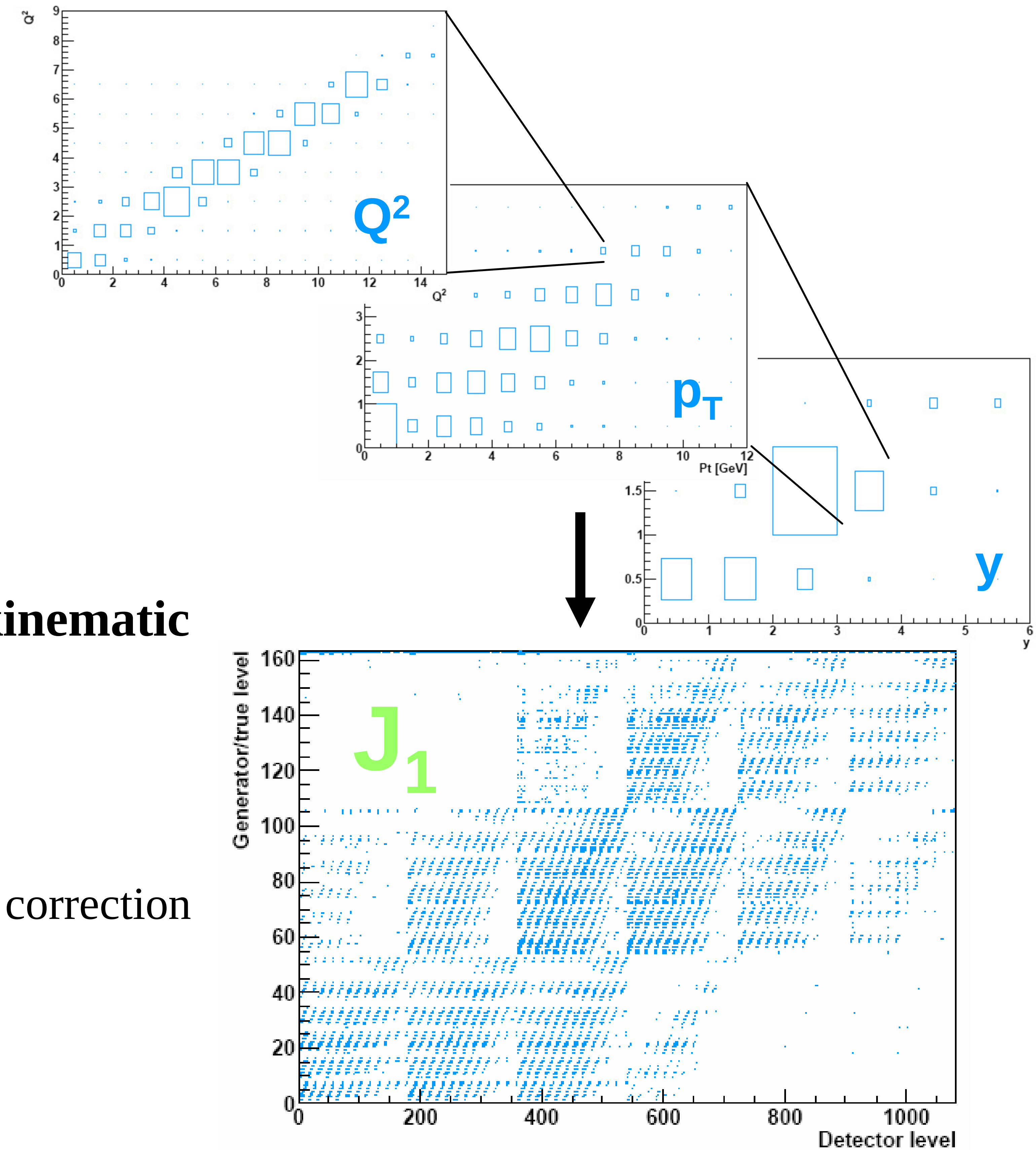
Multidimensional unfolding in kinematic variables

Q^2, p_T, y

Different jet multiplicities

'only' particle level jets -> Efficieny correction

'only' detector level jets ->
are estimated using NC-DIS events



Extending Migration Matrix with NC DIS Measurement

Neutral Current (NC) DIS

- Each single event
- Just electron kinematics
- 2-dim Unfolding: Q^2, y

Two Measurements

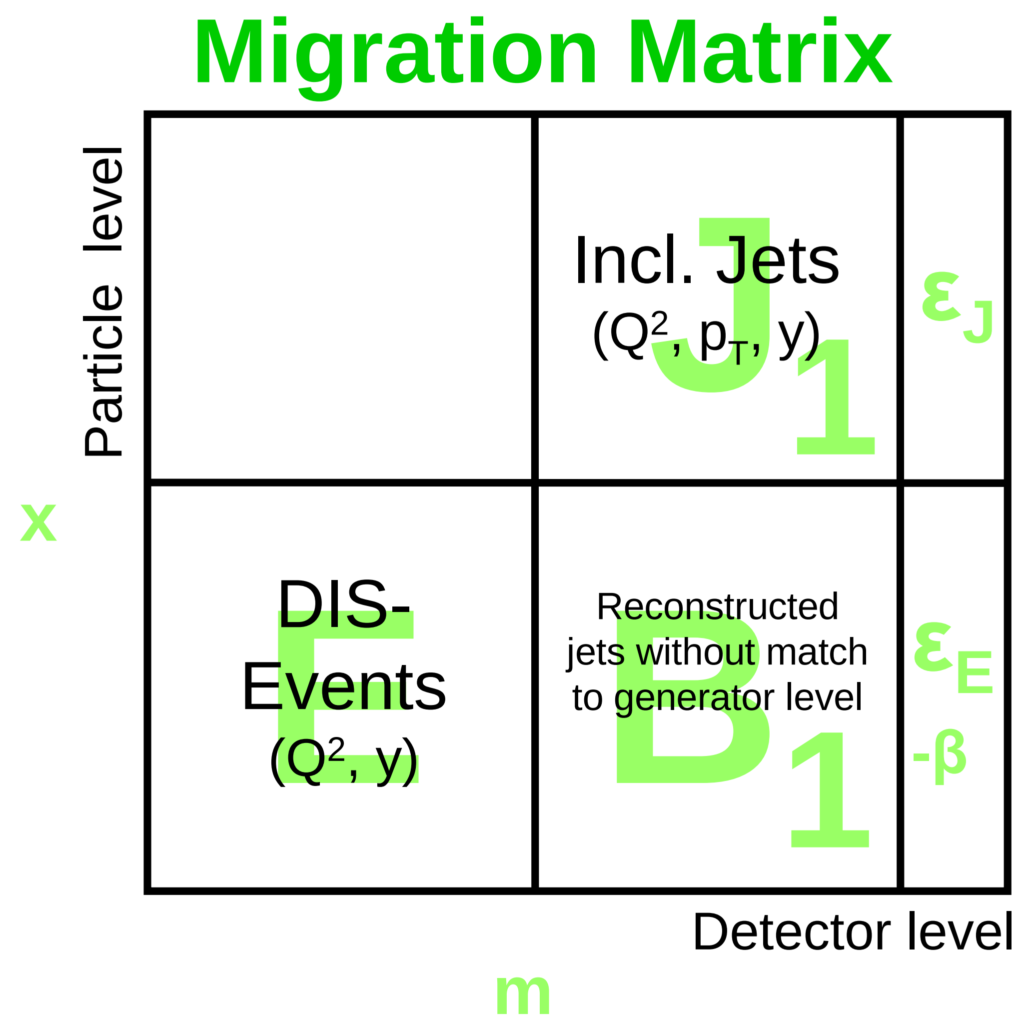
- Taking Correlations into account

'only Detector level' jets

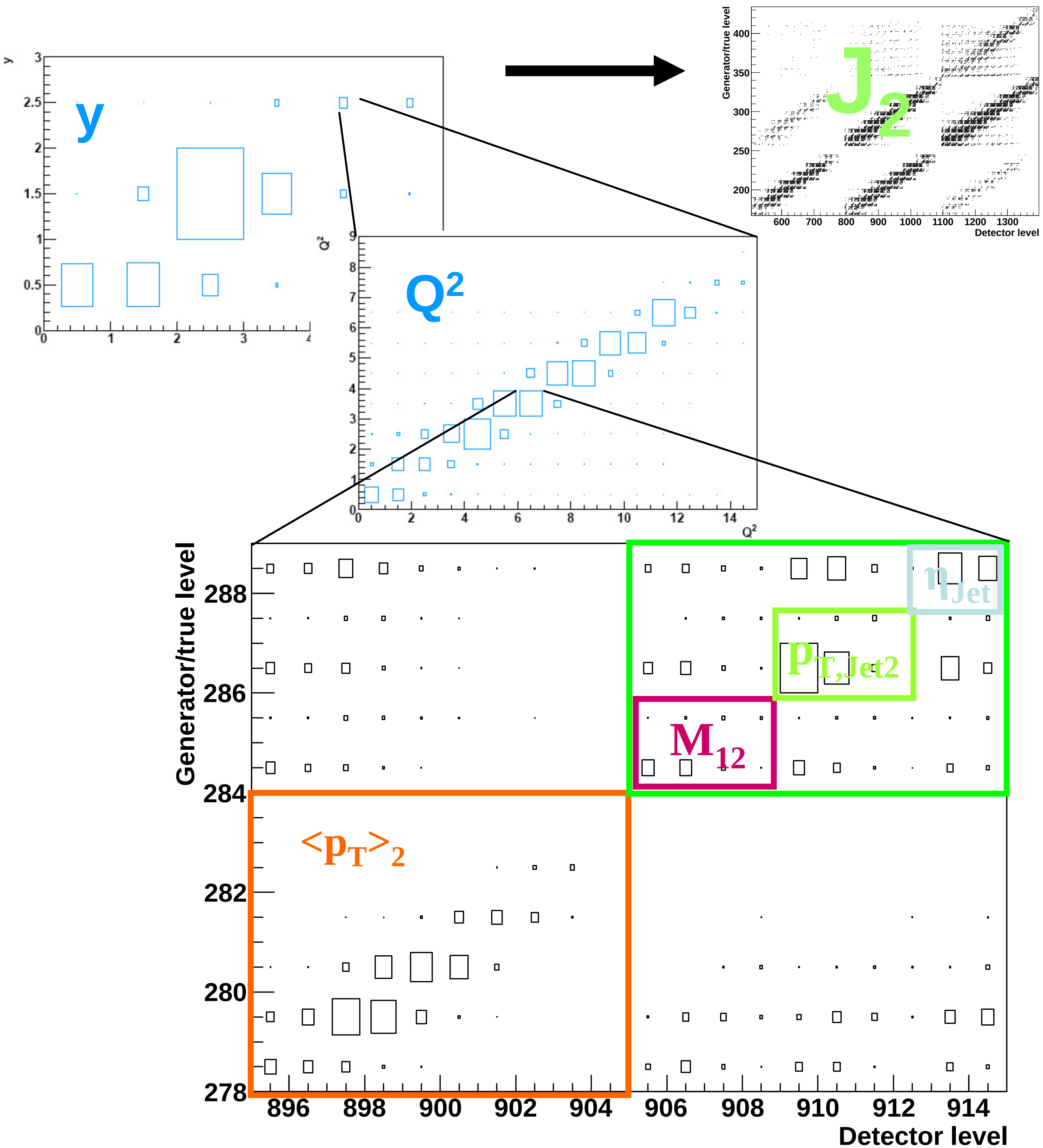
- Are estimated with electron kinematics
- Preserve normalization with $-\beta$

