Proton Structure from HERA

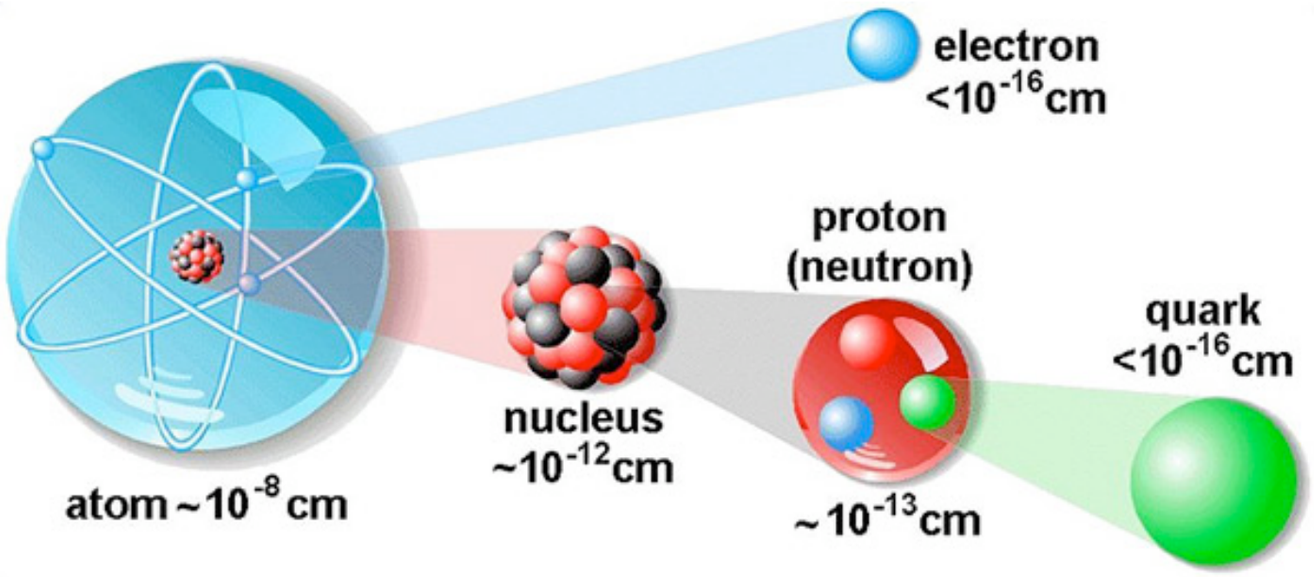
Voica A. Radescu
Heidelberg Physikalisches Institut

Trans-European School of High Energy Physics
July 2010

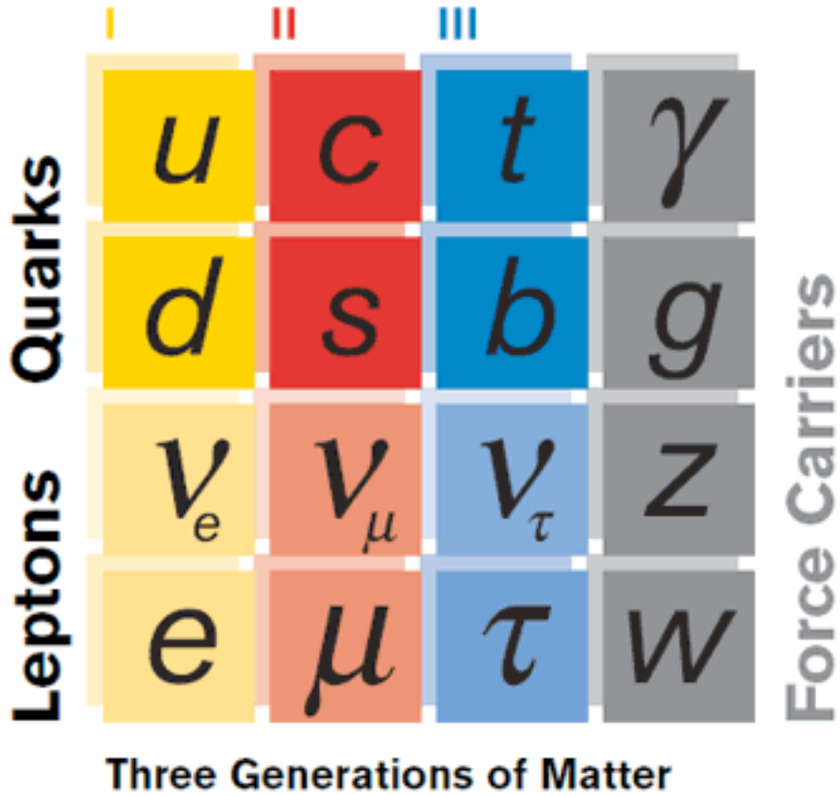
Outline

- Introduction
- Experimental Settings
- QCD Analysis Strategy
- Results and Comparisons
- Summary

Building Blocks



- Standard Model
  - Fermions (quarks and leptons)
    - constituents of matter
  - Bosons
    - force carrier particles
  - Arranged in three generations of doublets in increasing mass:



- SM is a triumph in particle physics, but there are still questions that remain unanswered.
- Large Hadron Collider will provide a rich physics potential ranging from more precise measurements of SM parameters to the search of new physics phenomena.
  - Understanding the complex structure of the proton is important for interpretation of the LHC results.

Probing the Proton Structure

- Proton can be best probed via elementary particles as:
  - neutrinos (fixed target experiments) - interact only weakly
  - electrons (fixed target and collider experiments) - interact electroweakly

- Deep Inelastic Scattering (DIS) is the cleanest way to study the substructure of nucleon
  - scattering of a lepton off the quarks within the proton resulting into a hadronic shower and a lepton

- Kinematic Variables:
  - virtuality of exchanged boson

$$Q^2 = -q^2 = -(k - k')^2$$

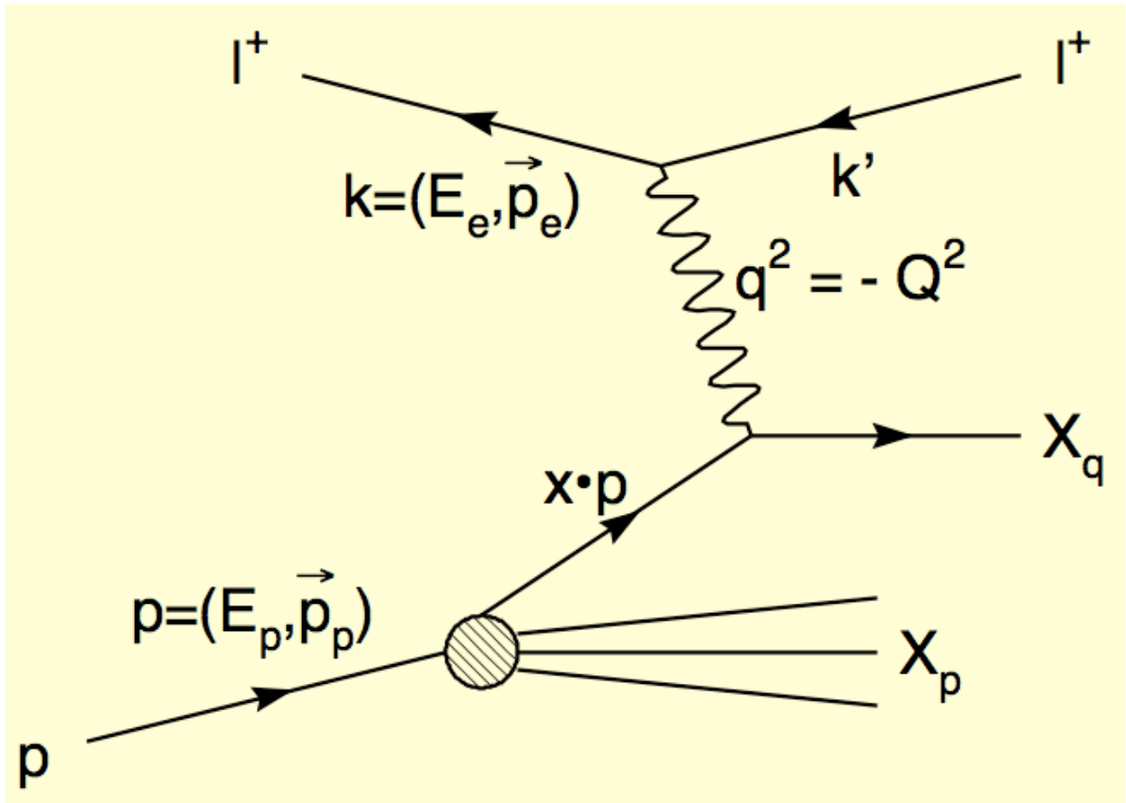
  - proton momentum fraction of the scattered quark (Bjorken scaling variable)

$$x = \frac{Q^2}{2p \cdot q}$$

  - inelasticity parameter:

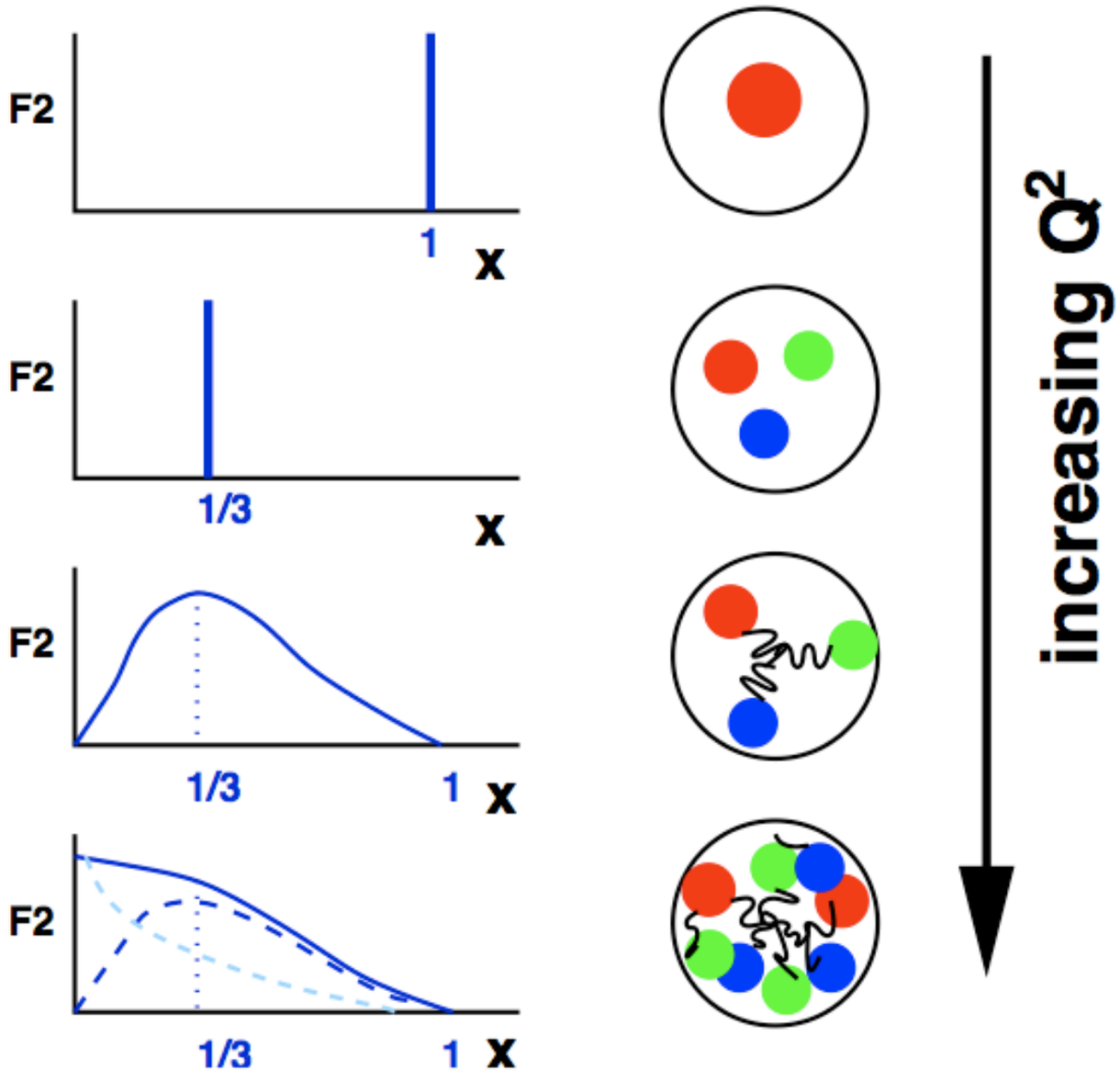
$$y = \frac{p \cdot q}{p \cdot k}$$

  - invariant centre of mass energy: $s = (k + p)^2 = \frac{Q^2}{xy}$



Structure of Proton

- The proton is a dynamical object and its observed structure depends on the resolution (Q^2) of the observation:
  - the higher the value of Q^2 the more detail we examine



DIS Cross Sections and Structure Functions

- General double differential cross-section for eN scattering:

$$\frac{\mathrm{d}^2\sigma^{\pm}_{NC}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+\tilde{F}_2 \mp Y_- x\tilde{F}_3 - y^2\tilde{F}_L\right] \equiv \frac{2\pi\alpha^2}{xQ^4}\,Y_+\tilde{\sigma}^{\pm}_{NC}$$

$$Y_\pm = 1 \pm (1-y)^2$$

- F_2, F_L, xF_3 are structure functions (hadronic part) which are related to the momentum distributions of quarks within the nucleon: Parton Distribution Functions (PDFs):
  - probability density that interacting parton carries a fraction x of the hadrons longitudinal momentum
    - valence quarks: carriers of charge
    - sea quarks and gluons: evolved by complex dynamics

At Leading Order (LO):

$$F_2 = x\sum e_q^2\left(q(x)+\bar{q}(x)\right)$$

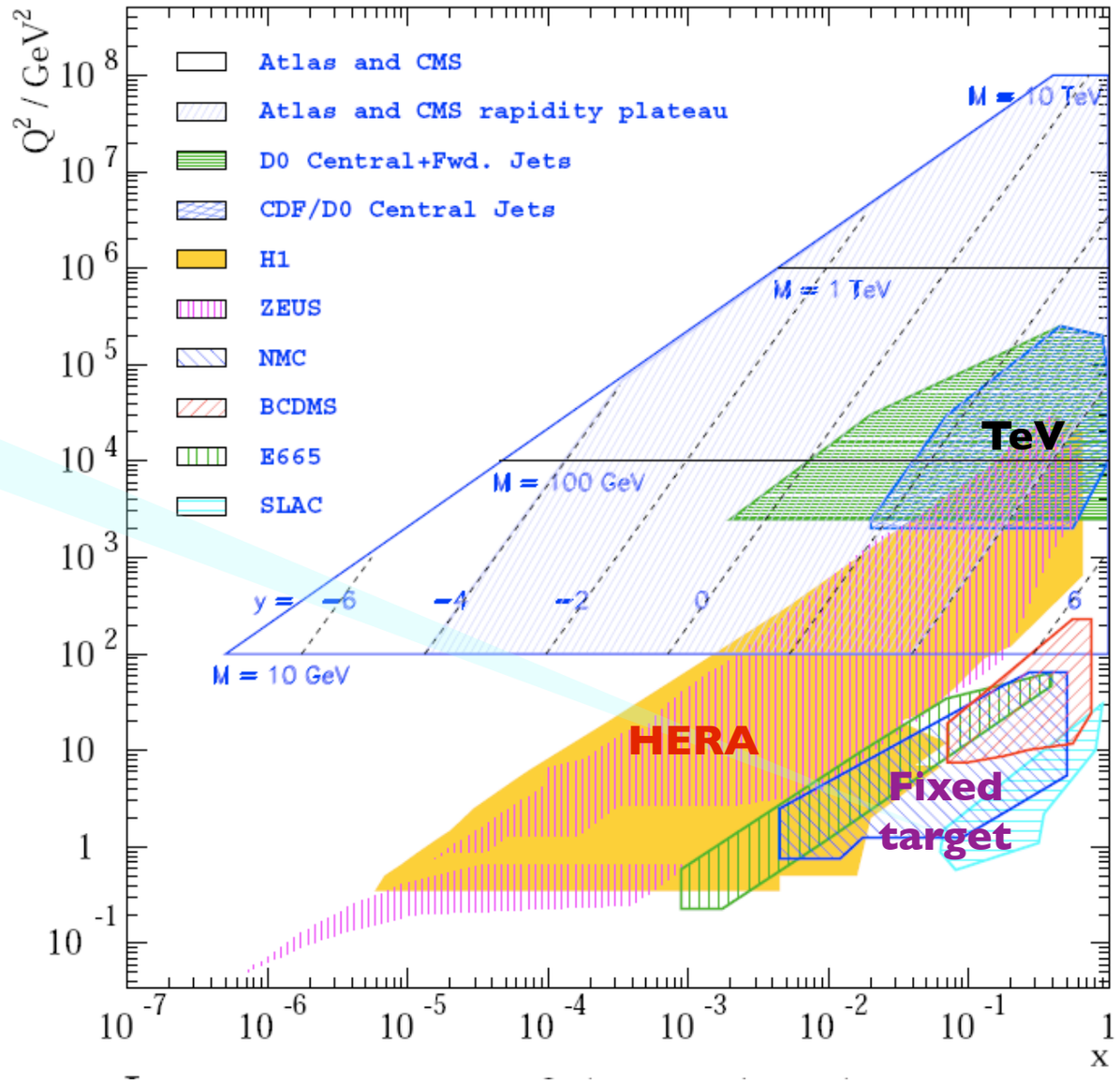
$$xF_3 = x\sum 2e_q a_q\left(q(x)-\bar{q}(x)\right)$$

- F_2 dominates
  - sensitive to all quarks
- xF_3
  - sensitive to valence quarks
- F_L
  - sensitive to gluons

- So, we can extract the proton structure functions experimentally by looking at the x, y, Q^2 dependence of the double differential cross-section.