Always 1.5 × more bins on detector level than particle level

Extended phase space on detector level



Schematic Definition of Migration Matrix



Dijet and Trijet Measurement

Measurement of 'event properties' of an event 'class'

$d\sigma/dQ^2d\langle p_T\rangle$

Migrations in 'kinematic' variable $\langle p_T\rangle$, Q^2 and y

Define 'Dijet'

M_{12}, η_{Jet} , $p_{T,Jet}$ phase space cuts
… and two jets

Respect Migrations into/out of these cuts

Full Schematic Migration Matrix

Migration Matrix

Particle level (x)

			Trijet Q^2, $<p_T>_3$, y, Trijet-cuts J_3	ε_{J3}
		Dijet Q^2, $<p_T>_2$, y, Dijet-cuts J_2		ε_{J2}
	Incl. Jet p_T, Q^2, y, (η) J_1			ε_J
DIS-Events (Q^2, y) E	Reconstructed jets without match to generator level B_1	Reconstructed Dijet events which are not generated as Dijet event B_2	Reconstructed Trijet events which are not generated as Trijet event B_3	ε_E $-\beta_1$ $-\beta_2$ $-\beta_3$

Detector level

m

Matrix dimension: 2205 × 671 Entries

Covariance Matrix: 2205 x 2205 Entries

Monte Carlo Test

Pull distributions

Corrected vs. true distribution

$$P_i = \frac{x_i^{Unfold} - x_i^{true}}{\Delta x_i}$$

Two Incl. DIS Models

Rapgap (MEPS)

Django (CDM)