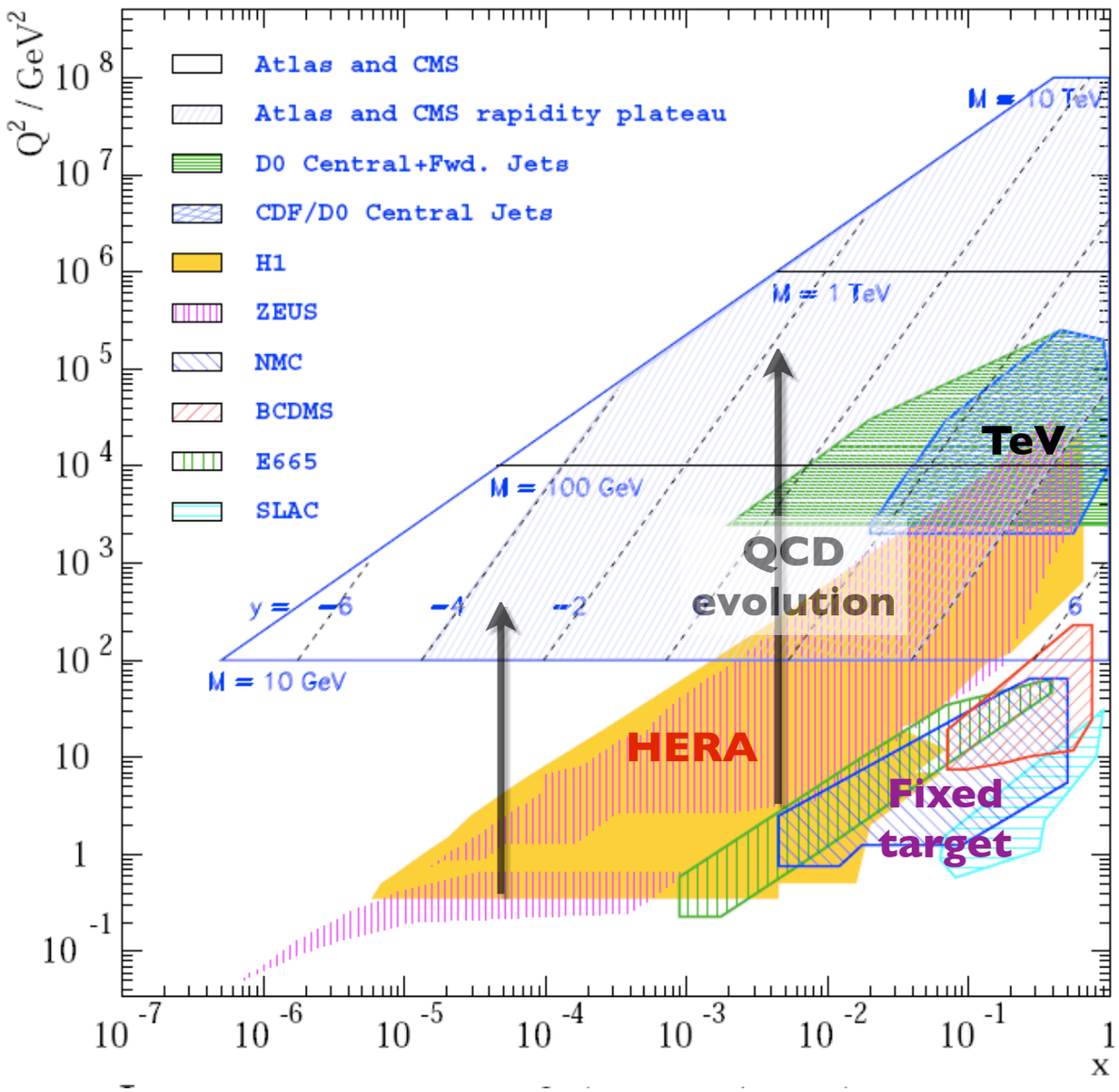
Experimental Data on Proton Structure

- Persistent experimental effort over the last 40 years both by fixed-target and collider experiments around the world supported by the theoretical developments
  - Large extension in kinematic space in x and Q^2 from the original SLAC measurements
- Currently, HERA measurements dominate the kinematic plane and, hence, provides the best basis for evolution to the scales relevant at the LHC:
  - central rapidity range for W/Z production at LHC is at low x (6×10^{-4} to 6×10^{-2})
- Under design: a new ep collider at TeV energies based on the LHC hadron beams: http://cern.ch/lhec



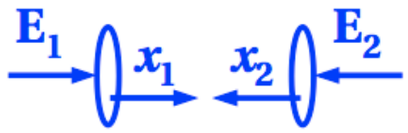
Experimental Data on Proton Structure

- Persistent experimental effort over the last 40 years both by fixed-target and collider experiments around the world supported by the theoretical developments
  - Large extension in kinematic space in x and Q^2 from the original SLAC measurements
- Currently, HERA measurements dominate the kinematic plane and, hence, provides the best basis for evolution to the scales relevant at the LHC:
  - central rapidity range for W/Z production at LHC is at low x (6×10^{-4} to 6×10^{-2})
- Under design: a new ep collider at TeV energies based on the LHC hadron beams: http://cern.ch/lhec



Experimental Data on Proton Structure

Proton-proton collisions:



Drell Yan scattering:

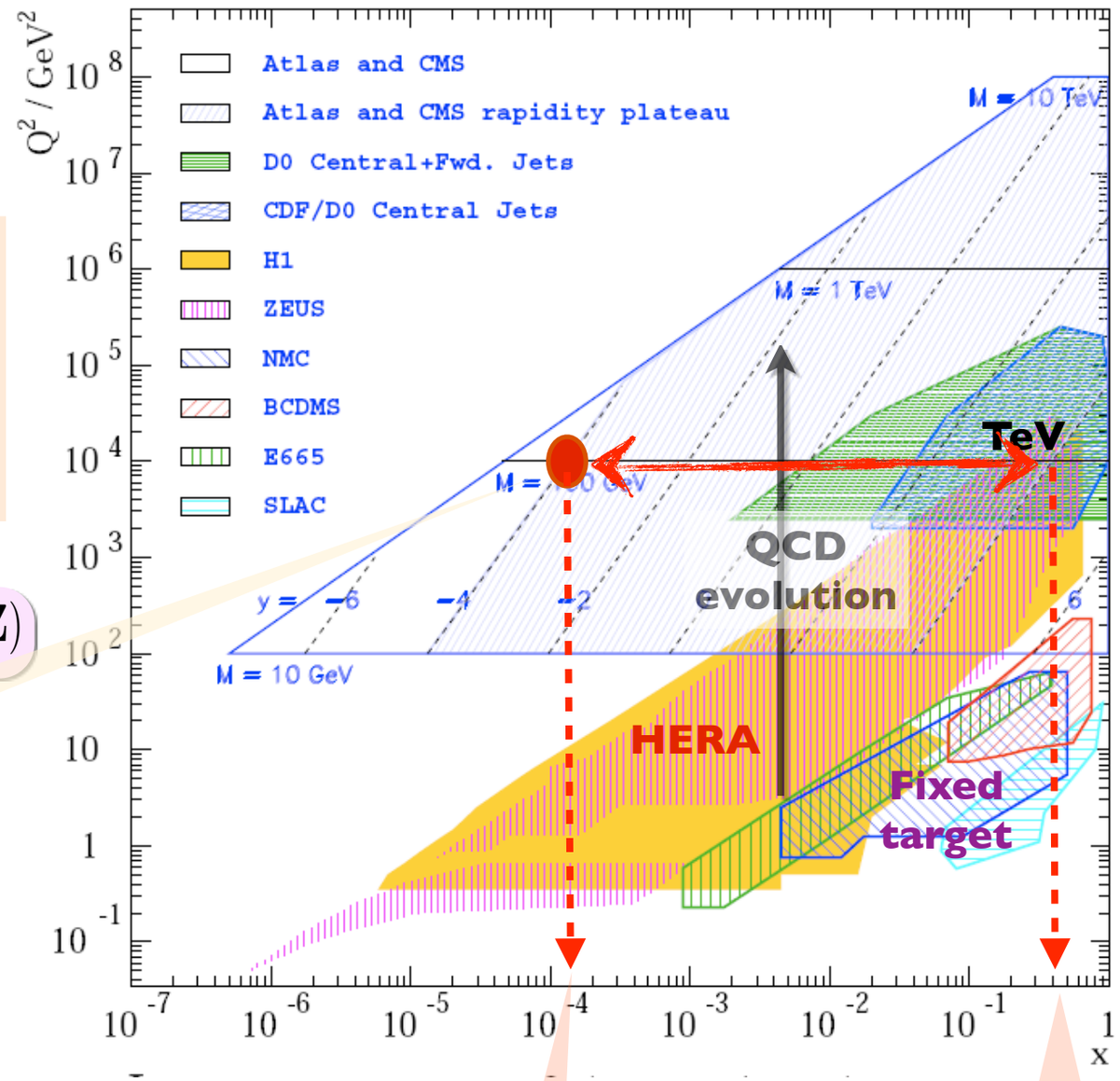
- $Q^2=M^2_{Z,W}$
- rapidity y
- $s=4E_p^2$

$$\text{rapidity } y = \frac{1}{2}\ln\frac{E+P_Z}{E-P_Z};$$

$$x_1 = \frac{M}{\sqrt{s}}e^{y}; x_2 = \frac{M}{\sqrt{s}}e^{-y};$$

$$\sigma(pp \to W, Z + X) \sim q, \bar{q}(x_1, M^2_{W,Z}) \times q, \bar{q}(x_2, M^2_{W,M}) \times \sigma(q\bar{q} \to W, Z)$$

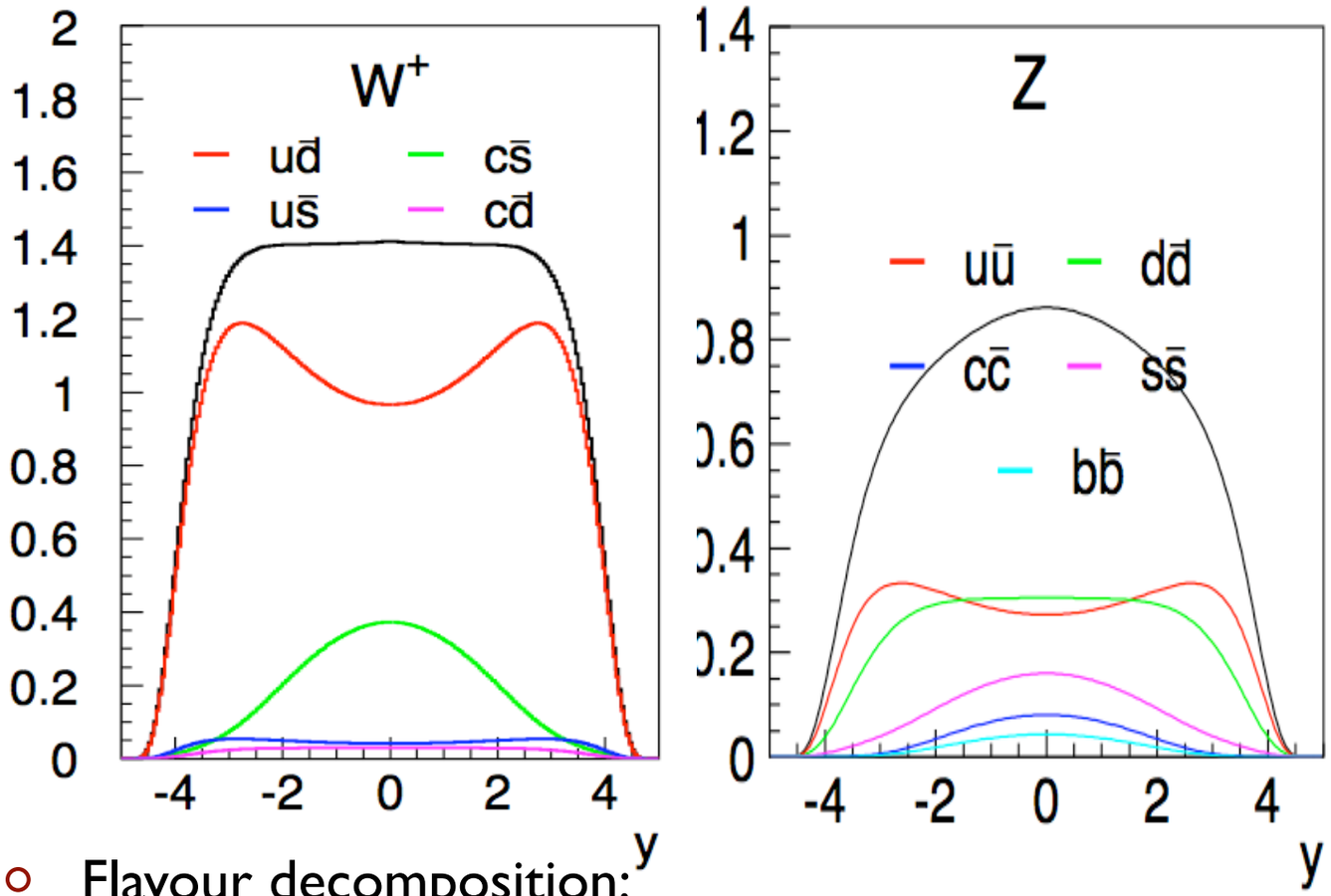
So, to predict Z or W prodution at LHC at some rapidity y=-4, need:

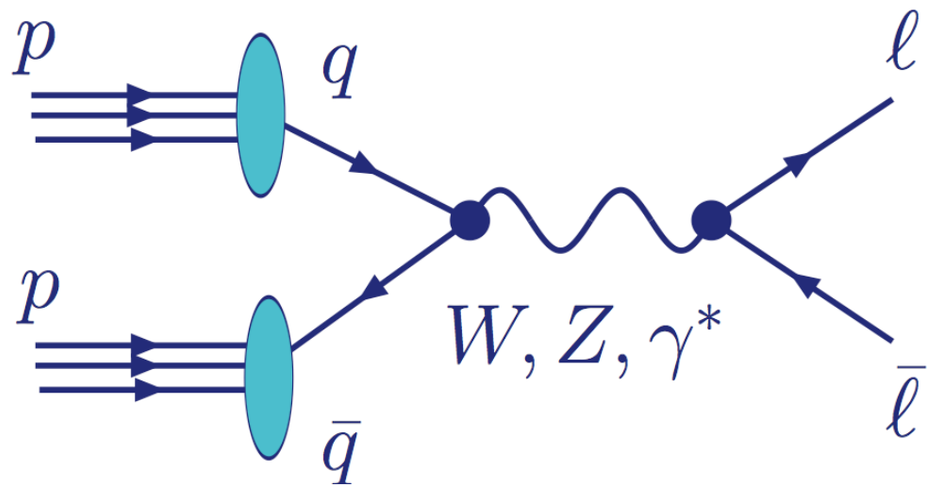


$q,q(x_1=10^{-4},Q^2=M^2_{W,Z})$

$q,q(x_1=0.3,Q^2=M^2_{W,Z})$

LHC's W and Z decompositions





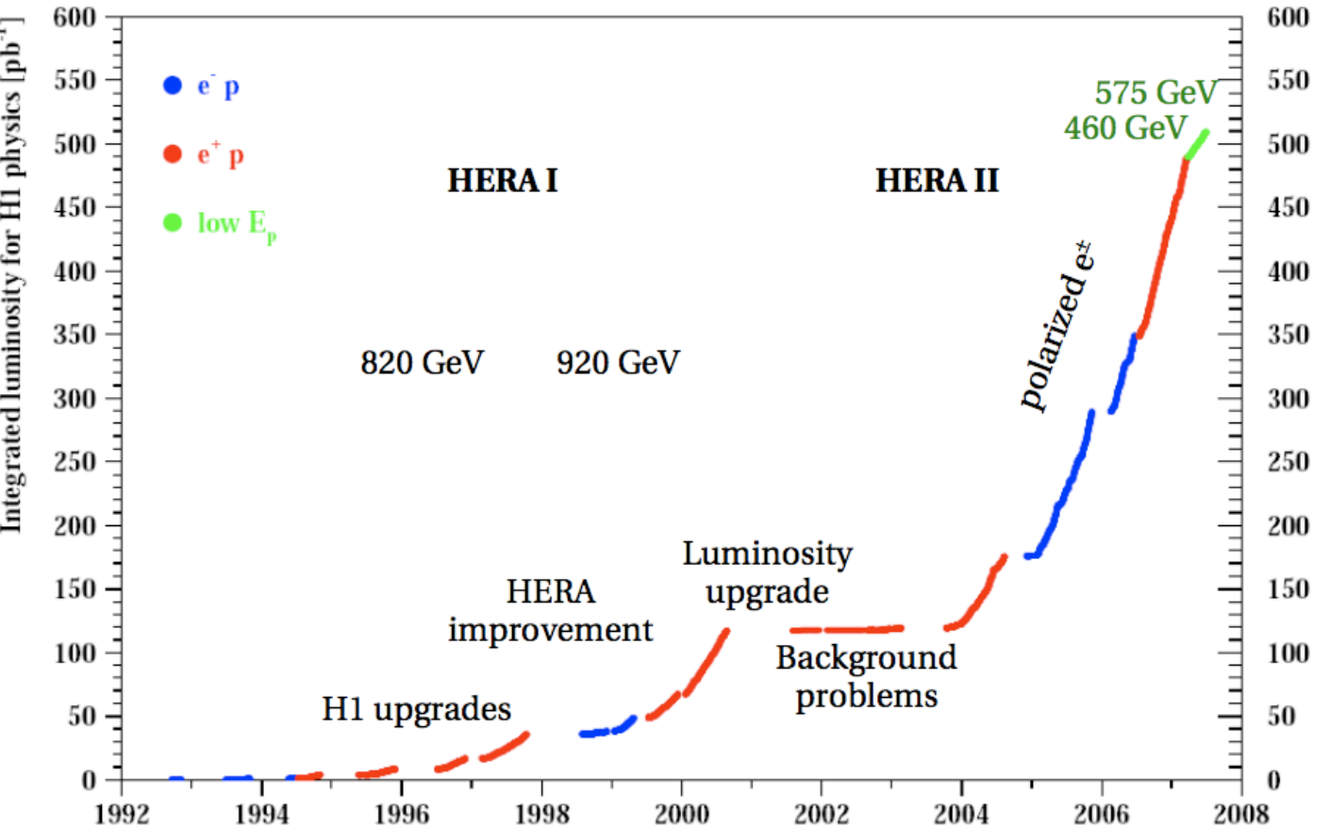
- Flavour decomposition:
  - For Ws:
    - ud dominates for W
    - u_{val} peaks at large rapidity
    - sc important at mid y
  - For Z:
    - all flavours contribute, even b is significant

That is why precise parton distributions are needed for LHC analyses.

Experimental Settings for Proton Structure Determination

HERA

- World's only $e^{\pm}p$ accelerator and collider
  - located at DESY, Hamburg - Germany:
  - In operation for 15 years (1992-2007)
  - H1 and ZEUS collider experiments
    - general purpose detectors

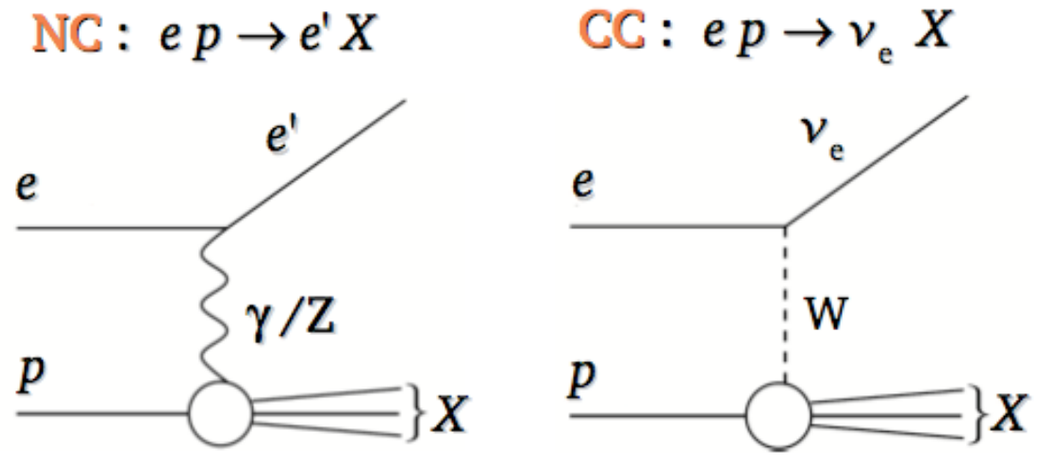


HERA-I	1992-2000	Ep=820, 920 GeV
HERA-II	2003-2007	Ep=920, 460, 575 GeV

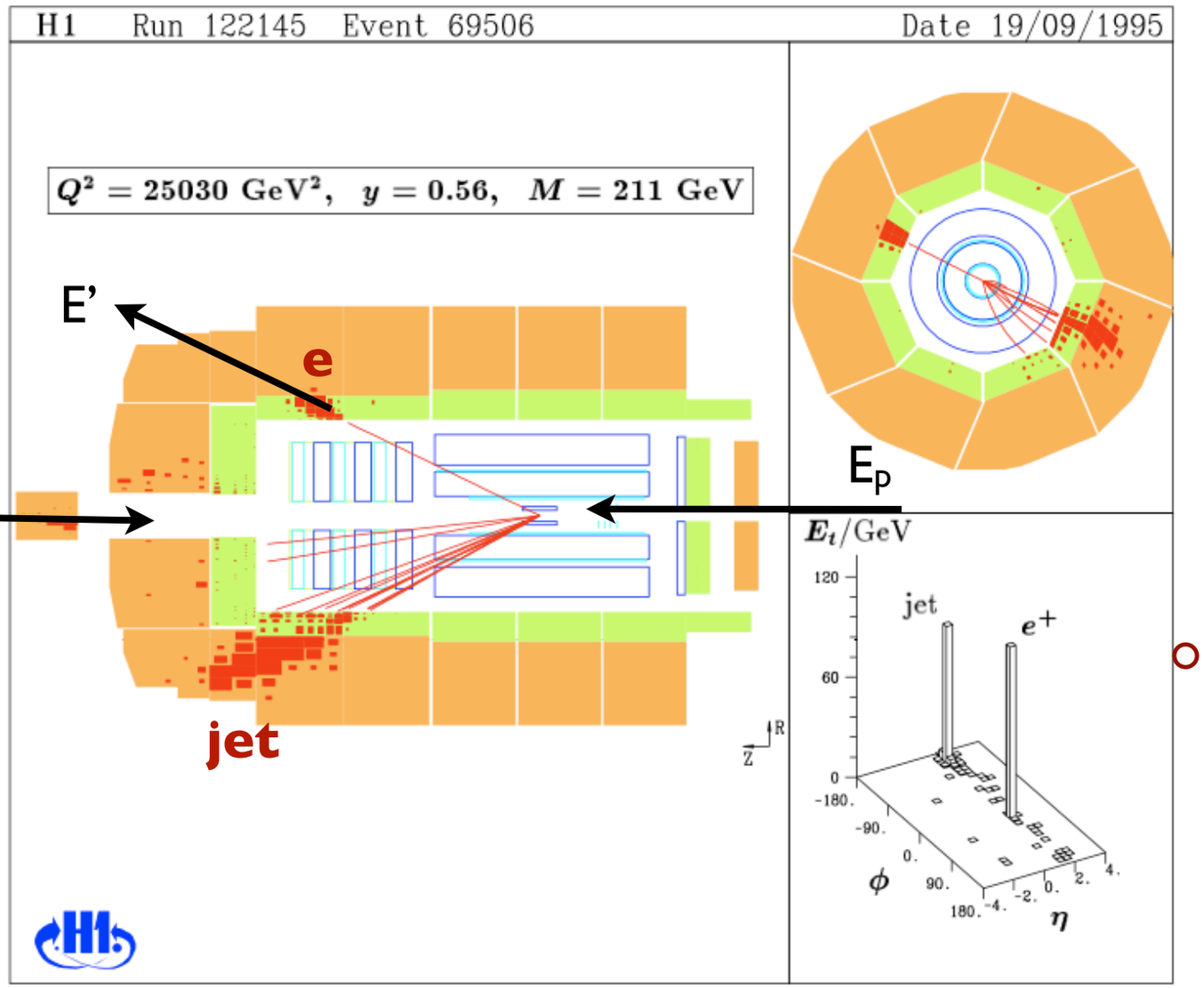
registered ~1fb^{-1} of integrated luminosity of physics data

Detector and Kinematics at HERA: NC DIS

- Processes:
  - Neutral Current (NC): Intermediate boson is neutral (Υ, Z)
  - Charged Current (CC): Intermediate boson is charged ($W^{\pm}$)



- Neutral Current event sample in H1 detector



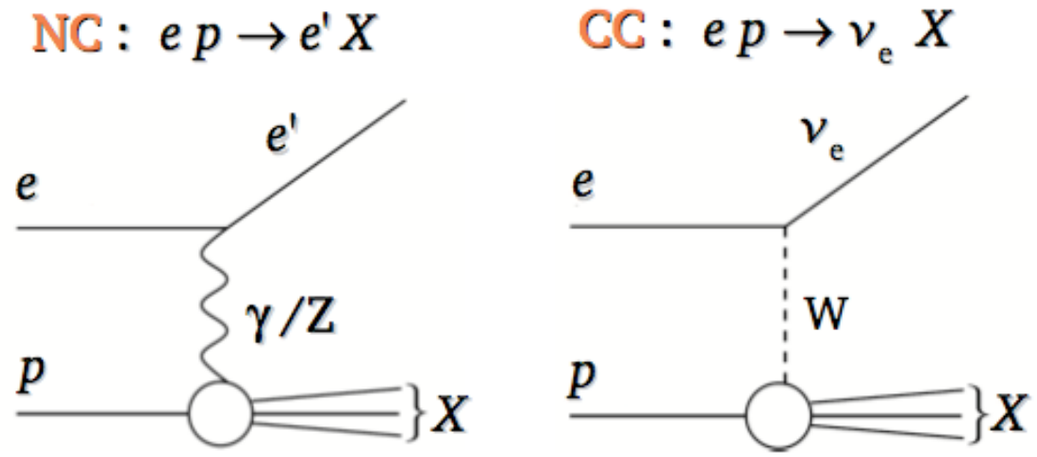
The invariant kinematic variables in terms of measurable variables in the lab frame:

$$s = 4E_eE_p$$
$$Q^2 = E_eE'(1+\cos\theta_e)$$
$$y = 1 - \frac{E'}{E_e}\frac{1}{2}(1-\cos\theta_e)$$
$$x = \frac{Q^2}{sy}$$

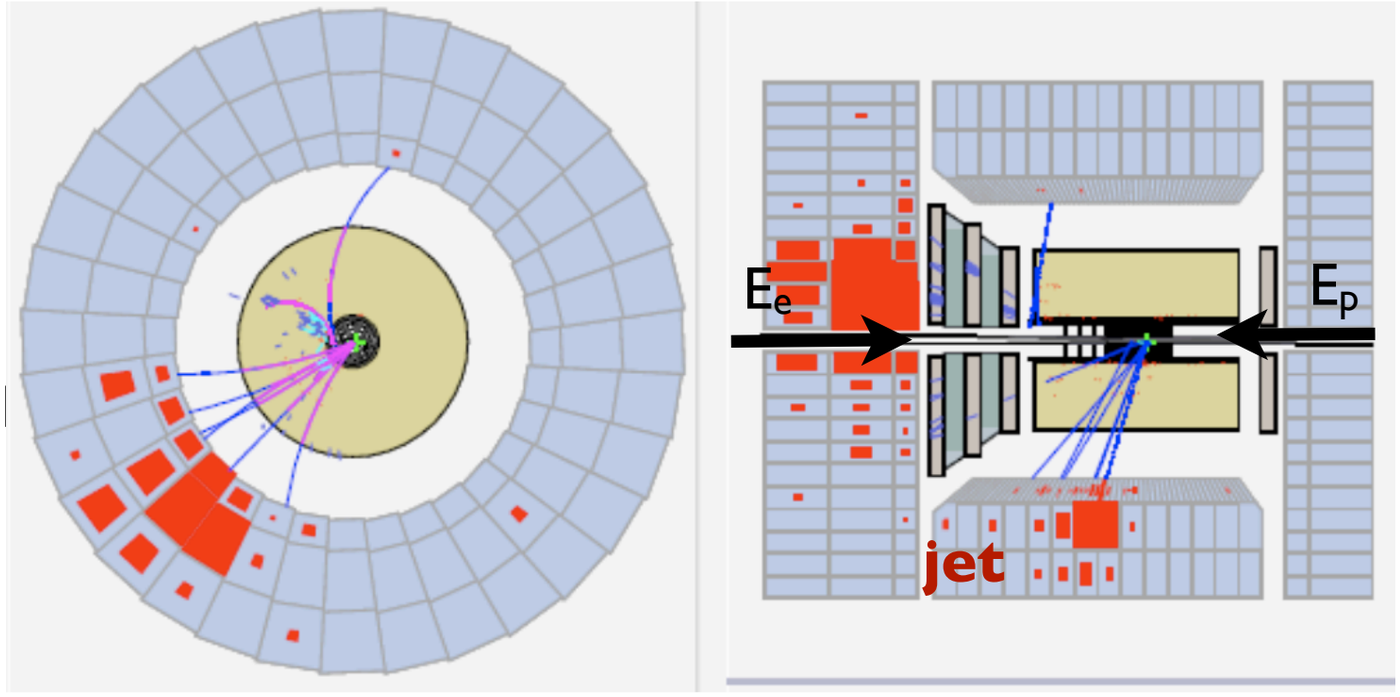
- Determination of the Event Kinematics:
  - using lepton information (E_e,E',θ_e)
  - using hadronic final state particles
  - using both lepton and hadronic final state variables

Detector and Kinematics at HERA: CC DIS

- Processes:
  - Neutral Current (NC): Intermediate boson is neutral (Υ, Z)
  - Charged Current (CC): Intermediate boson is charged ($W^{\pm}$)



- Charged Current event sample in ZEUS detector
  - Neutrino signature: missing transverse momentum