Statistically independent samples

Checking

Unfolding with same model

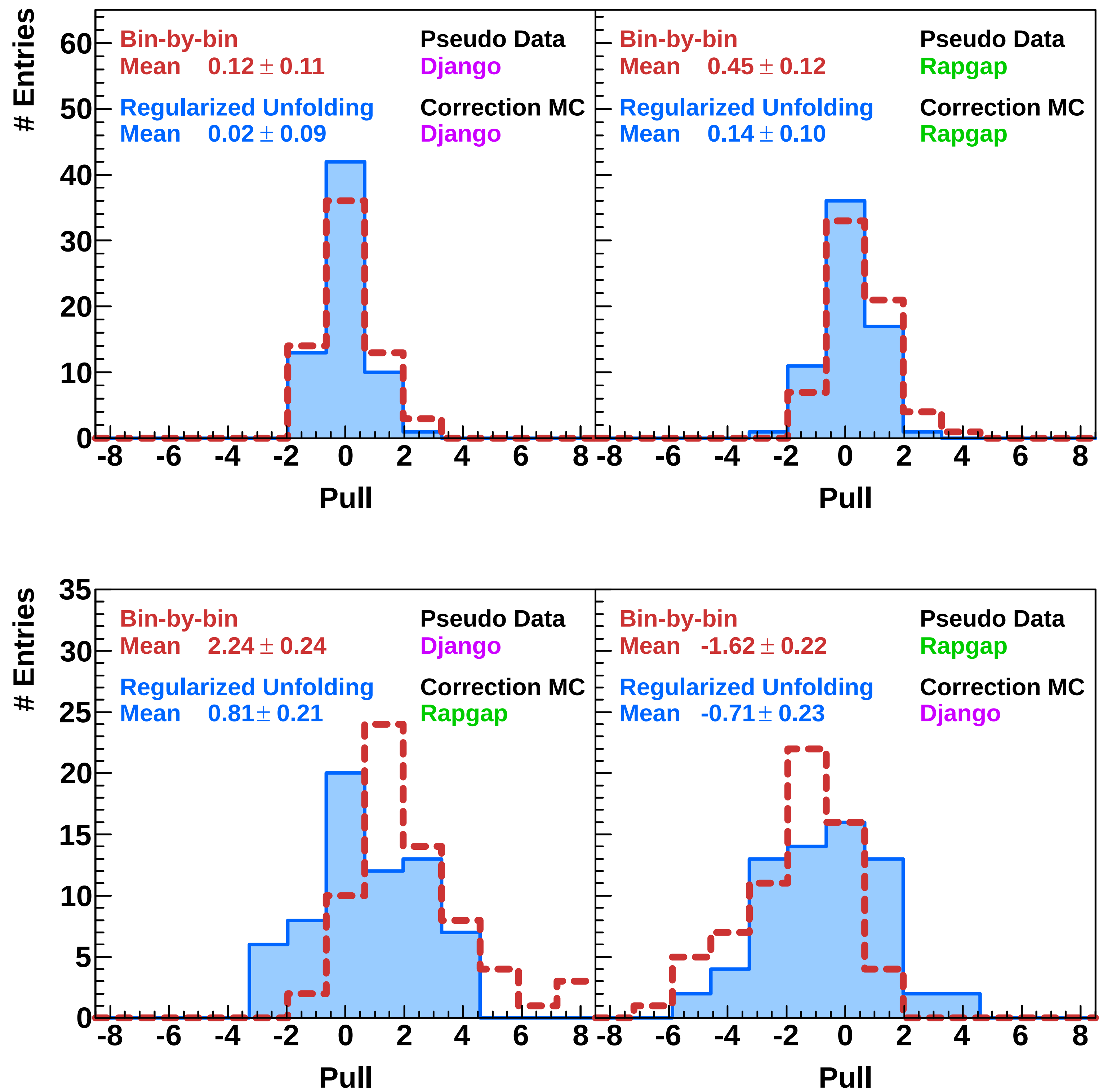
Unfolding with 'other' model

Compare

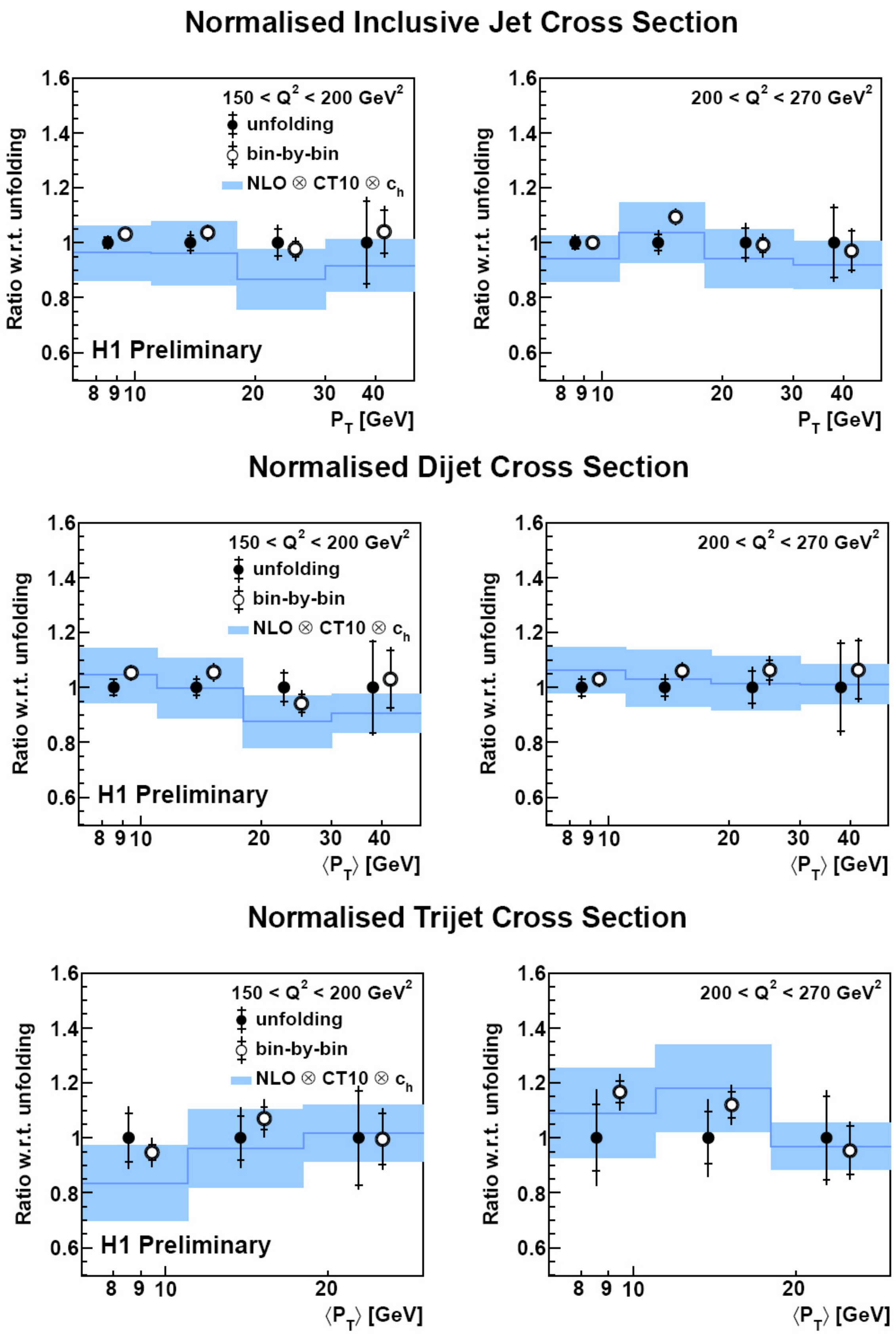
Bin-by-bin

Based on bin-wise correction factors

Regularized unfolding



Data unfolding
Comparison to bin-by-bin method



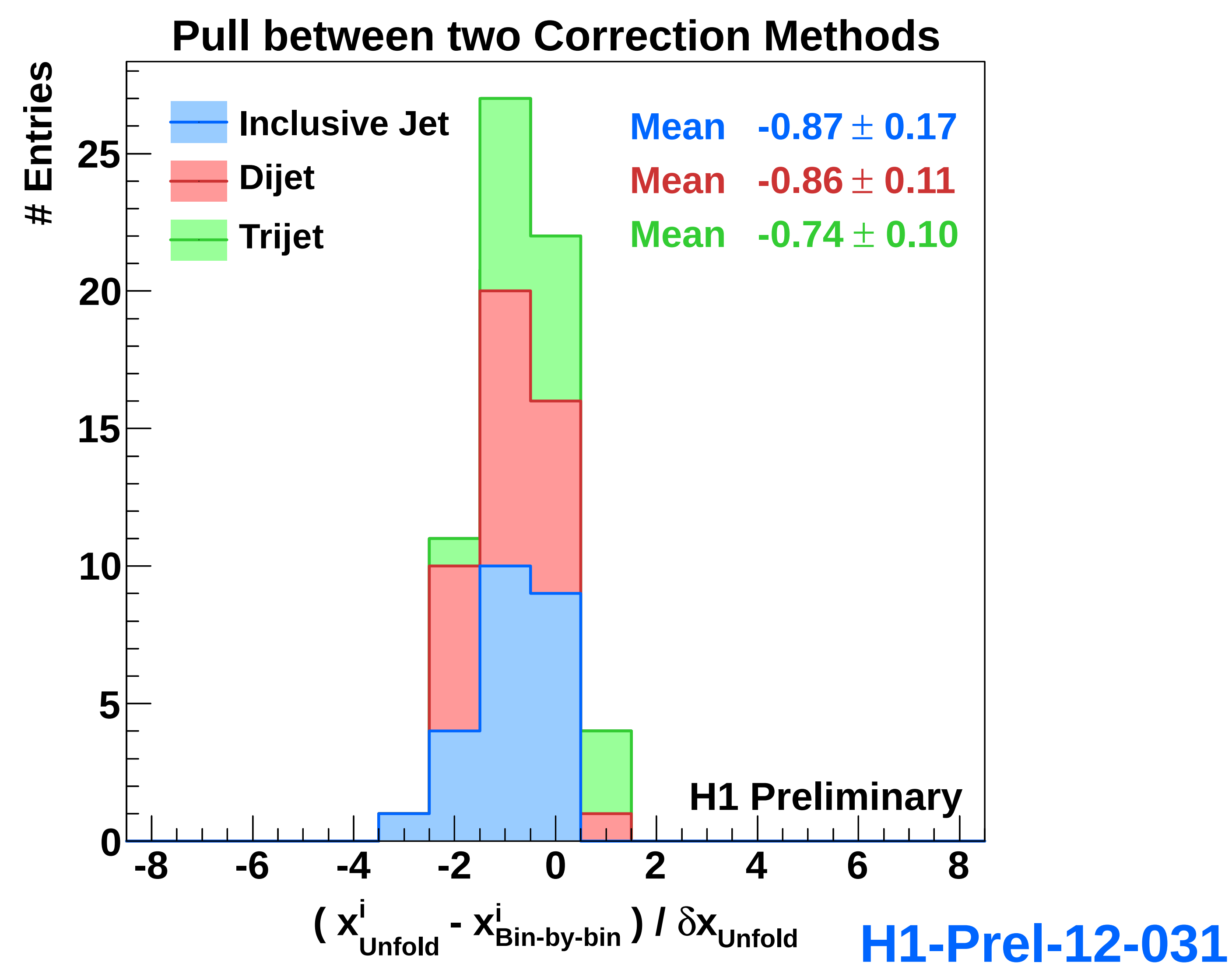
H1 Data

$A_{Unfold} = A_{Rapgap} + A_{Django}$

Bias also in data

Uncertainties are larger

-> But knowledge of correlations



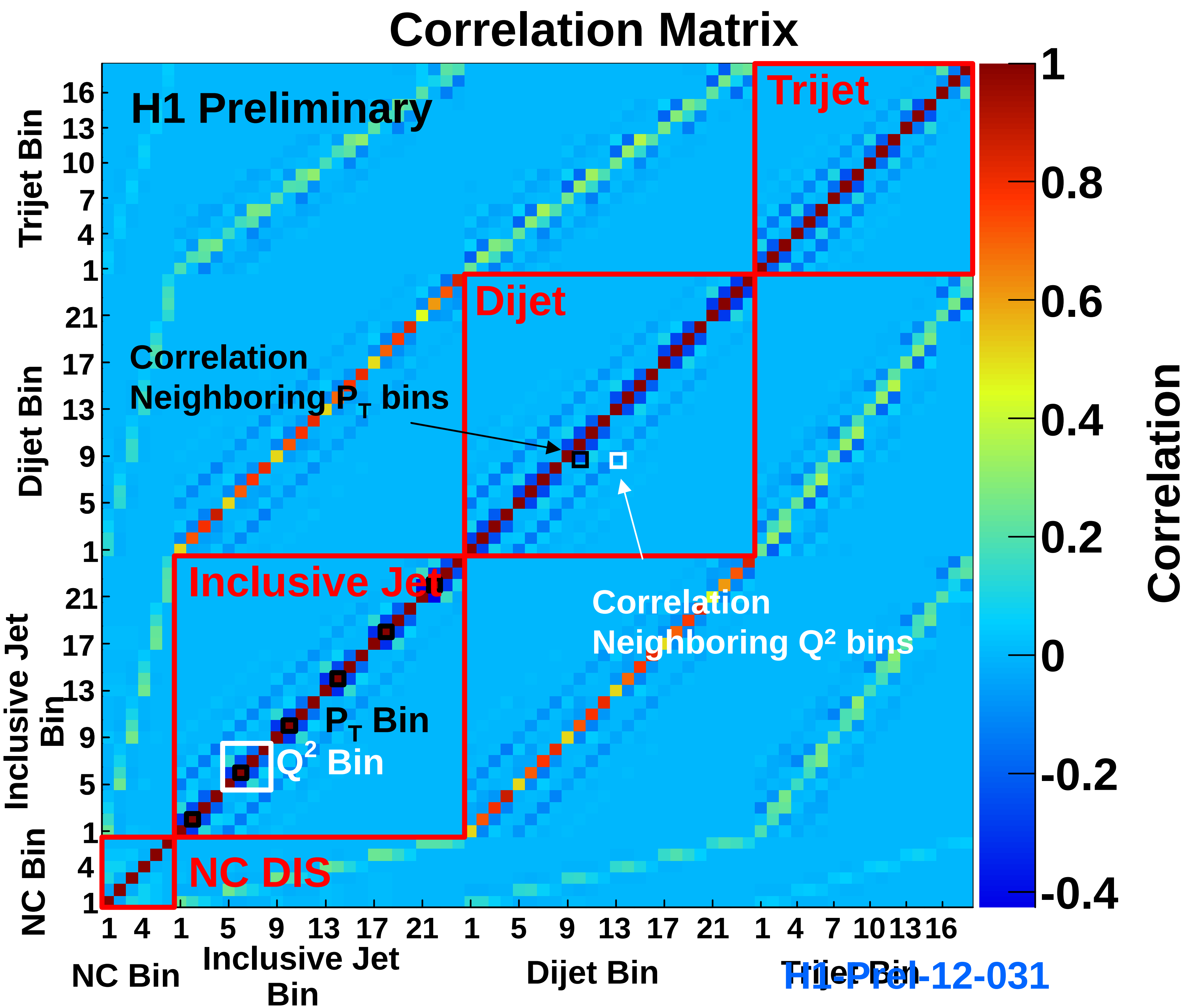
Correlation Matrix

Types of Correlations

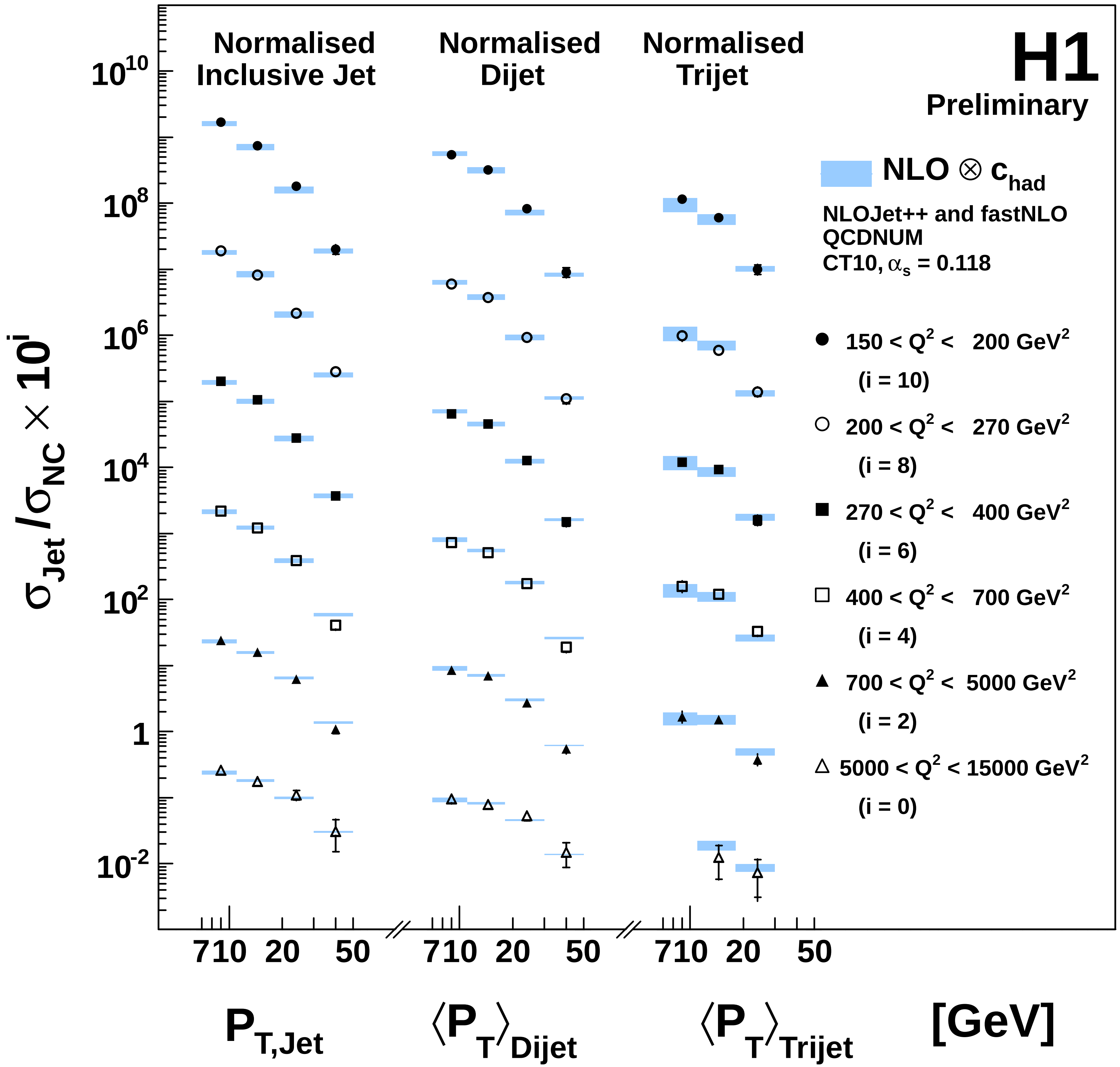
- Correlations resulting from unfolding
- Intrinsic physical correlations
  - - Between measurements
  - - Within Inclusive Jet

Useful for

- Normalized cross sections
  - $\sigma_{Jet} / \sigma_{NC}$
- Combined fit to all jet data



Normalized Multijet cross sections



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α_s Extraction: Fitting technique

NLO calculations depend on PDF and α_s

- Keep PDF fixed and determine $\alpha_s(M_Z)$

Jet cross sections

- NLOJET++, FastNLO v2.0
- $\mu_r^2 = (Q^2 + E_T^2)/2$
- $\mu_f^2 = Q^2$

NC-DIS cross sections

- QCDNUM
- $\mu_f^2 = \mu_r^2 = Q^2$

NLO × Had. corrections

PDF: CT10

Hessian Method

- Minimise χ^2
- TMinuit
- Comparable to Eur. Phys. J. C65, 363

$$\chi^2(\alpha_s, \varepsilon_k) = \vec{\tilde{\sigma}}^T \cdot V^{-1} \cdot \vec{\tilde{\sigma}} + \sum_k^{SysErr} \varepsilon_k^2$$

$$\tilde{\sigma}_i = \sigma_i^{Data} - \sigma_i^{Theo}(\alpha_s, f(\alpha_{s,fix})) \cdot \left(1 - \sum_k^{SysErr} \Delta_{i,k}(\varepsilon_k)\right)$$

Usage of full covariance matrix V from unfolding

- All correlations are respected in fit

Systematic errors as penalty terms

α_s Fits to Individual Measurements

Normalized Inclusive Jet

$\alpha_s = 0.1197 \pm 0.0008\,(\text{exp}) \pm 0.0014\,(\text{PDF}) \pm 0.0011\,(\text{had}) \pm 0.0053\,(\text{theo})$

χ^2 / ndf = 28.7/23 = 1.24

Fit Quality
- Reasonable χ^2 / ndf
- Penalty parameters $-1<\varepsilon<1$

Experimental Error
- Cancellations of some systematic errors
- All normalization uncertainties cancel

Theoretical/systematic errors

Repeated fit with offset method

PDF Error
- Error on **cross section** from PDF Eigenvectors
- Neglecting correlations from EV

Hadronisation Error
- Error on hadronisation correction
- Sherpa
  - Lund Fragmentation
  - Cluster Fragmentation

Theory Error
- Scan μ_r and μ_f independently
  - $0.5 < c < 2.0$
  - Largest and lowest cross section
- Add error on cross section from μ_r and μ_f variation in quadrature

α_s Fits to Individual Measurements

Normalized Inclusive Jet

$\alpha_s = 0.1197 \pm 0.0008\,(\text{exp}) \pm 0.0014\,(\text{PDF}) \pm 0.0011\,(\text{had}) \pm 0.0053\,(\text{theo})$

χ^2 / ndf = 28.7/23 = 1.24

Normalized Dijet

$\alpha_s = 0.1142 \pm 0.0010\,(\text{exp}) \pm 0.0016\,(\text{PDF}) \pm 0.0009\,(\text{had}) \pm 0.0048\,(\text{theo})$

χ^2 / ndf = 27.0/23 = 1.17

Normalized Trijet

$\alpha_s = 0.1185 \pm 0.0018\,(\text{exp}) \pm 0.0013\,(\text{PDF}) \pm 0.0016\,(\text{had}) \pm 0.0042\,(\text{theo})$

χ^2 / ndf = 12.0/16 = 0.75

Reasonable χ^2/ndf for each fit

Large tension between Incl. Jet and Dijet

Similar in previous H1 and ZEUS analyses

Combined Fit

$\alpha_s = 0.1177$ +/- 0.0008 (exp)

χ^2 / ndf = 104.608 / 64 = 1.634

Fit Quality Multijets

Very bad χ^2/ndf for combined fit

Because of tension between Incl. Jet and Dijet

α_s Fits to Multijet Measurements

Combined fit to

Normalized Inclusive Jet, Normalized Dijet, Normalized Trijet

Higher orders

k-factor can be interpreted as indicator for higher orders

$$k = \sigma_{NLO} / \sigma_{LO}$$

Inclusive Jets are more sensitive in pQCD than Dijets

Multijets: $1.05 < k < 1.45$

Normalized Multijet (k-factor < 1.3)

$\alpha_s = 0.1163 \pm 0.0011$ (exp) ± 0.0014 (PDF) ± 0.0008 (had) ± 0.0039 (theo)