The invariant kinematic variables in terms of measurable variables in the lab frame:

$$s = 4E_eE_p$$

$$Q^2 = \frac{p_t^2}{1-y}$$

$$y = \frac{\sum(E-P_Z)}{2E_e}$$

$$x = \frac{Q^2}{sy}$$

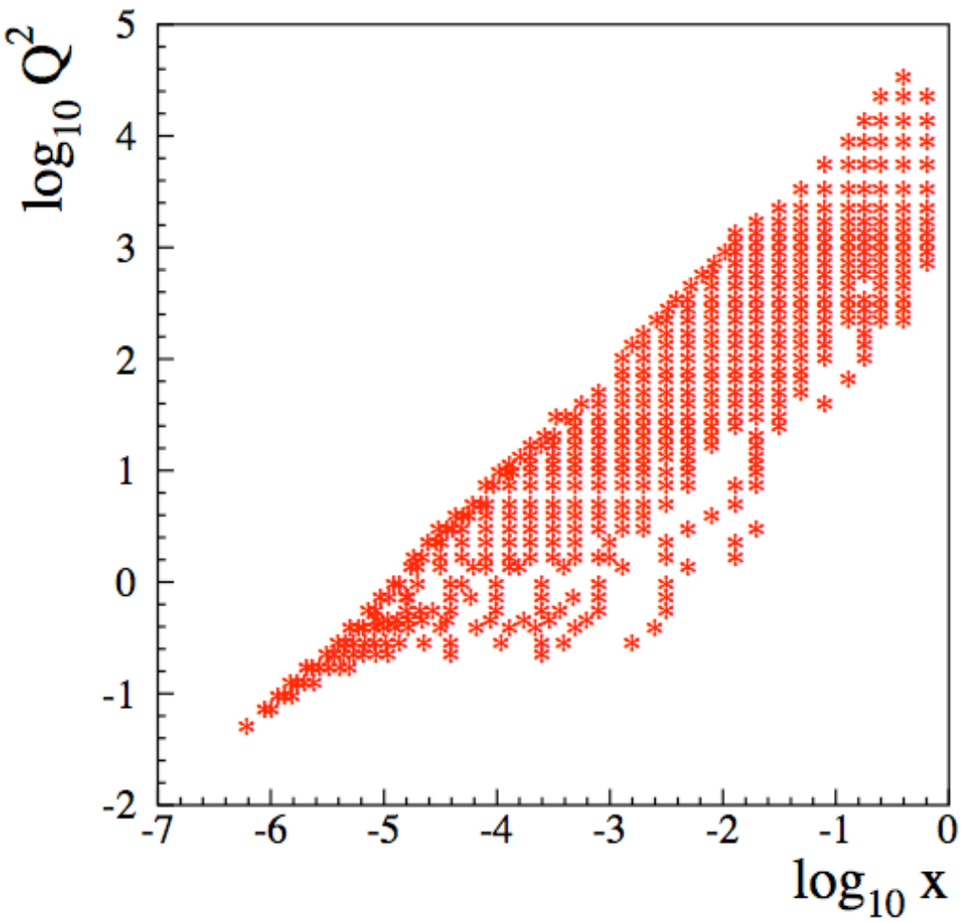
- Determination of the Event Kinematics:
  - using lepton information (E_e,E',θ_e)
  - using hadronic final state particles
  - using both lepton and hadronic final state variables

Latest Results on Proton Structure from HERA I

Measurements at H1 and ZEUS

- Published HERA-I inclusive NC and CC DIS data (1994-2000) is used for the QCD analysis
- Kinematic coverage:
  - $0.045<Q^2<30\,000$ GeV^2
  - $0.000006<x<0.65$

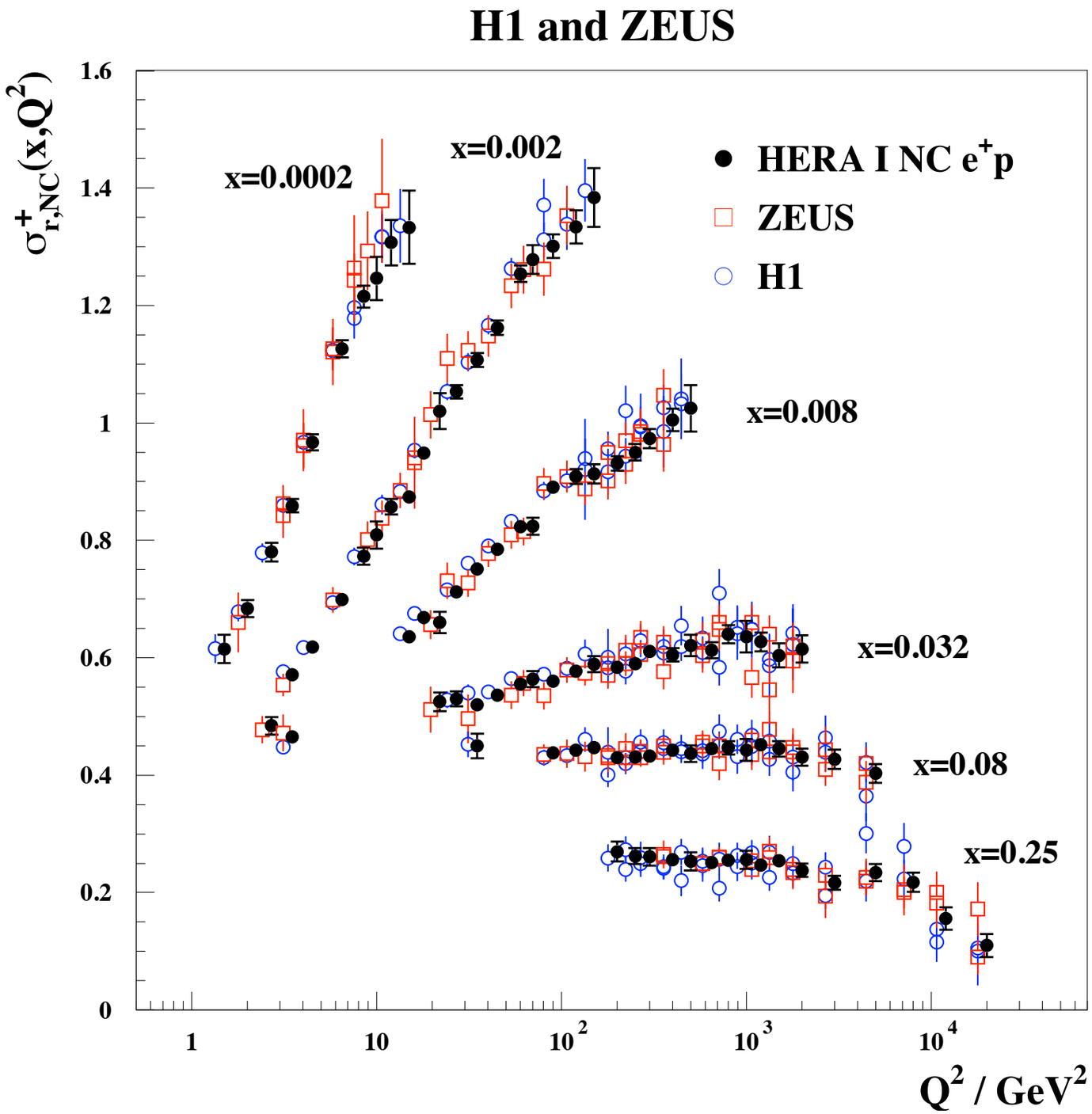
Data Set		x range		Q^2 range GeV^2		$\mathcal{L}$ pb^{-1}	$\sqrt{s}$ GeV
H1 svx-mb	95-00	5×10^{-6}	0.02	0.2	12	2.1	301-319
H1 low Q^2	96-00	2×10^{-4}	0.1	12	150	22	301-319
H1 NC	94-97	0.0032	0.65	150	30000	35.6	301
H1 CC	94-97	0.013	0.40	300	15000	35.6	301
H1 NC	98-99	0.0032	0.65	150	30000	16.4	319
H1 CC	98-99	0.013	0.40	300	15000	16.4	319
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	319
H1 NC	99-00	0.00131	0.65	100	30000	65.2	319
H1 CC	99-00	0.013	0.40	300	15000	65.2	319
ZEUS BPT	97	6×10^{-7}	0.001	0.045	0.65	3.9	301
ZEUS BPC	95	2×10^{-6}	6×10^{-5}	0.11	0.65	1.65	301
ZEUS SVX	95	1.2×10^{-5}	0.0019	0.6	17	0.2	301
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	301
ZEUS NC	96-97	6×10^{-5}	0.65	2.7	30000	30.0	301
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	319
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	319
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	319
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	319



[JHEP01 (2010) 109]

Combined HERA-I Data

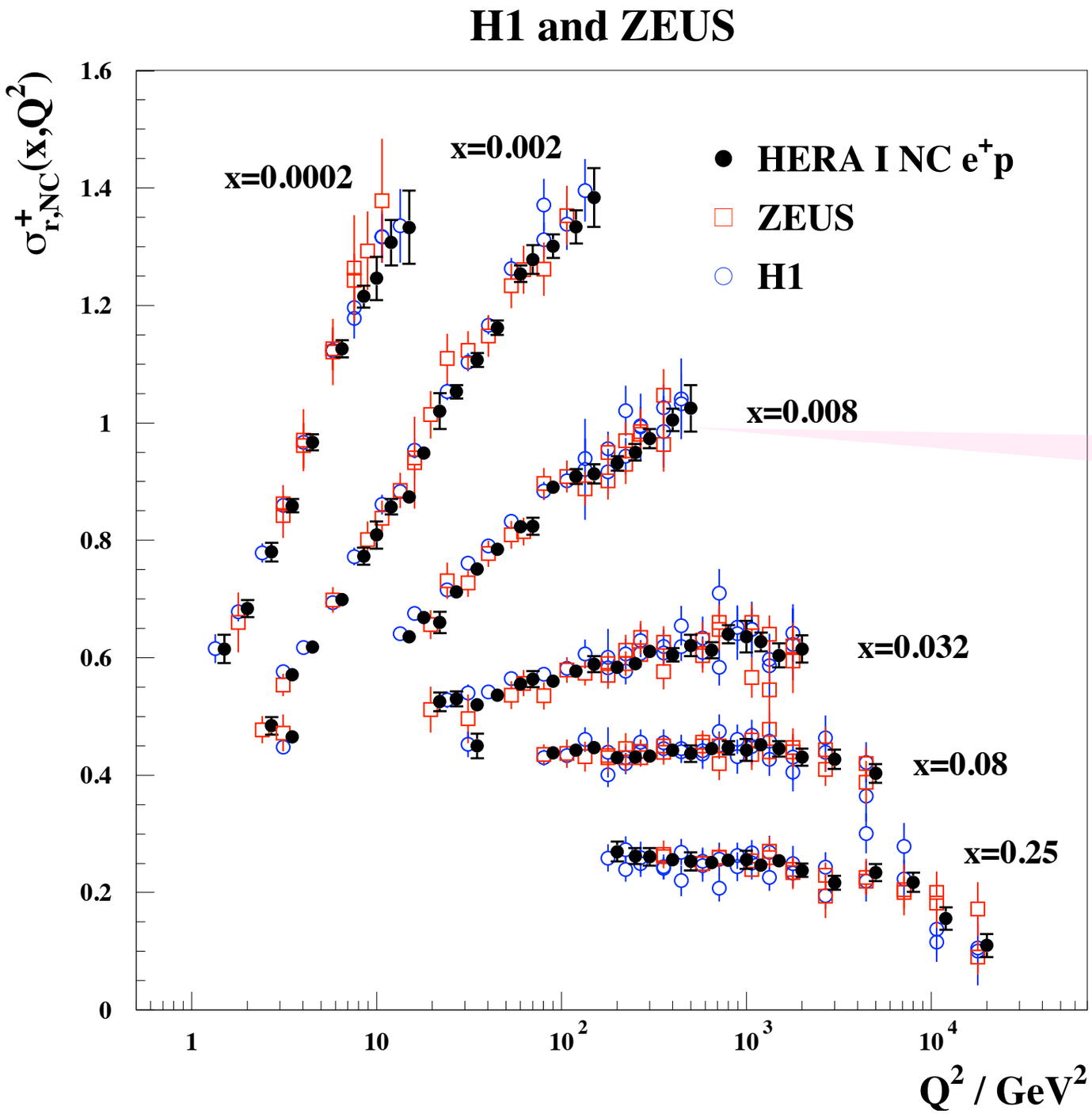
- Ultimate precision is obtained by combining H1 and ZEUS measurements
- Average H1 and ZEUS data is performed before QCD analysis:



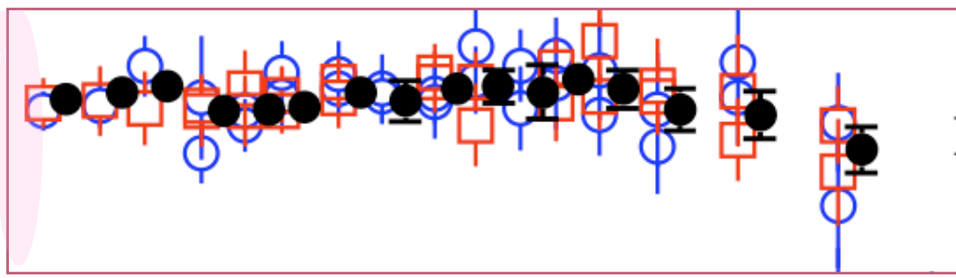
- Experiments cross calibrate each other:
  - H1 and ZEUS use different reconstruction methods which depend differently on the sources of experimental uncertainties, hence the combination procedure significantly reduces the errors.

Combined HERA-I Data

- Ultimate precision is obtained by combining H1 and ZEUS measurements
- Average H1 and ZEUS data is performed before QCD analysis:



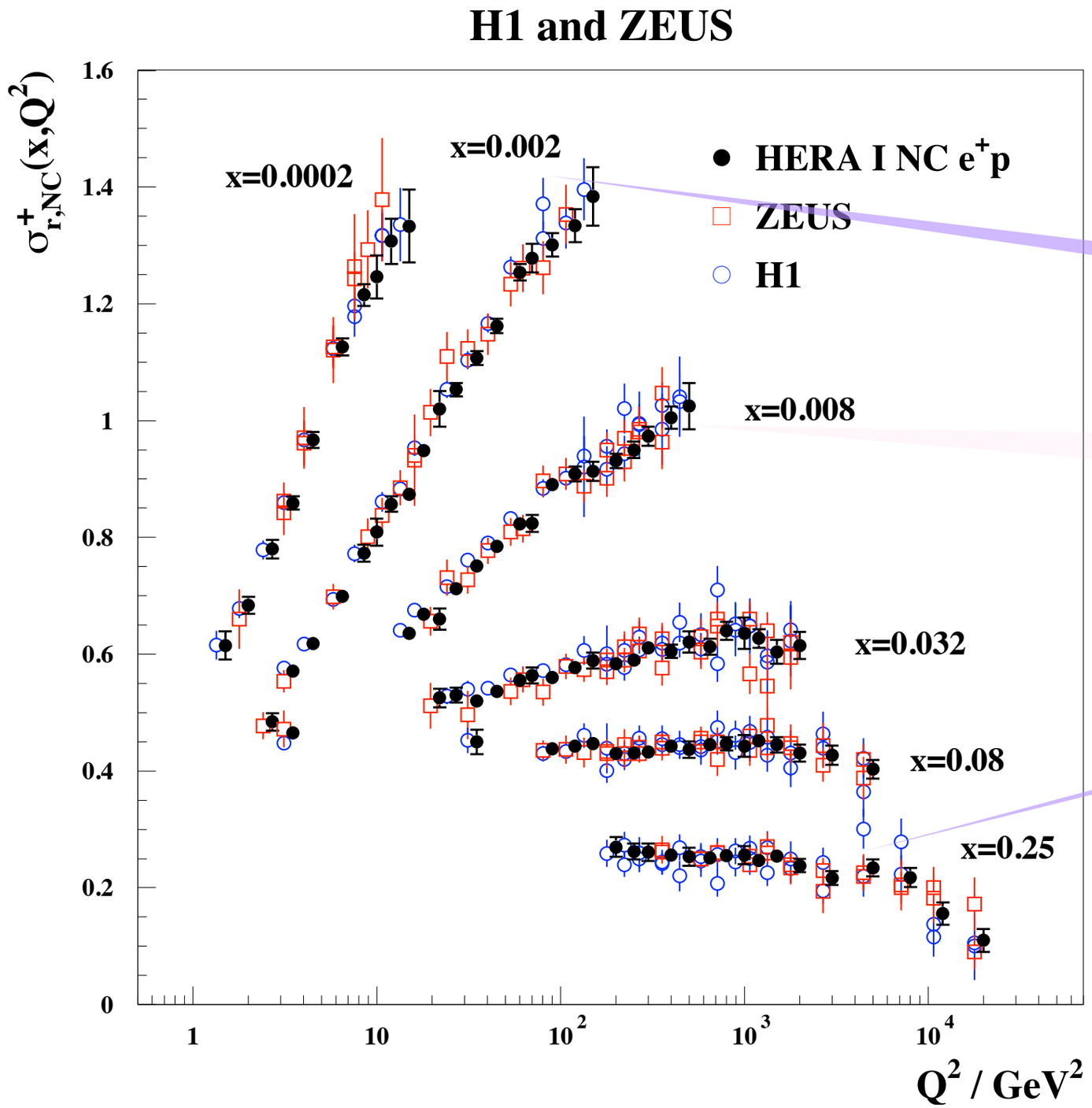
- Experiments cross calibrate each other:
  - H1 and ZEUS use different reconstruction methods which depend differently on the sources of experimental uncertainties, hence the combination procedure significantly reduces the errors.