χ^2 / ndf = 53.2 / 41 = 1.30

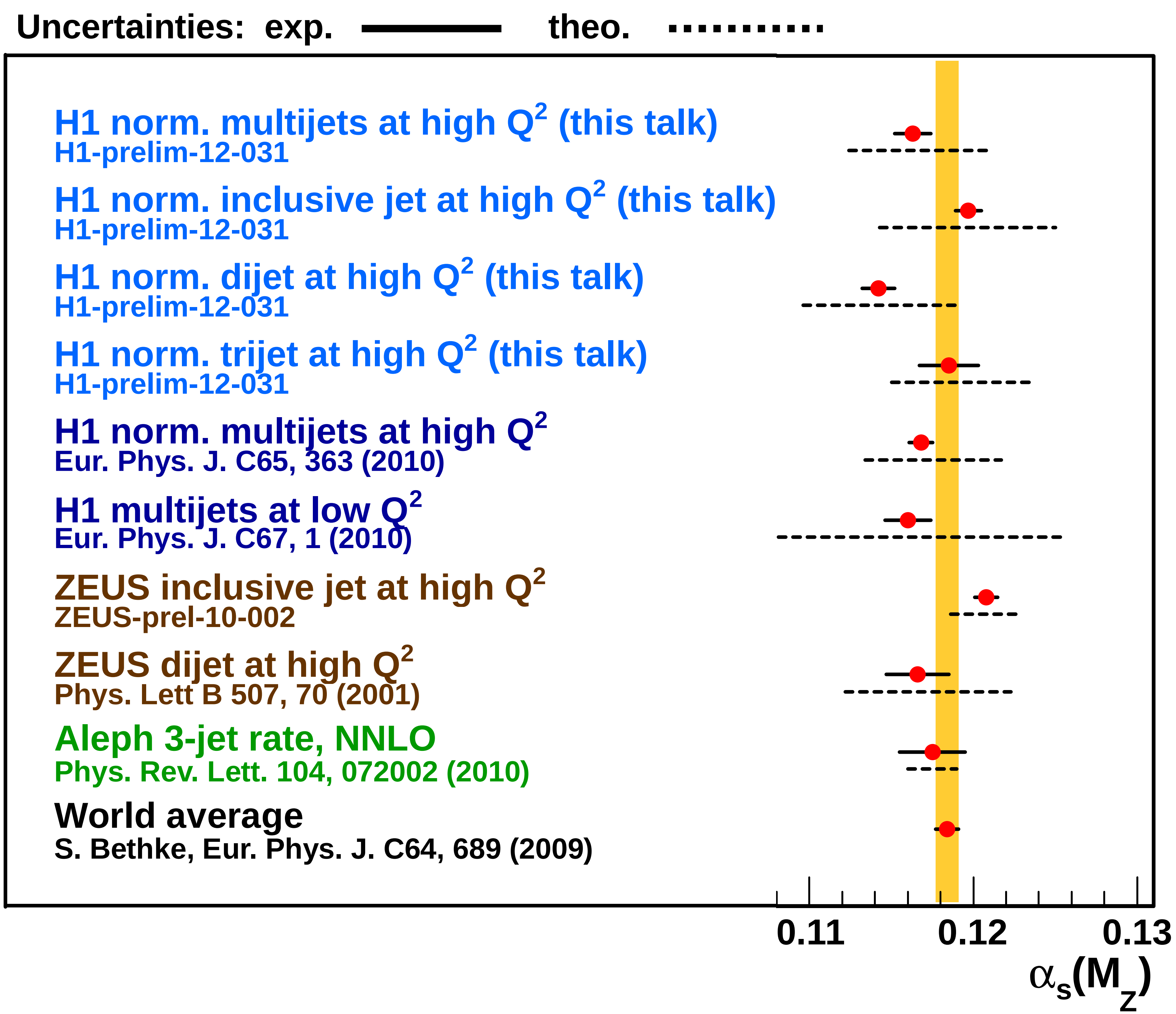
Cut on k-factor

Demanding NLO corrections < 30%

Trade-off between #bins and fast convergence of perturbative series

Keeping 42 out of 65 bins

Comparision of α_s values



Summary

Regularized unfolding

Simultaneous unfolding of multiple measurements

Unfolding of (Jet-)Multiplicities

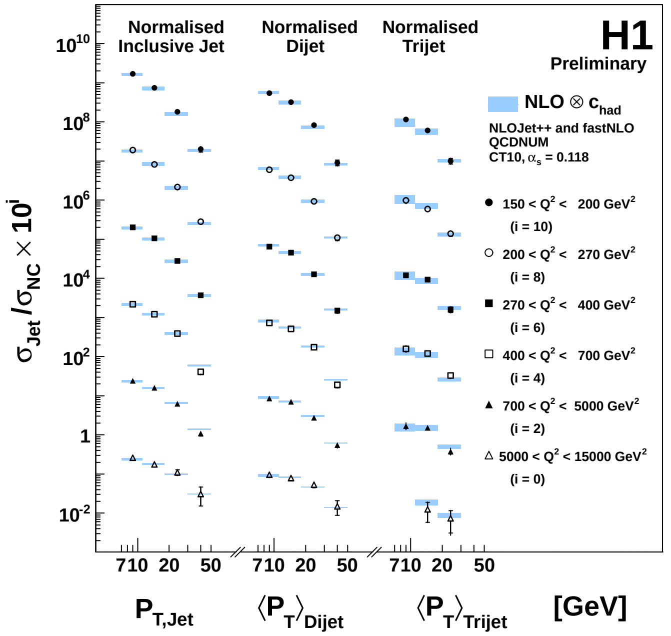
Observables of event classes

Normalized cross sections

Normalized Multijet Measurement

Normalized jet cross sections at High Q^2

- Normalized Inclusive Jet
- Normalized Dijet
- Normalized Trijet



α_s Determination

α_s Fit with unfolded data

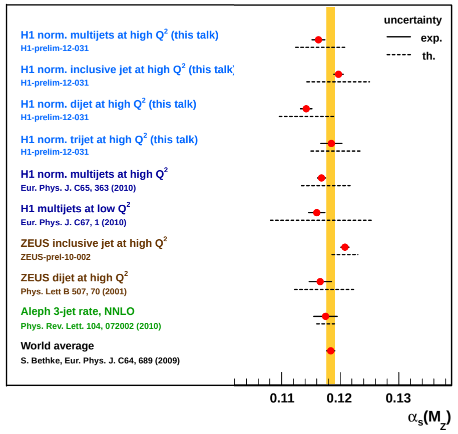
Full covariance matrix

Competitive experimental error

Theoretical errors dominate

0.1163

± 0.0011



Backup

Schematic Definition of Migration Matrix

Measurement	NC DIS	Inclusive Jet	Dijet	Trijet
Type	**Measurment of scattered electron kinematics (Event properties)**	**Measurement of jet multiplicities and jet kinematics**	**Measurement of event properties of an event class**	**Measurement of event properties of an event class**
Cross section **$0.2 < y < 0.7$** **$-1.0 < \eta < 2.5$**	**$d\sigma/dQ^2$**	**$d\sigma/dQ^2dp_T$**	**$d\sigma/dQ^2d\langle p_T\rangle_2$**	**$d\sigma/dQ^2d\langle p_T\rangle_3$**
Special characteristic	**Each event**	**(geometric) matching of 'jets' on on particle and detector level**	**Cuts for 'Dijet' definition:** **M_{12}, η_{Jet} , $p_{T,Jet2}$, $p_{T,Jet1}$**	**Cuts for 'Trijet' definition:** **M_{12}, η_{Jet}, $p_{T,Jet3}$, $p_{T,Jet1}$**
Considered migrations	**Q^2, y**	**Q^2, y, p_T**	**Q^2, y, $\langle p_T\rangle_2$, into/out of Dijet-class**	**Q^2, y, $\langle p_T\rangle_3$, into/out of Trijet-class**
Through NC DIS estimated migrations		**Measured jets without matching jet on particle level**	**Measured Dijet events which do not fulfill any Dijet class definition**	**Measured Trijet events which do not fulfill any Trijet class definition**

Regularized Unfolding using TUnfold

Aim

Cross section on particle level

Correction for detector effects

Detector Response

Steeply falling P_T spectra

Boost to Breit frame

Finite resolution

-> Migrations are getting relevant

Bin-by-bin correction

Detector correction based on bin-wise correction factors

Some error propagation from migrations

Regularized Unfolding

(Migration) Matrix A describes detector response

$$\vec{m} = A \cdot \vec{x}$$

m: measured distribution (Detector level)

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Determine particle level x by analytic minimization of χ^2 as function of x

$$\chi^2(x) = \tfrac{1}{2}(m - Ax)^T V^{-1}(m - Ax) + \tau^2 \cdot \chi_L^2(x)$$

TUnfold v17 (S.Schmitt)

Regularisation parameter τ suppresses large fluctuations/errors in comparision to direct matrix inversion

Unfolding of four measurements at once

NC-DIS, Inclusive Jet, Dijet und Trijet

-> same dataset, same events

-> Covariance matrix V contains correlations

α_s Extraction: Fitting technique

NLO calculations depend on PDF and α_s

- Keep PDF fixed and determine $\alpha_s(M_Z)$
- Neglet correlations between gluon and α_s

Theory Cross Sections

- Jet cross sections
  - NLOJET++
  - FastNLO v2.0
  - $\mu_r^2 = (Q^2 + E_T^2)/2$
  - $\mu_f^2 = Q^2$
- NC-DIS cross sections
  - QCDNUM
  - $mu_f^2 = mu_r^2 = Q^2$
- NLO × Had. corrections
- LHAPDF
  - PDF: CT10
  - α_s evolution code (M_Z etc…)

Hessian Method

- Minimise χ^2
- TMinuit
- Comparable to Eur. Phys. J. C65, 363

$$\chi^2(\alpha_s, \varepsilon_k) = \vec{\tilde{\sigma}}^T \cdot V^{-1} \cdot \vec{\tilde{\sigma}} + \sum_k^{SysErr} \varepsilon_k^2$$

$$\tilde{\sigma}_i = \sigma_i^{Data} - \sigma_i^{Theo}(\alpha_s, f(\alpha_{s,fix})) \cdot \left(1 - \sum_k^{SysErr} \Delta_{i,k}(\varepsilon_k)\right)$$

$$\Delta_{i,k}(\varepsilon_k) = \frac{|\Delta_{i,k}^+| + |\Delta_{i,k}^-|}{2} \cdot \varepsilon_k + \frac{|\Delta_{i,k}^+| - |\Delta_{i,k}^-|}{2} \cdot \varepsilon_k^2$$

Usage of full covariance matrix V from unfolding

- All correlations are respected in fit

Systematic errors as penalty terms

Multijet Measurement

Kinematics

DIS Event = scattered electron

$Q^2 = -q^2 = (k - k')^2$

$y = q \cdot P / k \cdot P$ (Inelasticity)

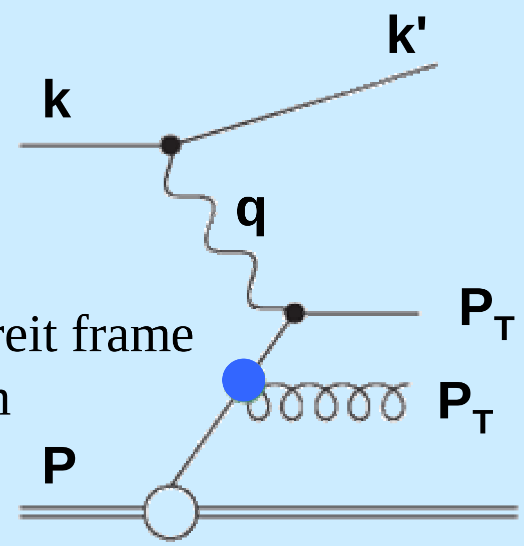
Jet kinematics

P_T = jet transverse momentum in Breit frame

$\langle P_T \rangle$ = Average transverse momentum of 2 (resp. 3) jets

η = pseudorapidity of Jet

M_{12} = invariant mass of two leading jets



Phase Space

Neutral current phase space

$150 < Q^2 < 15000$ GeV^2

$0.2 < y < 0.7$

Jet phase space

$-1.0 < \eta_{Lab} < 2.5$

Inclusive Jet

$7.0 < P_T < 50$ GeV

Dijet and Trijet

$5 < P_T < 50$ GeV

$7 < \langle P_T \rangle 50$ GeV

$M_{12} > 16$ GeV

Four Measurements

(inclusive) DIS Measurement

$d\sigma/dQ^2$

Inclusive Jets

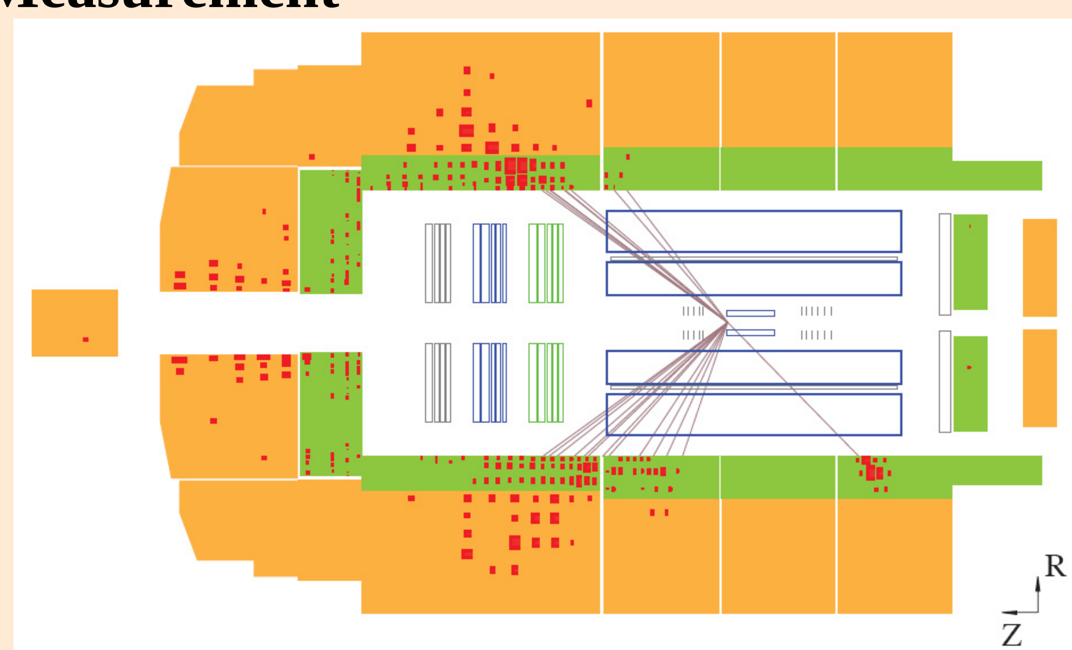
$d\sigma/dQ^2dP_T$

Dijet

$d\sigma/dQ^2d\langle P_T \rangle$

Trijet

$d\sigma/dQ^2d\langle P_T \rangle$



Detector Correction

Aim

Cross section on particle level

-> Correction of detector effects

Bin-by-bin correction

Detector correction based on bin-wise correction factors

Error propagation for 'gain' events

Regularized unfolding

This talk, this measurement

Bin-by-bin versus Unfolding

Inclusive jet cross sections, four P_T bins in the first Q^2 bin

bin-by-bin

bin label	σ_{jet} pb	δ_{stat} %	δ_{sys}^{unc} %	δ_{tot} %
1α	$7.25\ 10^1$	0.9	1.5	3.9
1β	$3.22\ 10^1$	1.3	2.0	4.4
1γ	$7.48\ 10^0$	2.6	2.9	5.8
1δ	$8.95\ 10^{-1}$	7.6	3.7	9.7

unfolding

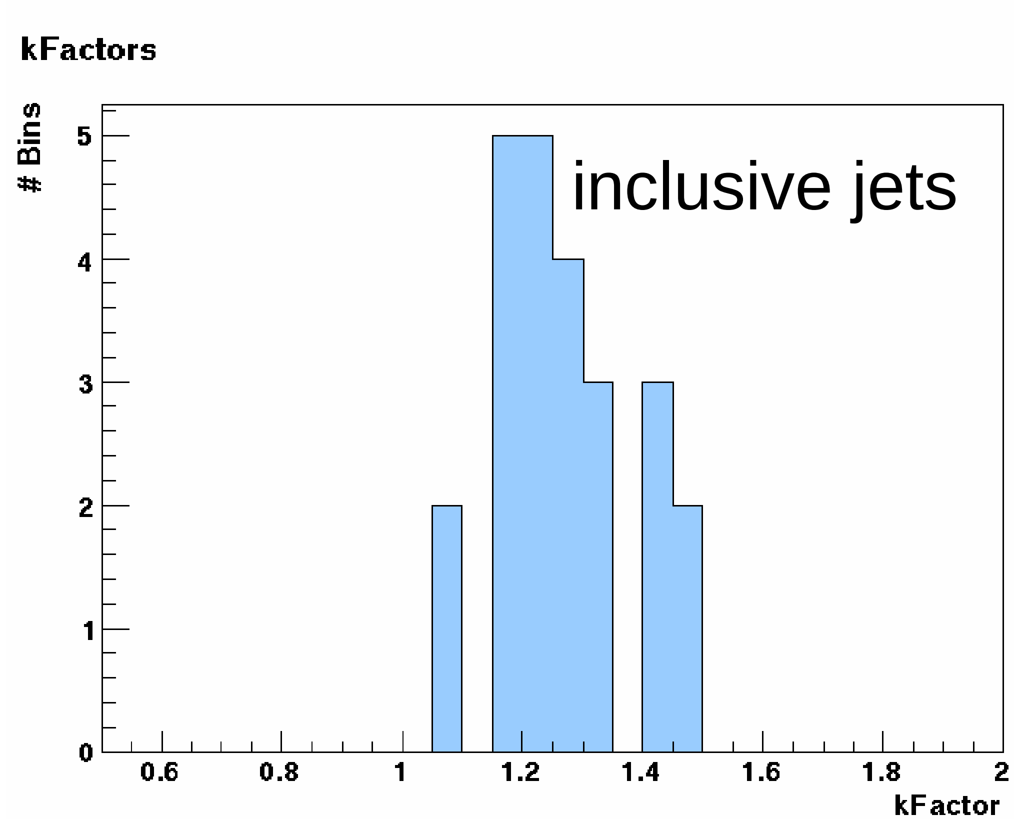
bin label	σ_{jet} pb	δ_{stat} %	δ_{sys}^{unc} %	δ_{tot} %
1α	$6.98\ 10^1$	2.4	1.8	4.8
1β	$3.11\ 10^1$	3.1	2.7	6.0
1γ	$7.64\ 10^0$	5.5	3.5	8.1
1δ	$8.58\ 10^{-1}$	16.8	5.5	18.8

In case of unfolding the statistical uncertainty increased by more than 100%.

But correlated and anti-correlated bins, so comparison is not totally meaningful

The systematic uncertainties are very similar between the unfolded and bin-by-bin corrected results.

k- factors


kFactors
Bins
inclusive jets
0
1
2
3
4
5
0.6
0.8
1
1.2
1.4
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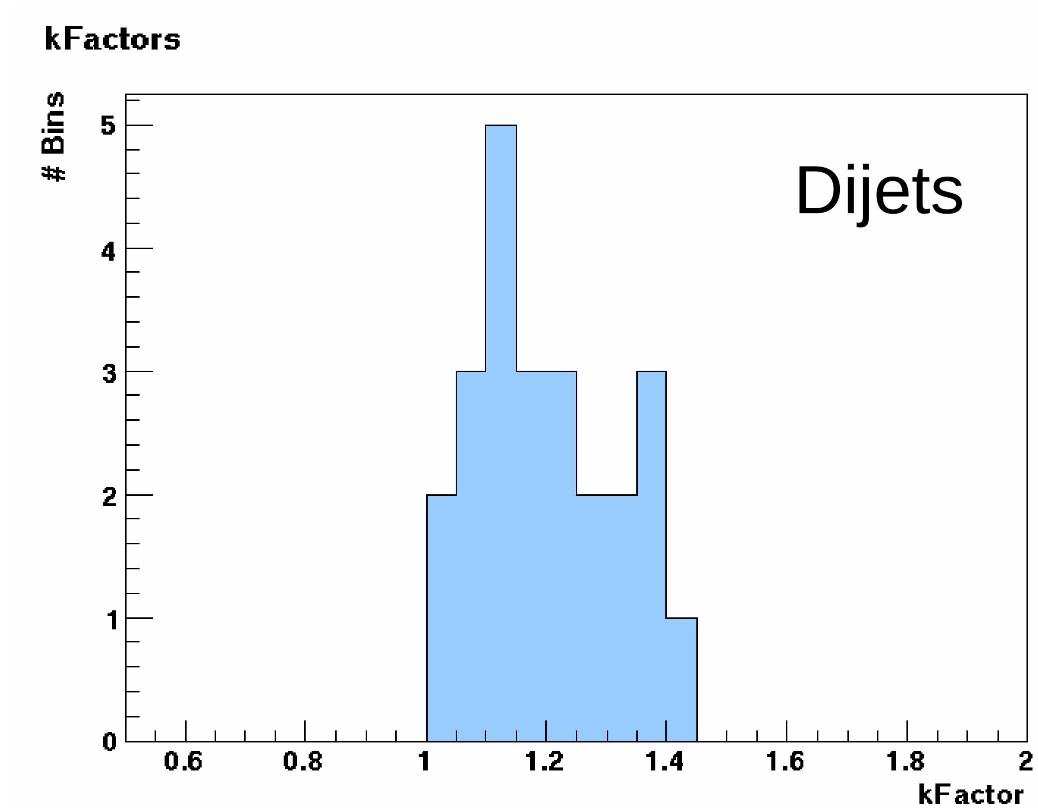
1.8
2
kFactor


kFactors
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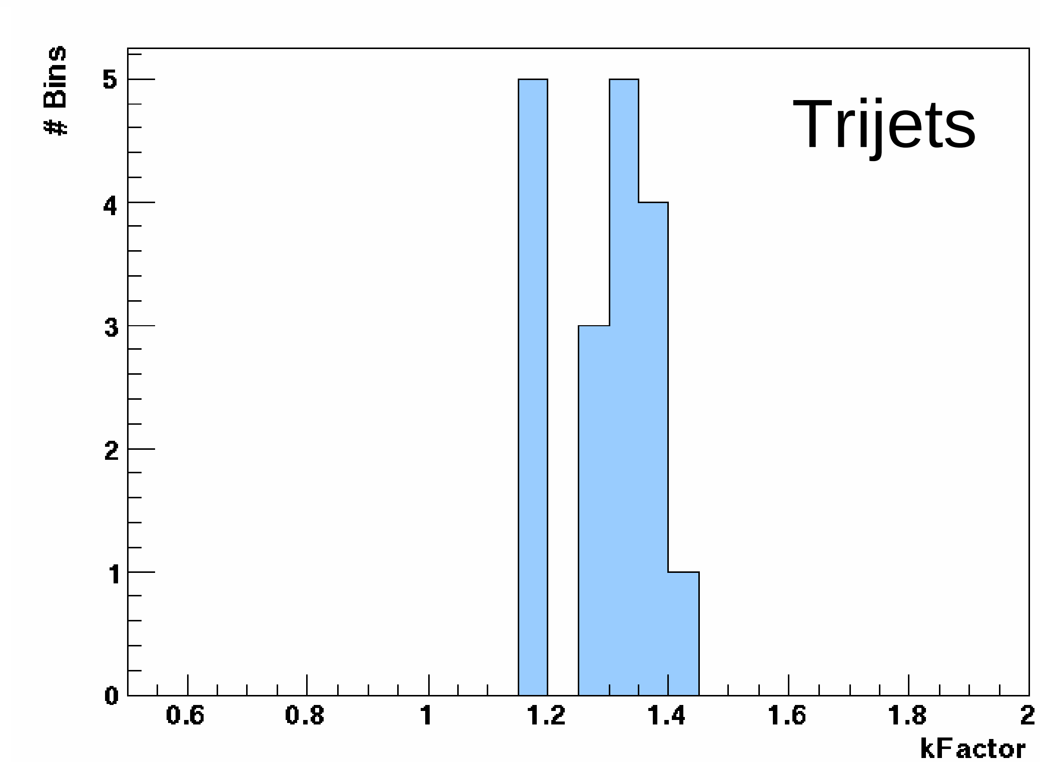
kFactor


Bins
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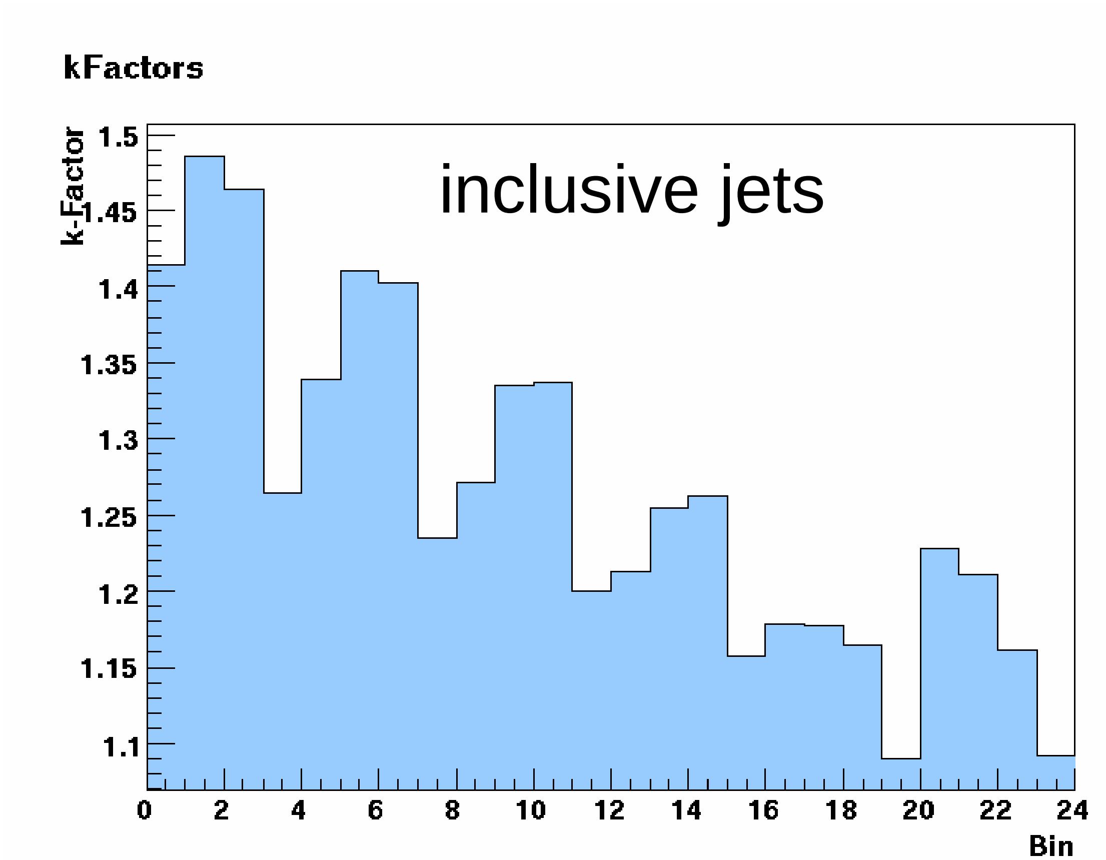
kFactor