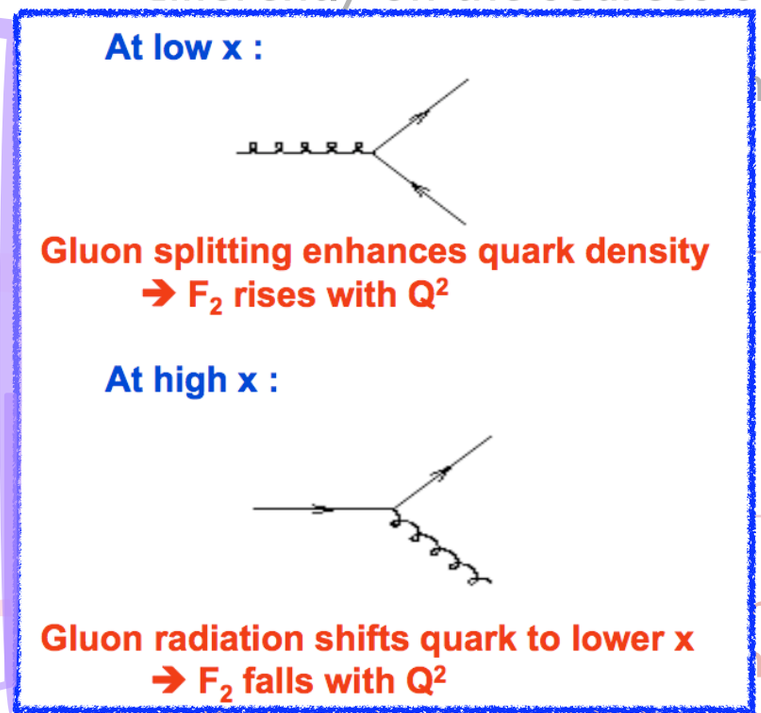
- Before combination, the systematic errors are ~3 times larger than statistical for $Q^2<100$ GeV^2
- After combination, the systematic errors are of same precision as the statistical errors, reaching 1% total precision!

Combined HERA-I Data

- Ultimate precision is obtained by combining H1 and ZEUS measurements
- Average H1 and ZEUS data is performed before QCD analysis:



- Experiments cross calibrate each other:
  - H1 and ZEUS use different reconstruction methods which depend differently on the sources of … hence the … ificantly



  - … natic … statistical for Q^2 < 100 GeV
  - After combination, the systematic errors are of same precision as the statistical errors, reaching 1% total precision!

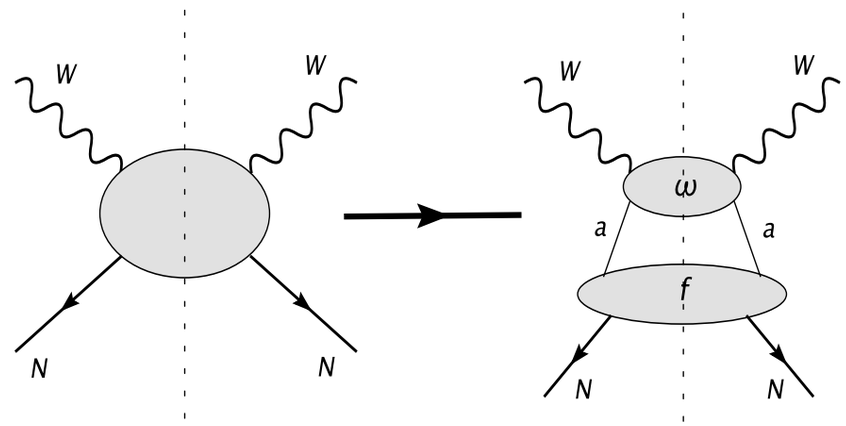
QCD Analysis Strategy

QCD features

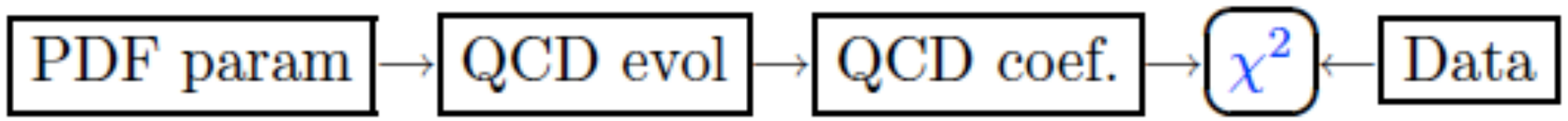
- Quantum Chromo Dynamics is theory of strong interactions among quarks and gluons
- The strength of interactions is given by the strong coupling constant.
  - Quarks are bound inside protons, strongly coupled, cannot measure directly their distributions: confinement
  - At large scattering scales the coupling constant of strong force decreases and quarks become quasi-free partons: asymptotic freedom
    - interactions of quarks and gluons at large scales can be calculated perturbatively in strong coupling constant alpha_S (pQCD in alpha_S)
- Factorisation Theorem
  - short and long distances processes are separable
  - PDFs are universal

⇨Structure Functions (F_i) are a convolution of PDFs (f_a) with hard scattering coefficient function

$$F_i = \sum_a \omega_i^a \otimes f_a$$



Schematics of PDF extractions



- PDFs are extracted from QCD fits to double differential cross section data:
  - Parametrise PDFs at a starting scale by smooth functions with sufficient parameters;
  - Evolve PDFs to other scales by the evolution equations (DGLAP)
  - Compute cross sections for DIS (or other processes) at NLO (NNLO)
  - Calculate χ^2 measure of agreement between data and theory model
  - Obtain the best estimate of the PDFs by varying the free parameters to minimize χ^2

HERA PDF Parametrisation for the Central Fit

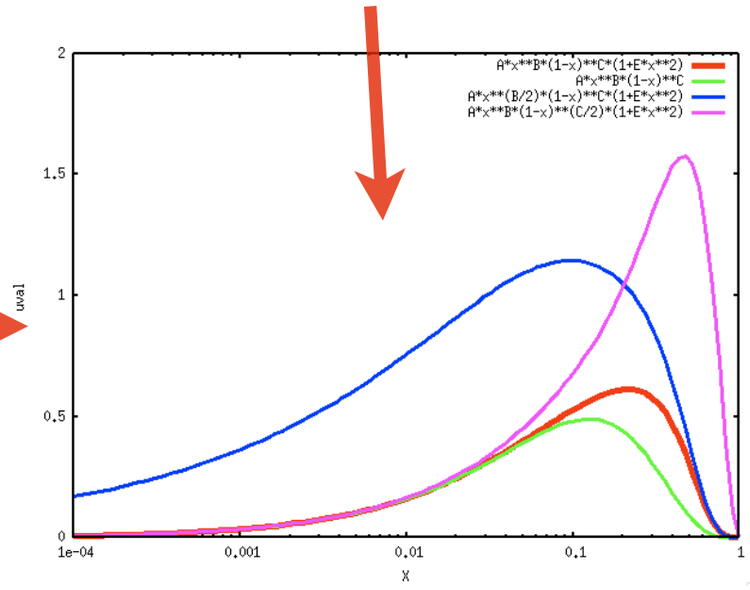
- PDFs that are parametrised at a starting scale Q_0^2=1.9 GeV^2 (below M_c^2) are:

$$xg, xu_v, xd_v, x\bar{U} = x\bar{u}, x\bar{D} = x\bar{d} + x\bar{s}$$

- A standard functional parametrisation form is used:

$$xf(x, Q_0^2) = Ax^B(1-x)^C(1 + Dx + Ex^2)$$

  - A - normalisation
  - B - low x behaviour
  - C - high x behaviour
  - D,E - medium x tuning



- It describes the shape of PDFs with few input parameters
  - The number of parameters are chosen by saturation of the χ^2

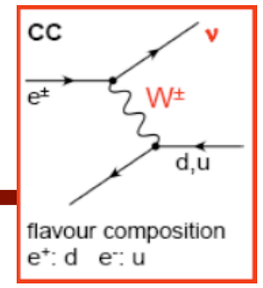
- The parametrisation for the best fit (central fit):

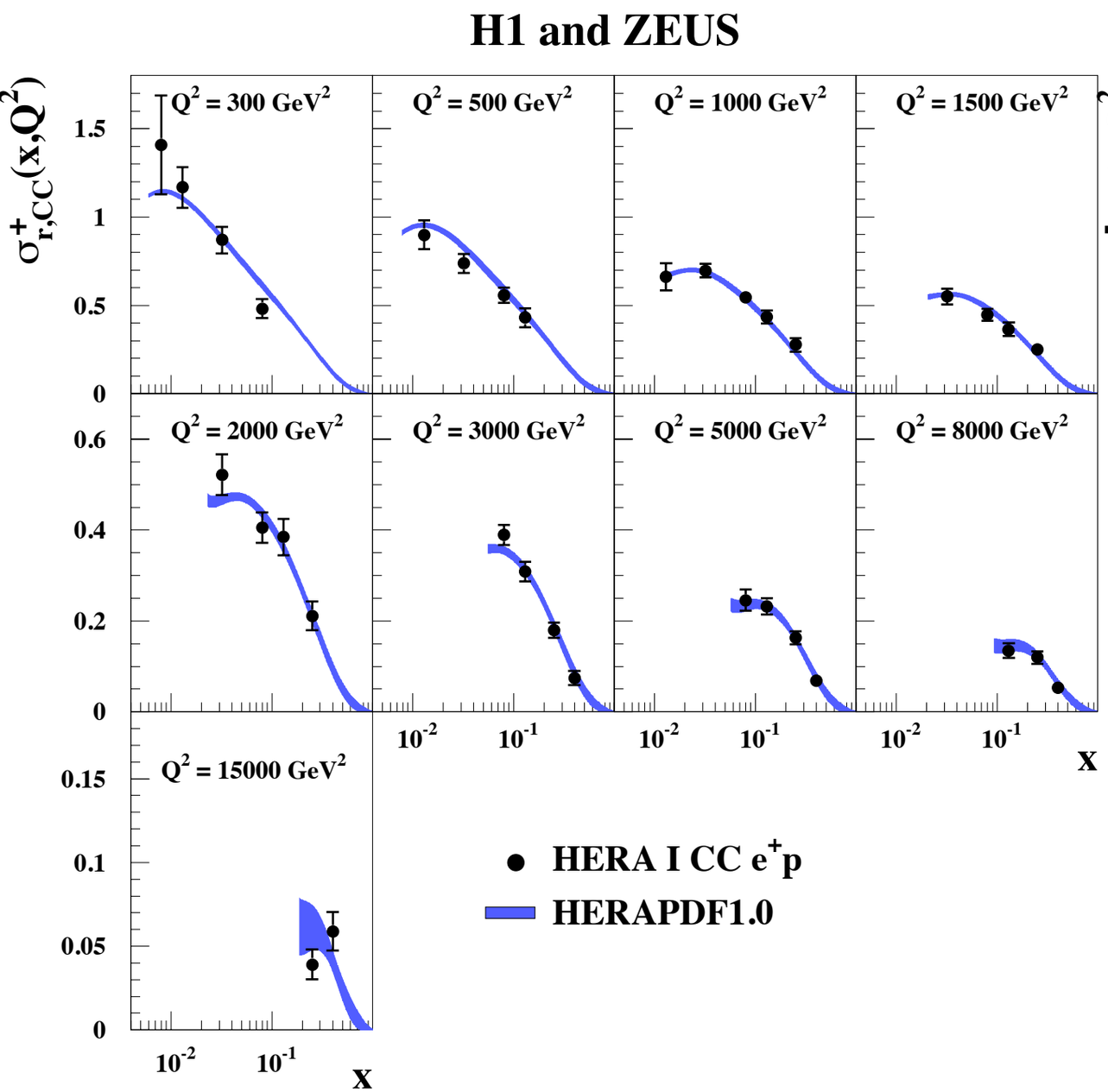
$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1 + E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}, \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

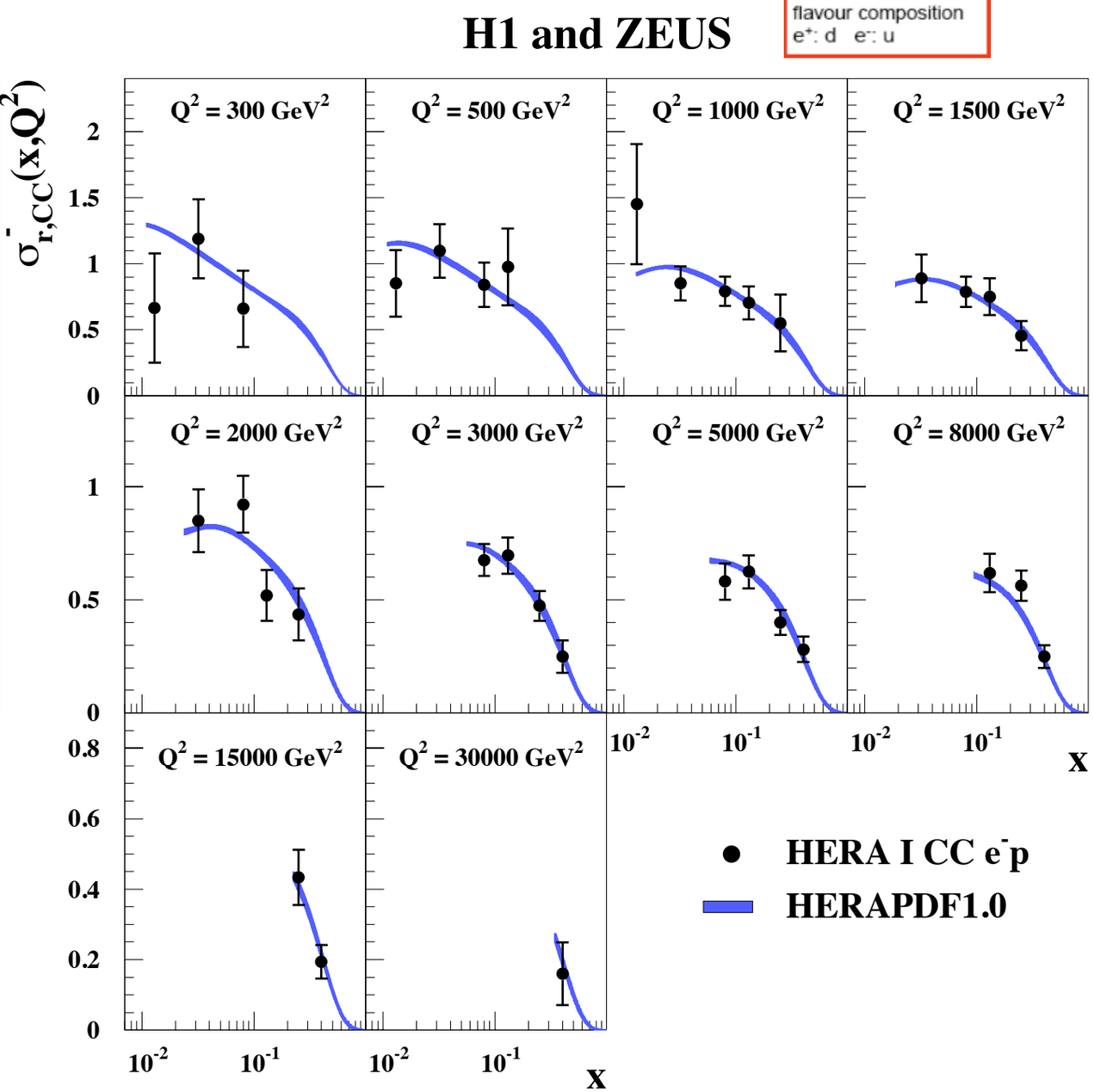
- The number of free parameters is reduced by the physics constraints such as:
  - normalisation parameters A_g, A_{uv}, A_{dv} by the quark number and momentum sum-rules
  - B parameters so that there is one for sea and another one for valence distributions
  - Ensure that $x\bar{u} \to x\bar{d}$ as $x \to 0$.