k-factors


kFactors
inclusive jets
k-Factor
1.5
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22
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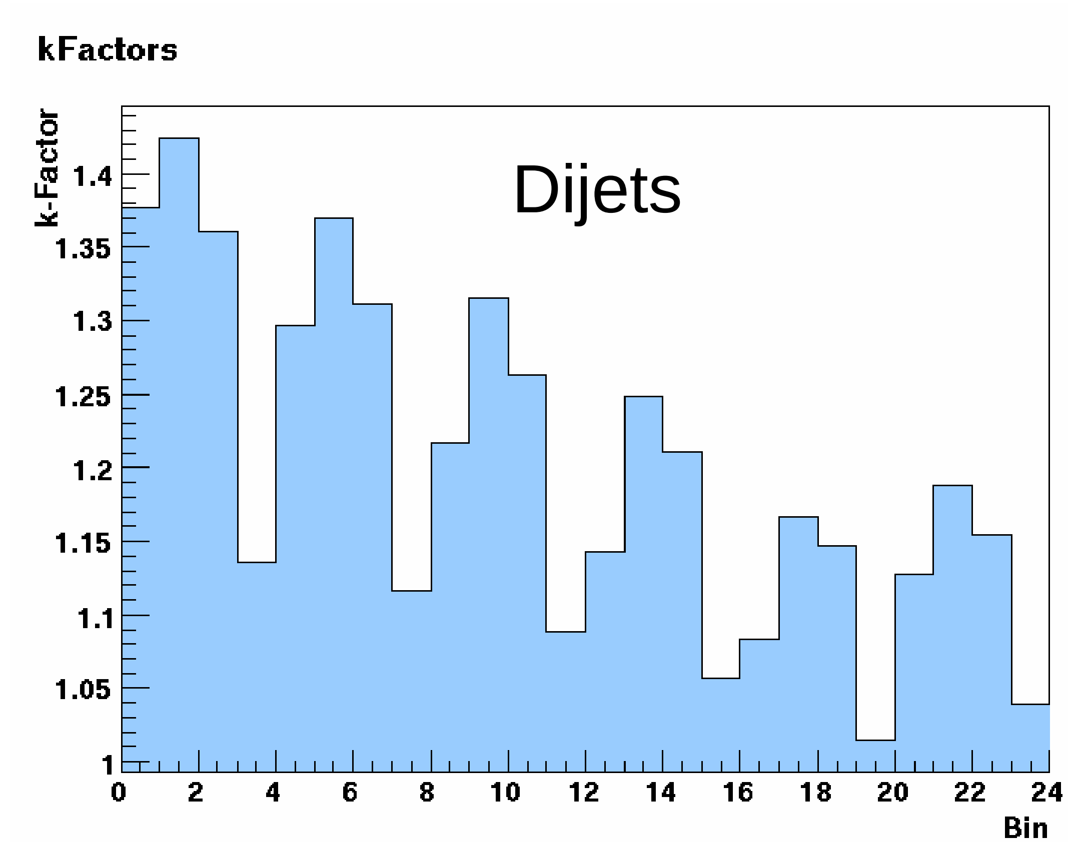
Bin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Dijets
k-Factor
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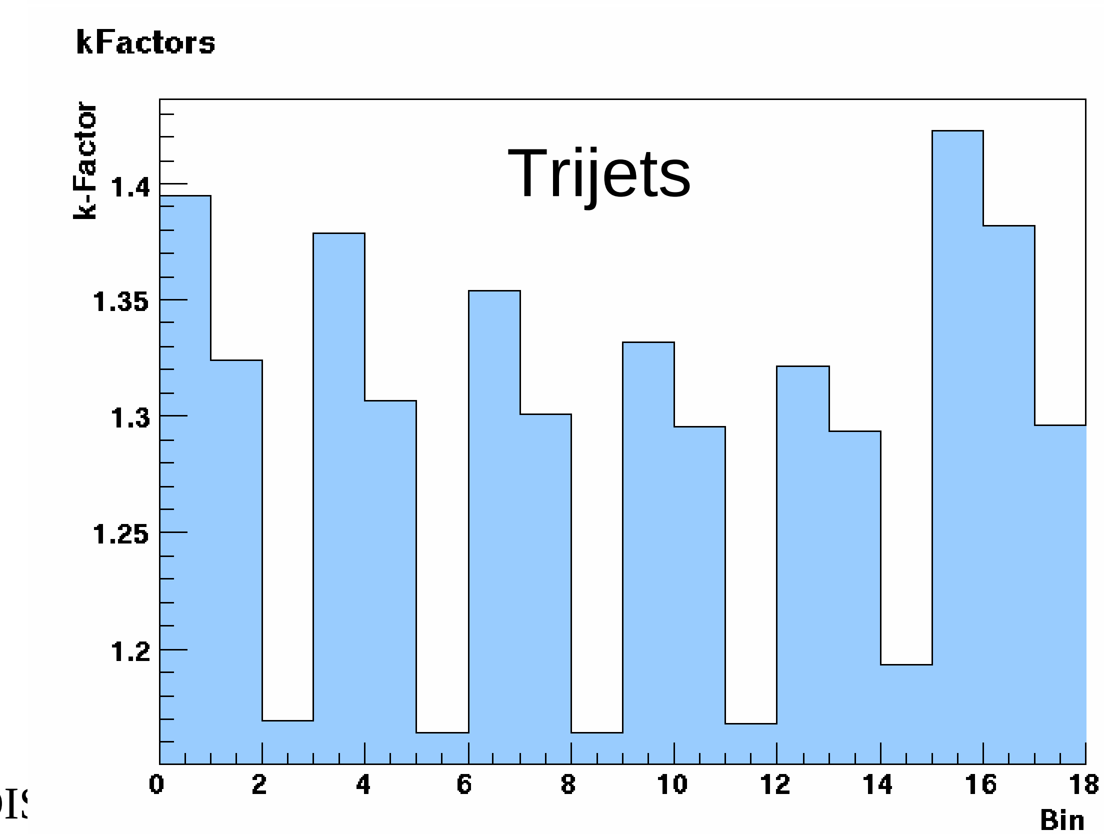
Bin


kFactors
Trijets
k-Factor
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1.35
1.3
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6
8
10
12
14
16
18
Bin

α_s Fits to Individual Measurements

Experimental Error
- Cancellations of some systematic errors
- All normalization uncertainties cancel

Fit Quality
- Reasonable χ^2 / ndf
- Penalty parameters $-1<\varepsilon<1$

Normalized Inclusive Jets

$\alpha_s = 0.1197 \pm 0.0008\,(\text{exp}) \pm 0.0014\,(\text{PDF}) \pm 0.0011\,(\text{had}) \pm 0.0053\,(\text{theo})$

χ^2 / ndf = 28.7/23 = 1.24

Normalized Dijets

$\alpha_s = 0.1142 \pm 0.0010\,(\text{exp}) \pm 0.0016\,(\text{PDF}) \pm 0.0009\,(\text{had}) \pm 0.0048\,(\text{theo})$

χ^2 / ndf = 27.0/23 = 1.17

Normalized Trijets

$\alpha_s = 0.1185 \pm 0.0018\,(\text{exp}) \pm 0.0013\,(\text{PDF}) \pm 0.0016\,(\text{had}) \pm 0.0042\,(\text{theo})$

χ^2 / ndf = 12.0/16 = 0.75

PDF Error
- Error on **cross section** from PDF Eigenvectors
- Neglecting correlations from EV

Hadronisation Error
- Error on hadronisation correction
- Sherpa
  - Lund Fragmentation
  - Cluster Fragmentation

Theory Error
- Scan mu_r and mu_f independently
  - $0.5 < c < 2.0$
  - Largest and lowest cross section
- Add error on cross section from mu_r and mu_f variation in quadrature

α_s Fits to Individual Measurements

Normalized Inclusive Jets

α_s = 0.1197 ± 0.0008 (exp) ± 0.0014 (PDF) ± 0.0011 (had) ± 0.0053 (theo)

χ^2 / ndf = 28.7/23 = 1.24

Fit Quality
- Reasonable χ^2 / ndf
- Penalty parameters $-1<\varepsilon<1$

Experimental Error
- Cancellations of some systematic errors
- All normalization uncertainties cancel

Theoretical/systematic errors

Repeated fit with offset method

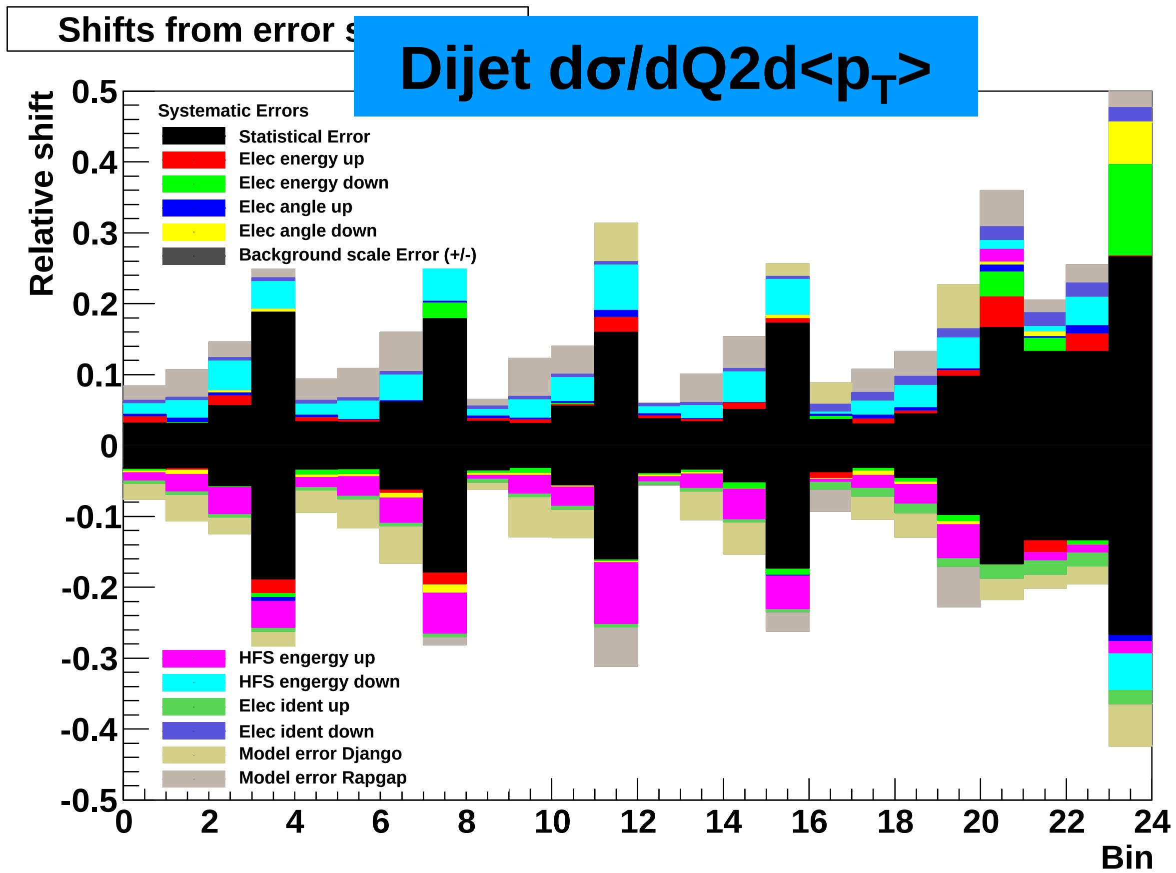
PDF Error
- Error on **cross section** from PDF Eigenvectors
- Neglecting correlations from EV

Hadronisation Error
- Error on hadronisation correction
- Sherpa
  - Lund Fragmentation
  - Cluster Fragmentation

Theory Error
- Scan mu_r and mu_f independently
  - $0.5 < c < 2.0$
  - Largest and lowest cross section
- Add error on cross section from mu_r and mu_f variation in quadrature

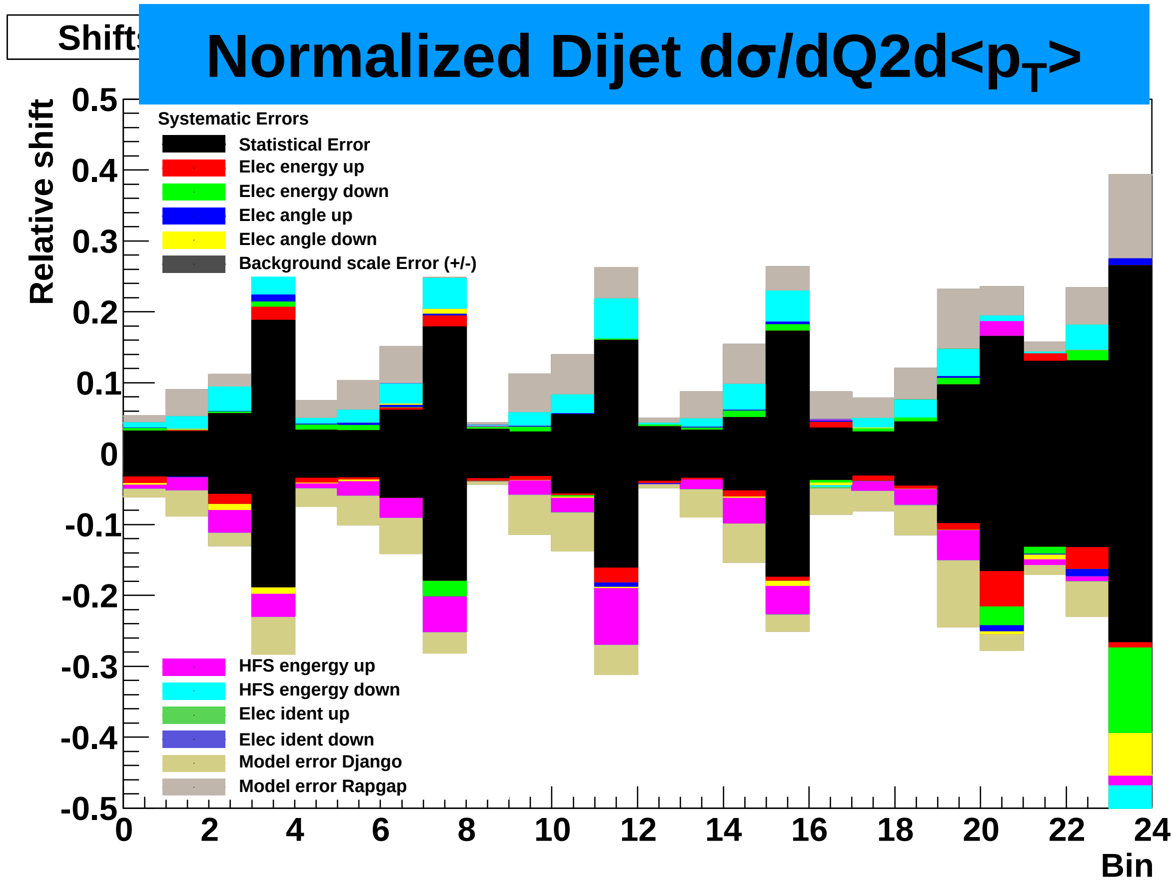
Errors on normalized cross sections

Some systematic errors cancel for normalized measurement

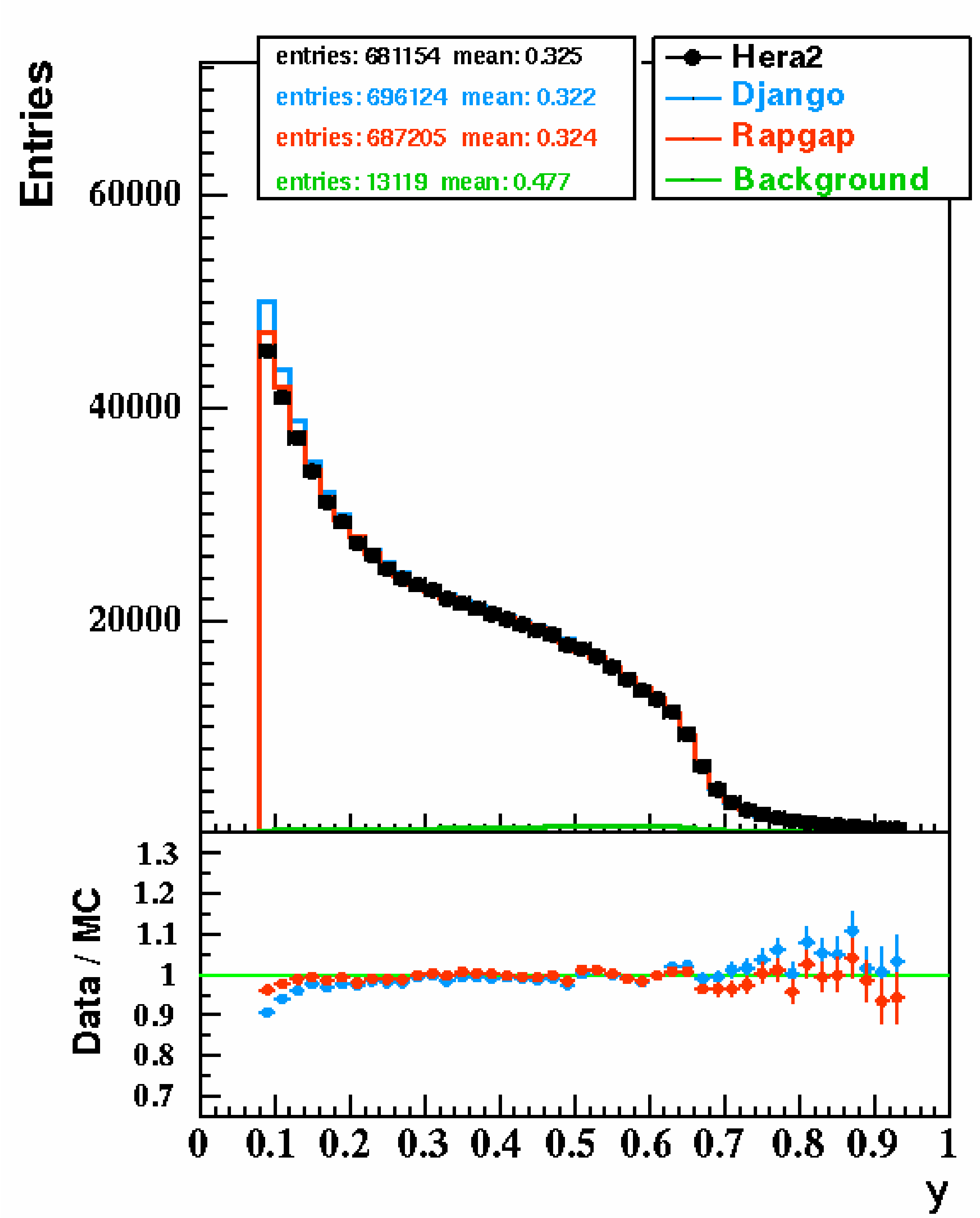


Here

Dijet dQ^2dp_T normalized with NCDIS



The y-distribution (inelasticity)



Binning on detector level

- 0.08, 0.15 , 0.45 , 0.70 , 0.95

very little statistics above 0.7

Electron energy cut on 11.0 GeV

- causes "cutoff" in y ~0.7
- change would imply a restudy of the trigger efficiency
- lowering of el. energy cut increases radiative corrections

In unfolding we basically would have to fill

- 1.5 to 2 yps bin
- 15 Q2 bins
- 10, 9, 7 bins for pT
- from this little statistics ???

Problem:

- almost every event that is generated with y>0.7 is not reconstructed
- We would have to unfold quite some information out of almost no data

Solution:

- and we don't "generate jets" above 0.7
- we don't measure jets (pt) for high yps side bin