- The best fit results in 10 free parameters

HERA Fit Results vs CC Data







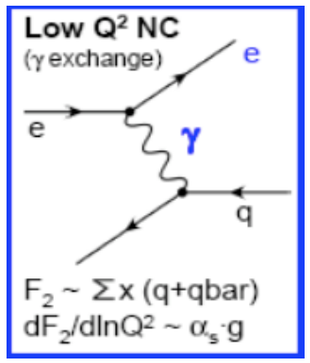
- HERA CC data give flavour information:
  - e^+p CC process sensitive to d_v at high x
  - e^-p CC process sensitive to u_v at high x

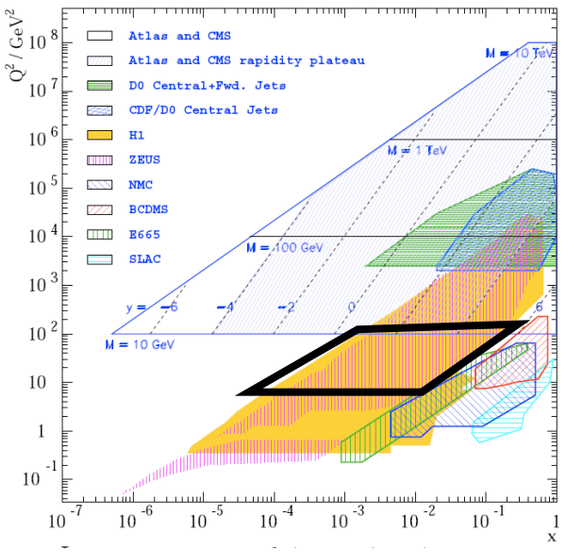
$$\sigma^{+}_{CC} \sim x(\bar{u}+\bar{c}) + x(1-y)^2(d+s)$$

$$\sigma^{-}_{CC} \sim x(u+c) + x(1-y)^2(\bar{d}+\bar{s})$$

- Good agreement between QCD Fit and Data

HERA Fit Results vs NC low Q^2 data

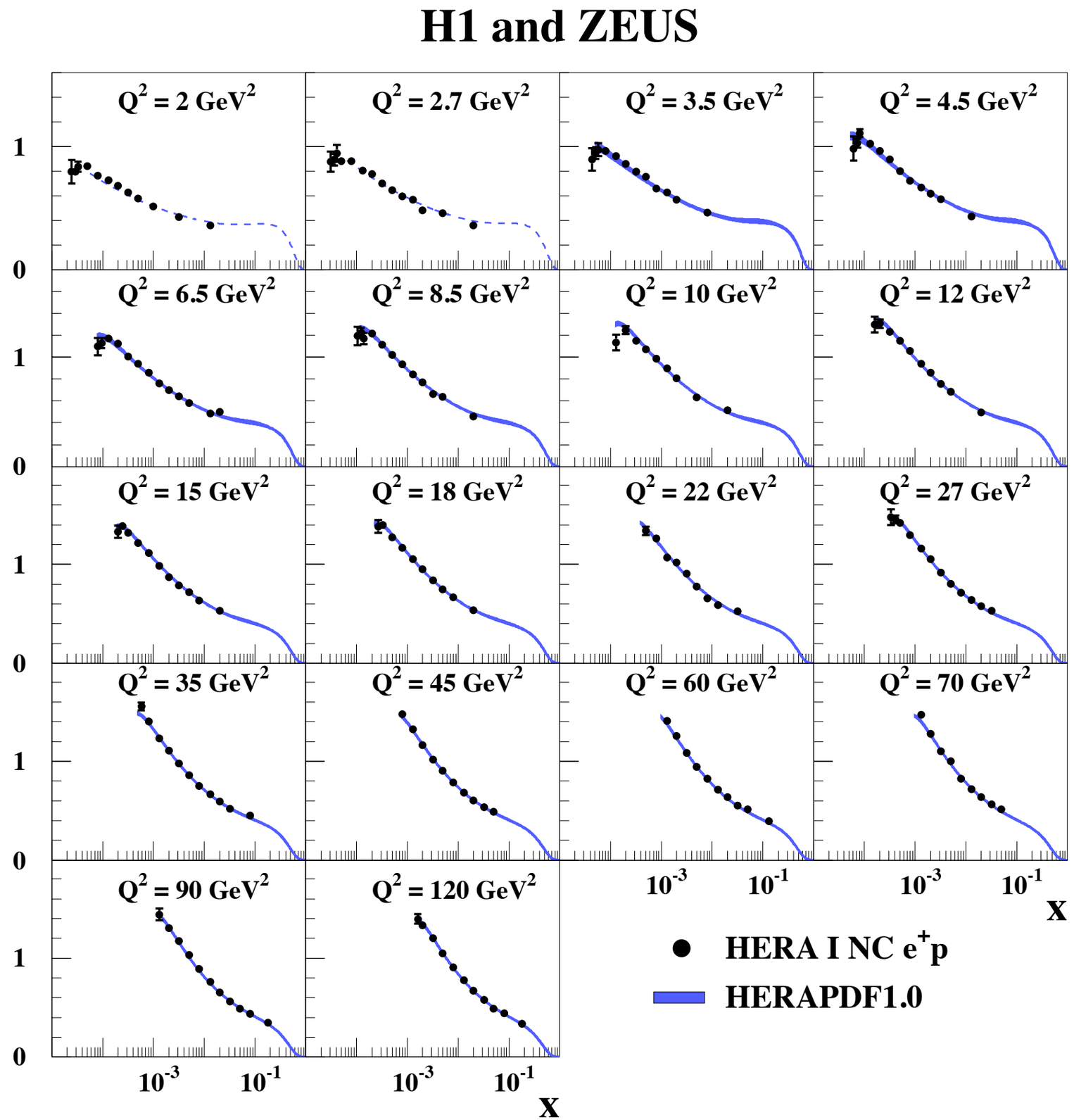




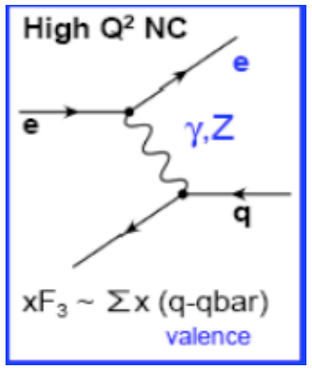
- At low Q^2, DIS cross section is described by F_2 and F_L:

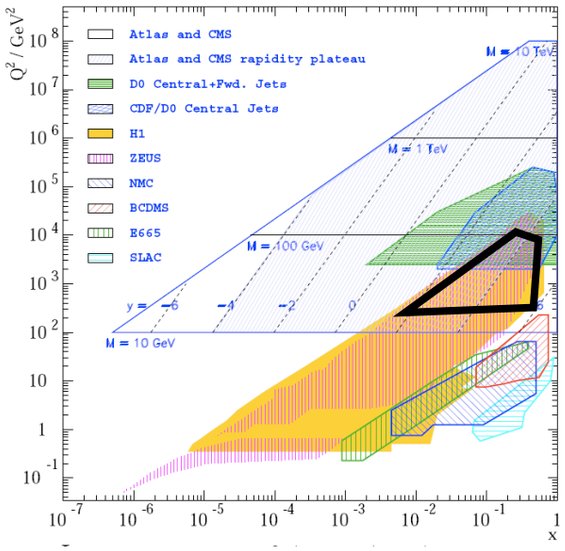
$$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$$

  - turn-over given by F_L
- Precise data is well described by the NLO HERA fit
  - even for $Q^2<3.5$ GeV^2 which is not included in the fit
  - fit line includes all errors
  - ~1% data precision
- Strong rise as x goes to 0, increases with increasing Q^2

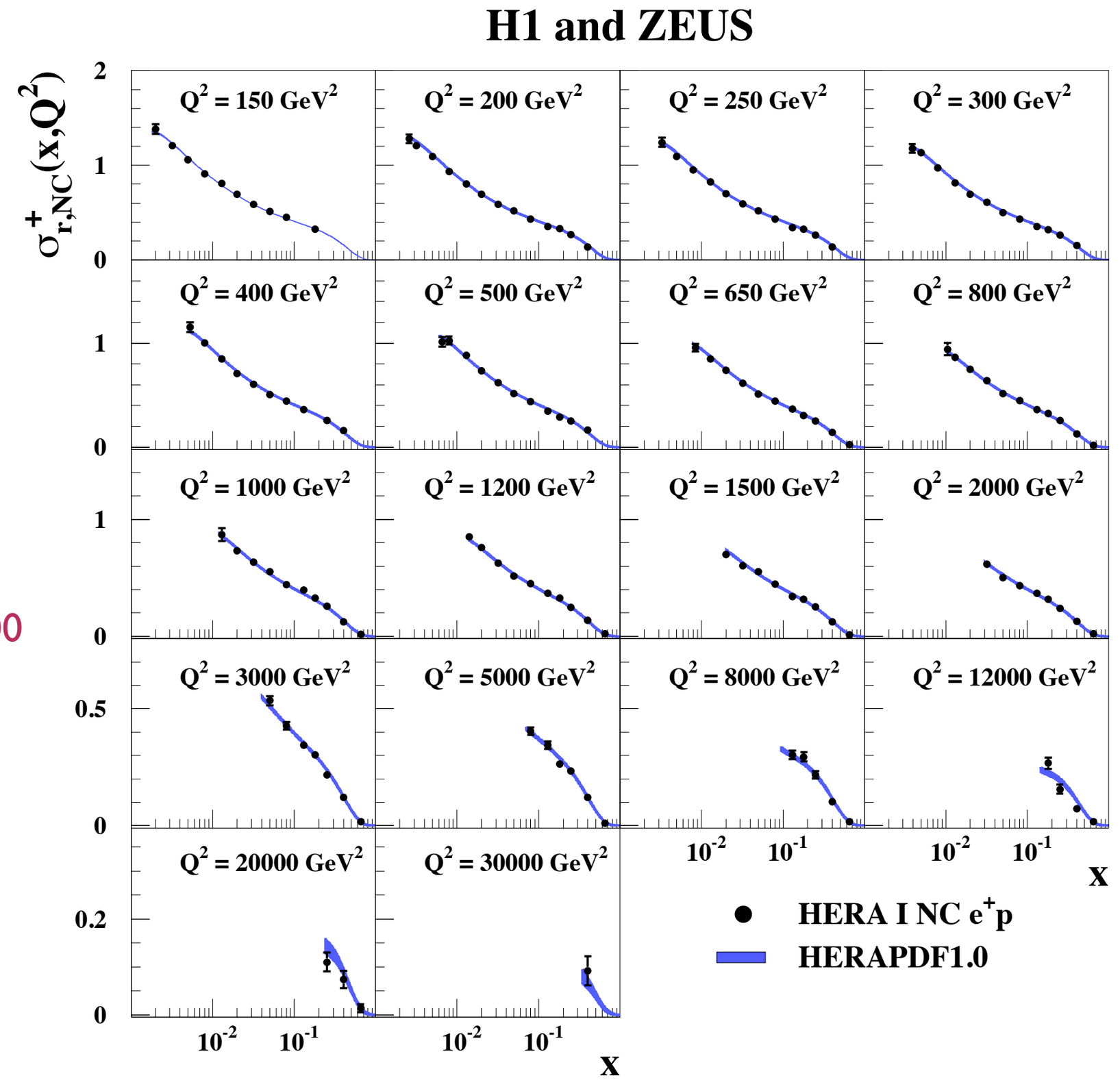


HERA Fit Results vs NC high Q^2 data





- Precise data very well described by the HERA fit
  - ~2% data precision for $Q^2<500$ GeV^2
  - <5% data precision for $Q^2<5000$ GeV^2
- χ^2/dof = 574/582



Sources of PDF uncertainties

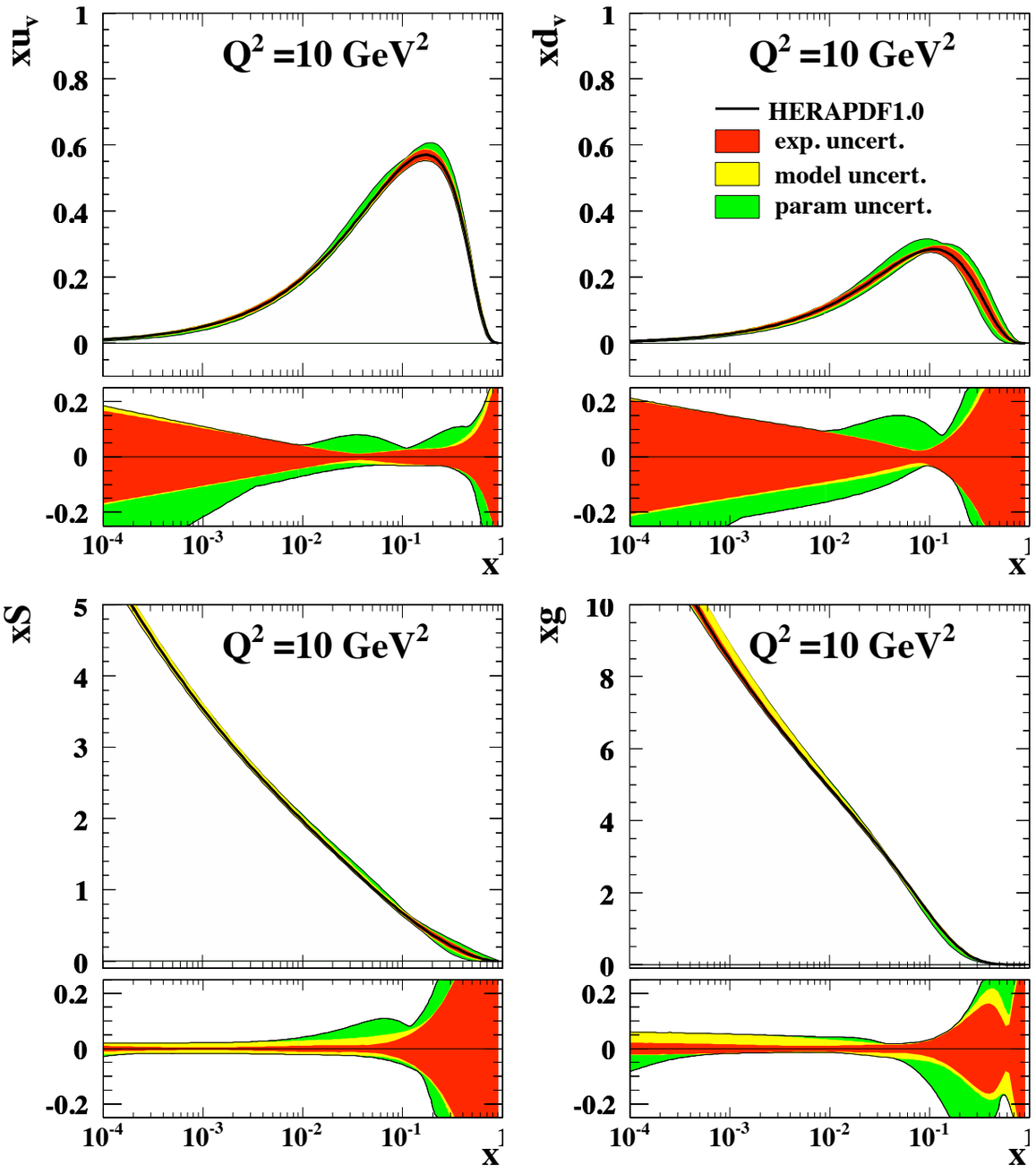
- PDFs are needed for precise estimation of cross sections at the LHC
  - Therefore, it is essential to understand and improve the precision on PDFs.

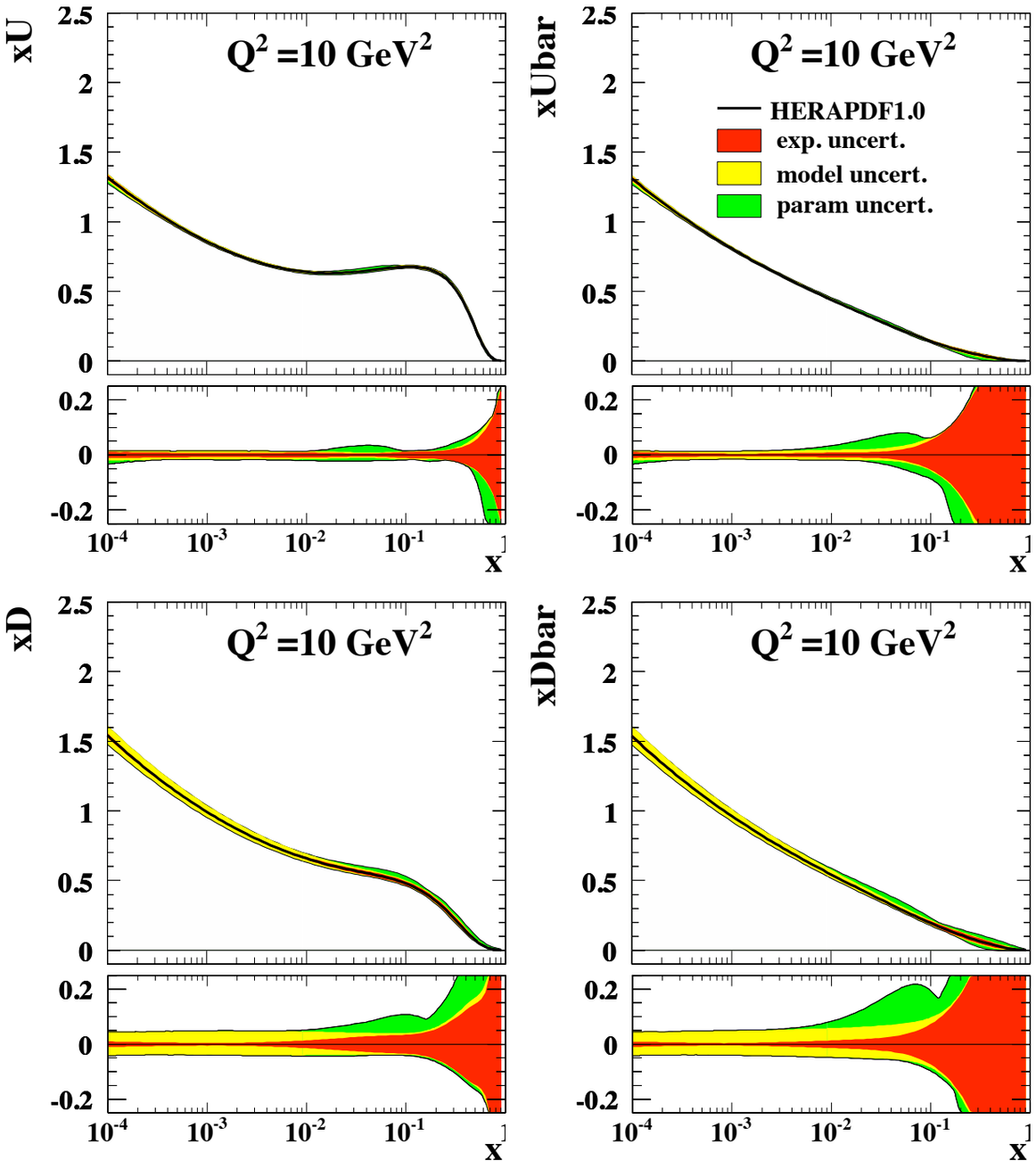
- At HERA, sources of the PDF uncertainty are classified as:

  - Experimental uncertainties propagated to PDFs
  - Model/Theory uncertainties:
    - Related to the choices of model input and theory assumptions
  - Parametrisation uncertainties:
    - Scanning the parameter space given by standard parametrisation form

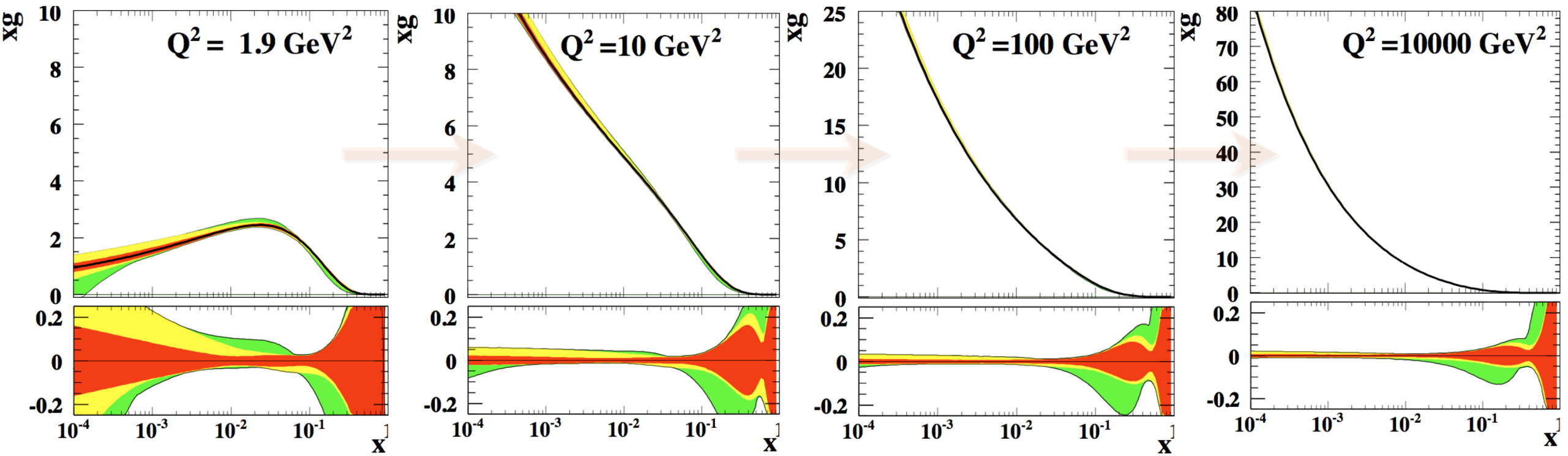
HERA PDF distributions at Q^2=10 GeV^2

The break-up of the PDFs into different flavours: xuv, xdv, xS=2x($\bar{U}$+$\bar{D}$), xg,
xU=x(u+c), xD=x(d+s), x$\bar{U}$, x$\bar{D}$ are closely related to measurements → well constrained at low x





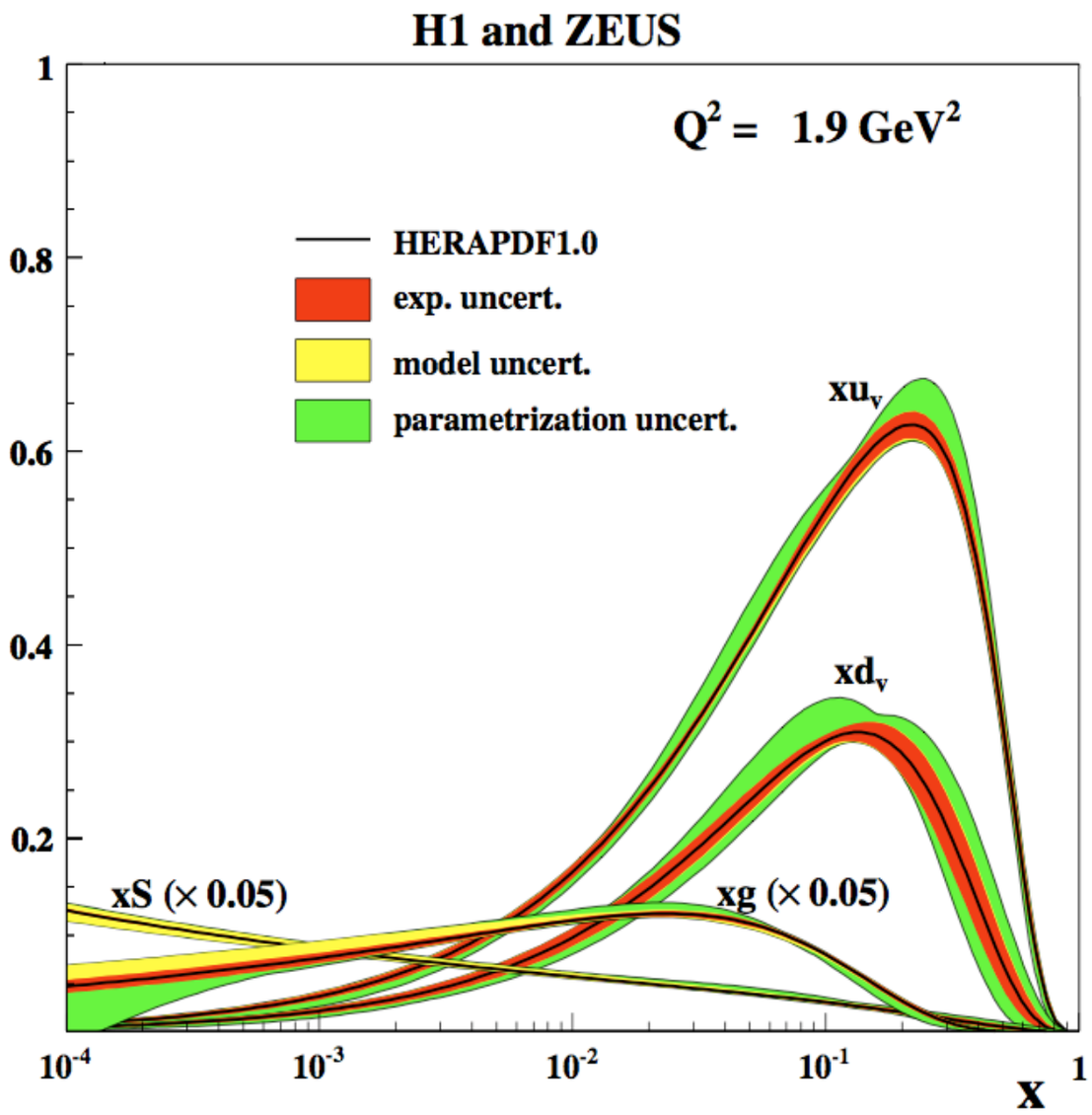
Evolution of Gluon with Q^2



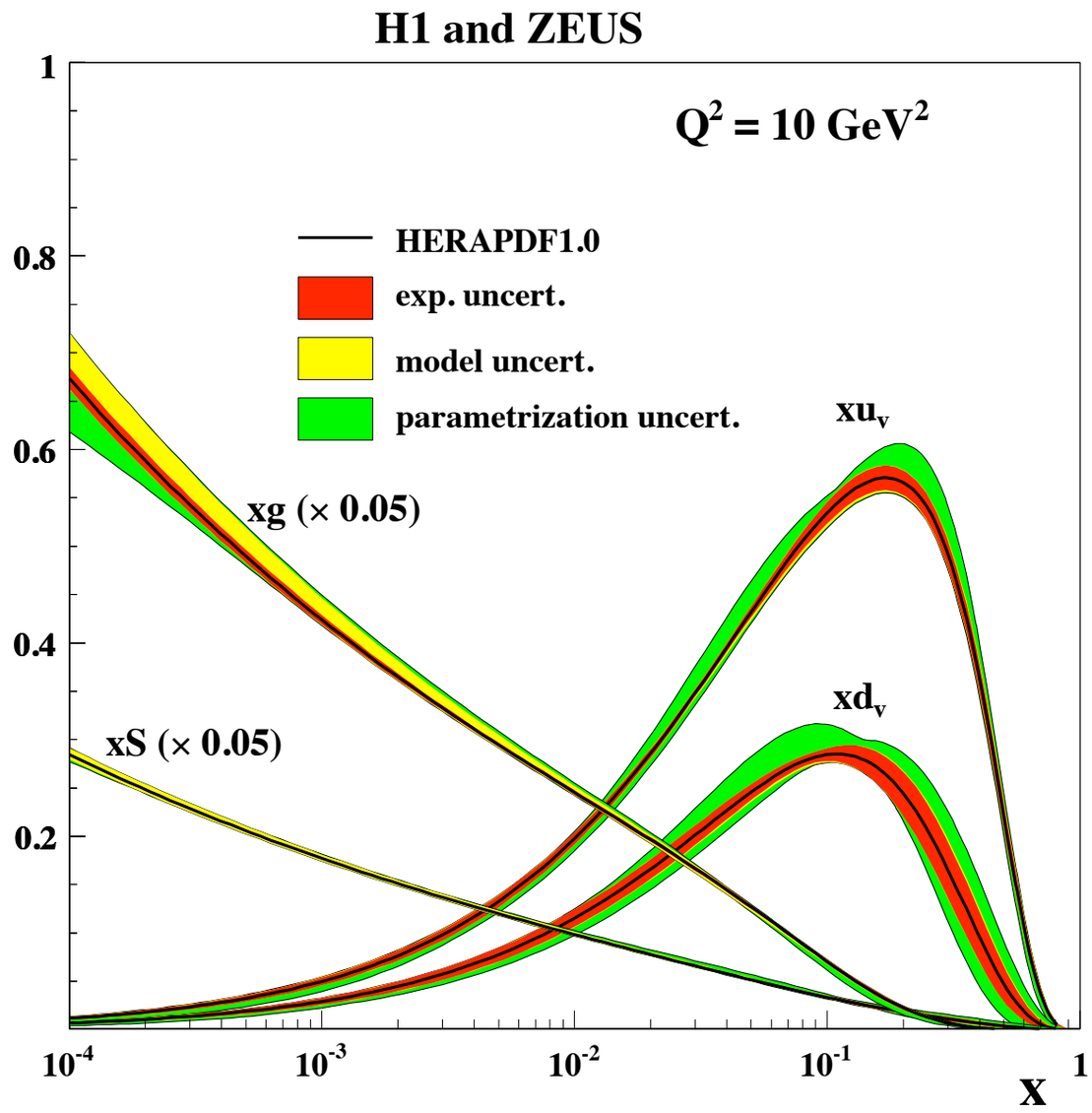
- Different sources of systematics are shown.
- Near the starting scale gluon is valence like:
  - at the starting scale the model uncertainty is large at low x
    - dominated mostly by the variation of Mc
- PDF Parametrisation uncertainty dominates the high x region
- The precision of gluon PDF is very good in the low x region at the scale relevant to LHC!

Summary Plot of HERAPDF1.0

Starting scale:



10 GeV^2:

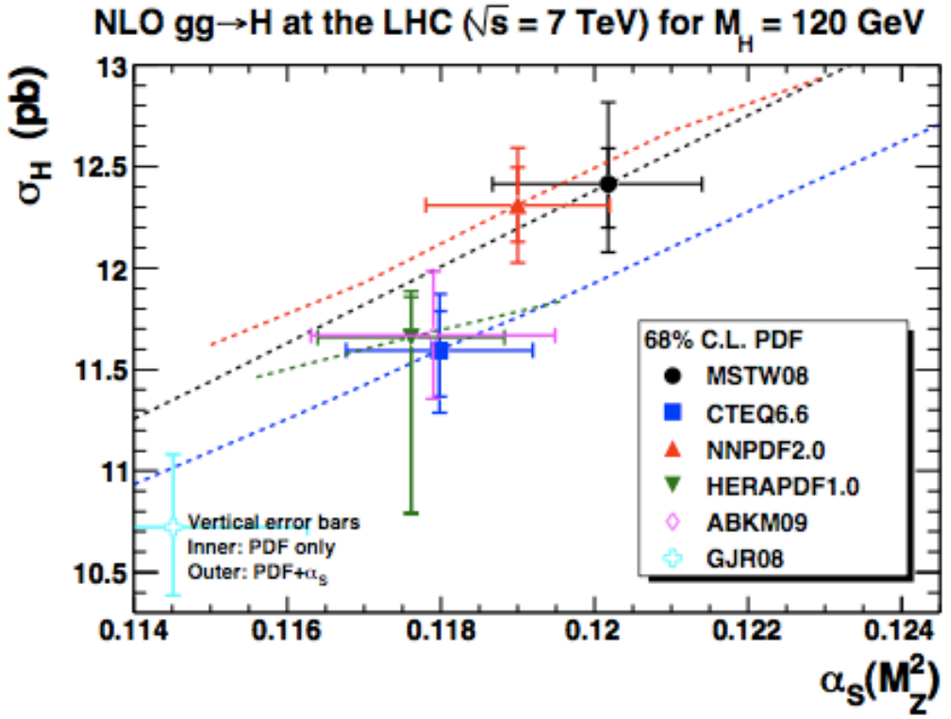


Global Fit Analysis Groups

- Various data sets have constraining powers on PDFs:
  - fixed target experiments - high x
  - HERA - low x
  - Tevatron - high x

- Following global fit groups are active:
  - CTEQ (Coordinated Theoretical-Experimental Project on QCD)
  - MSTW (Martin-Stirling-Thorne-Watt)
  - NNPDF (Neural Network PDFs)
  - ABKM (Alekhin-Bluemlein-Klein-Moch)
  - GJR (Glueck-JimenezDelgado-Reya)
    - different parametrisation
    - different arrangements of perturbative series
    - different alphas
    - different data sets
    - different treatments of the heavy quarks

Higgs ($M_H = 120$ GeV) total cross section vs. $\alpha_S(M_Z^2)$

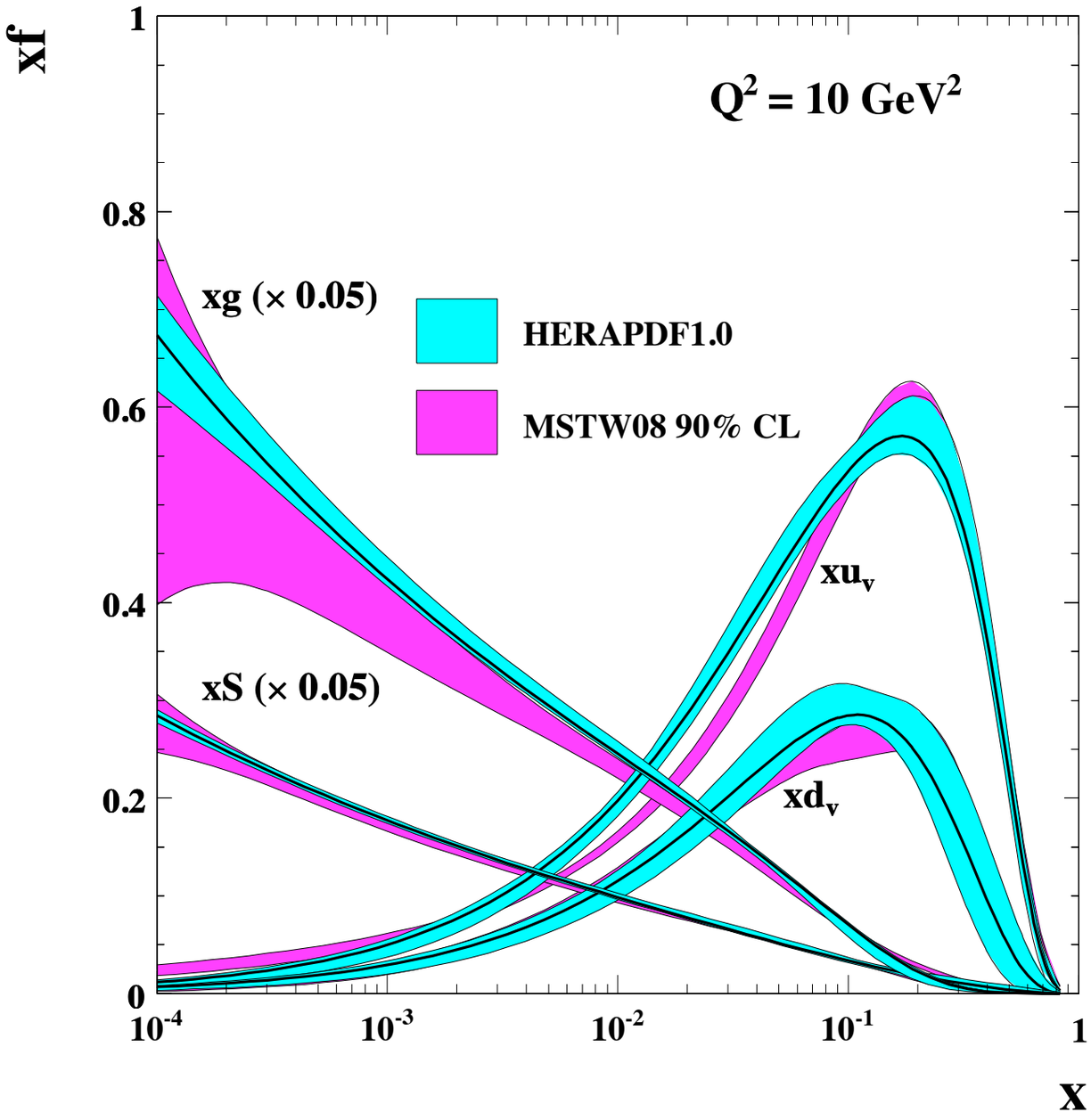


- The PDF sets from the above groups can all be used for LHC predictions, including HERAPDF sets. The remaining differences still need to be studied and understood.

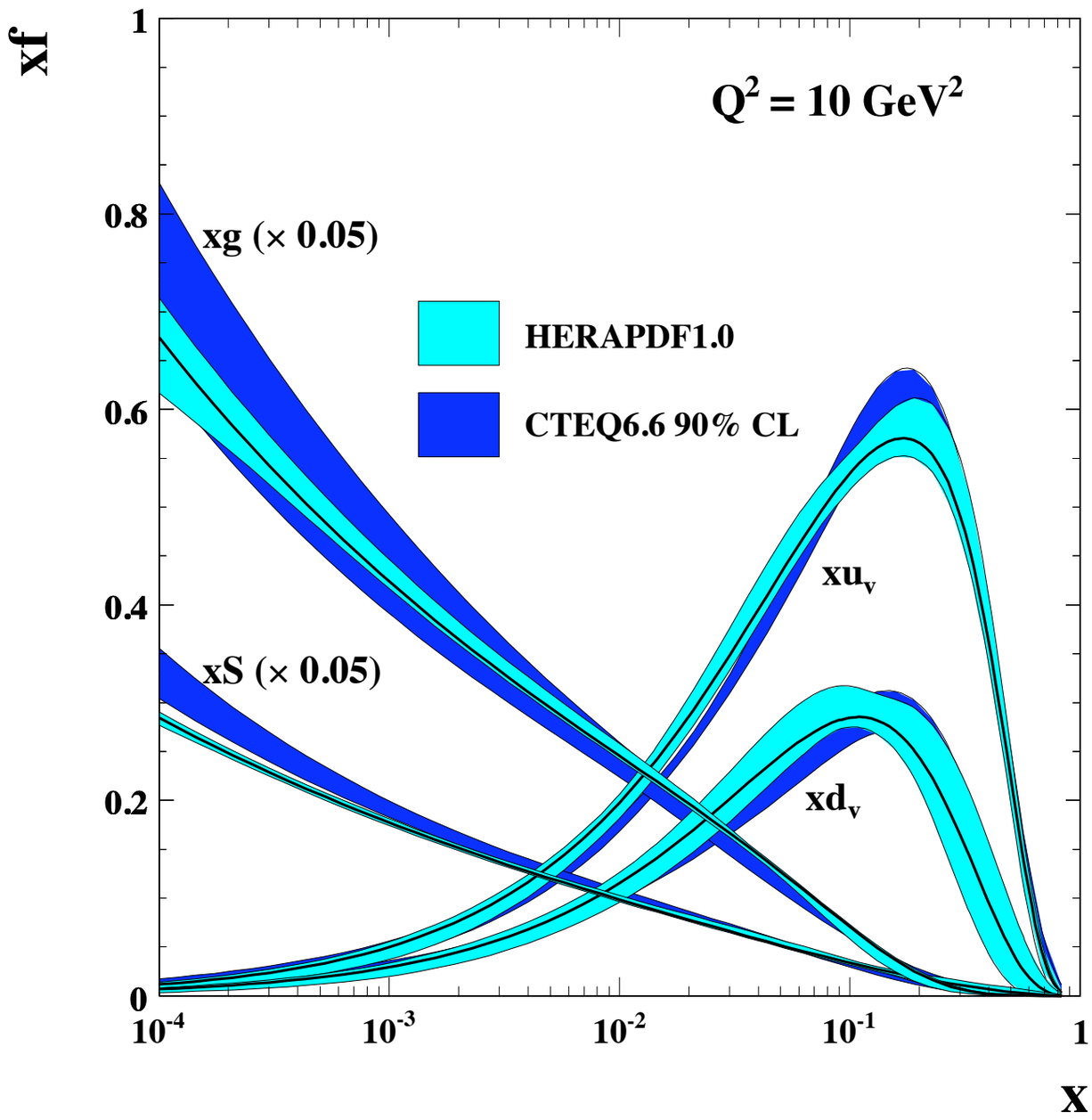
Comparison with Global Fits at Q^2=10 GeV^2

Impressive precision of gluon HERAPDF!

MSTW08



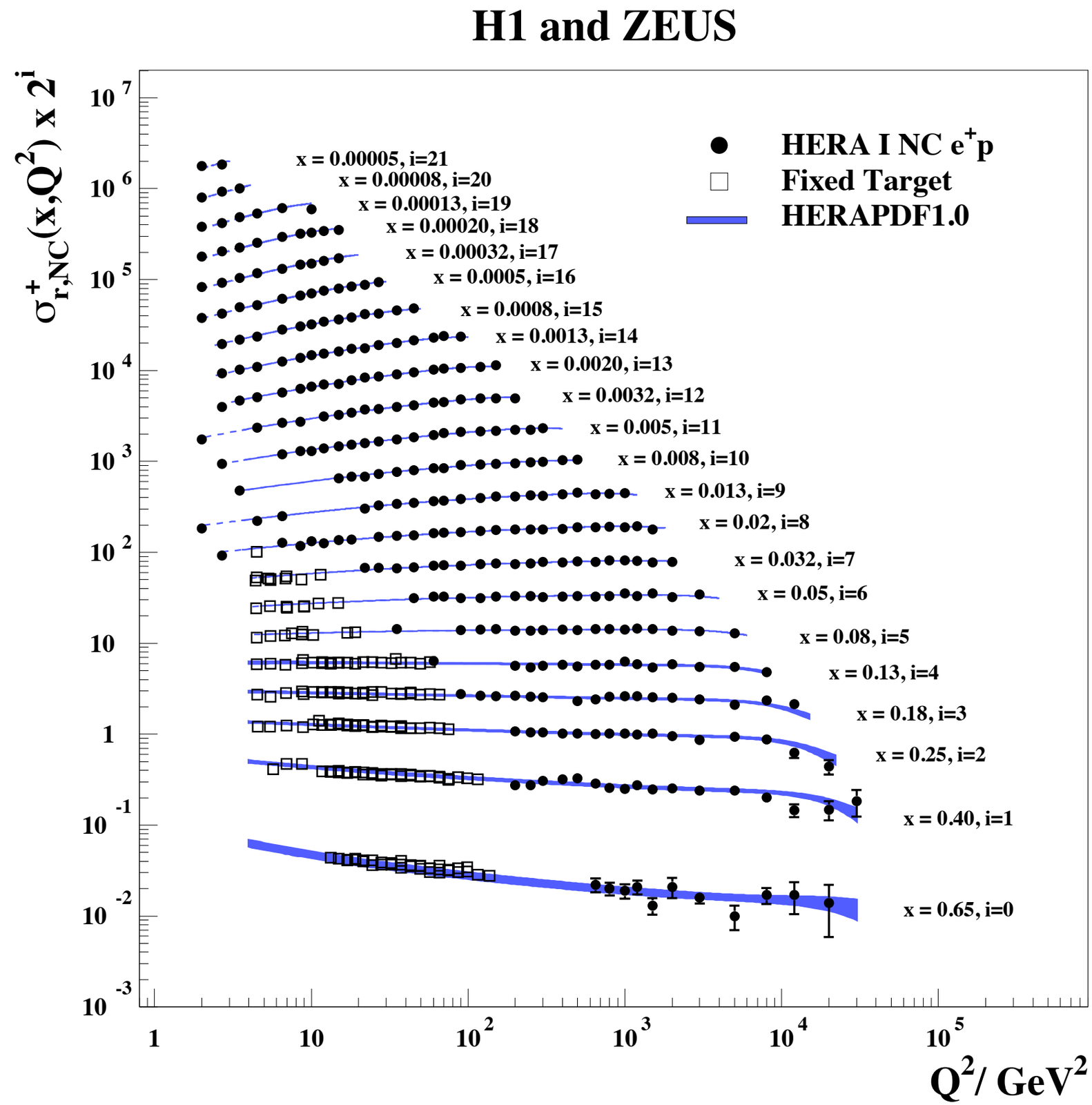
CTEQ6.6



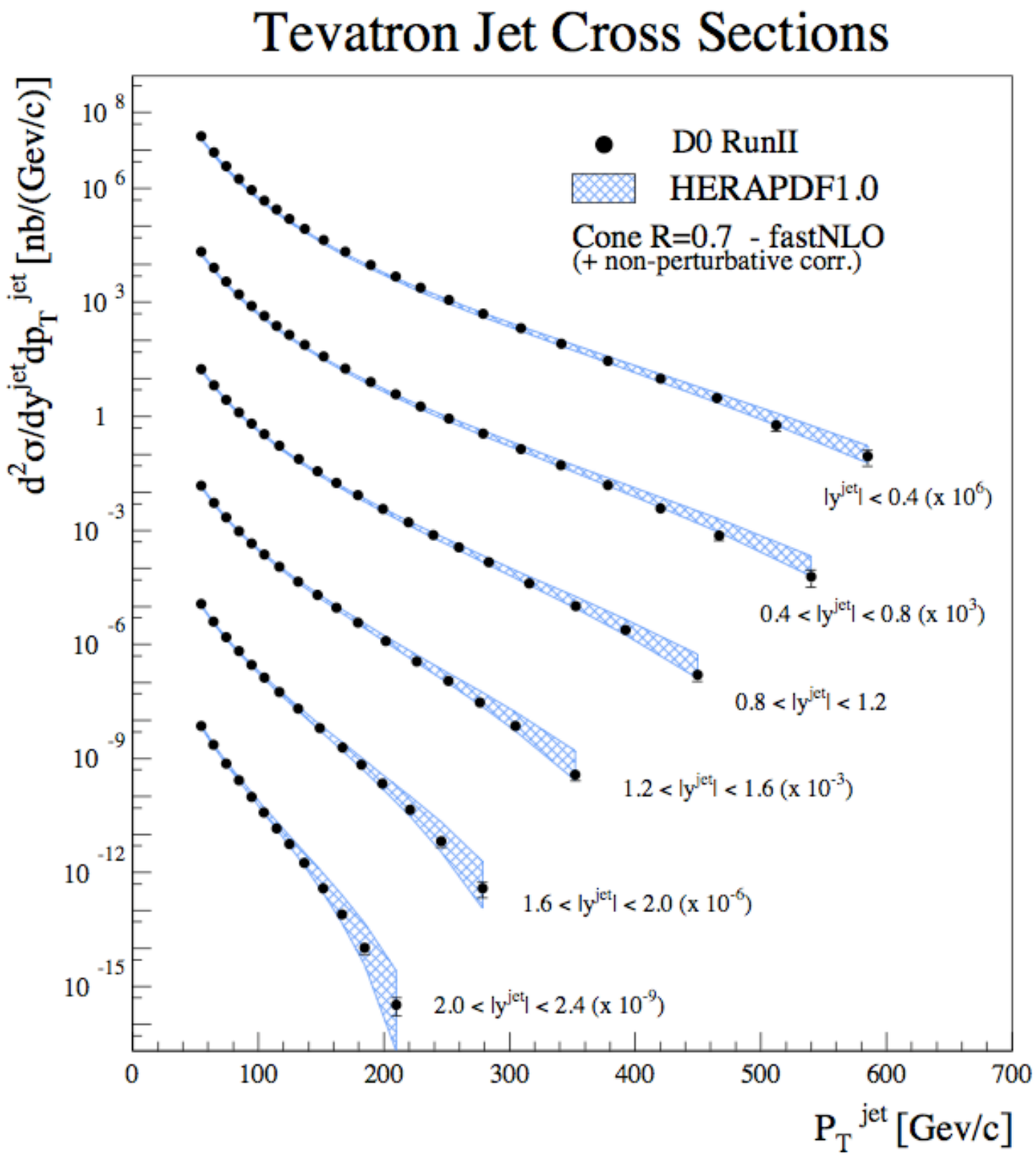
Predictions based on HERA PDFs

HERA PDFs vs fixed target data

- Plots show the extended kinematic range of the HERA data as compared to the fixed target measurements:
  - experimental errors included
  - fit line includes total error
- HERA precision similar to the fixed target experiments
- Extrapolation of the fit agrees well with fixed target data



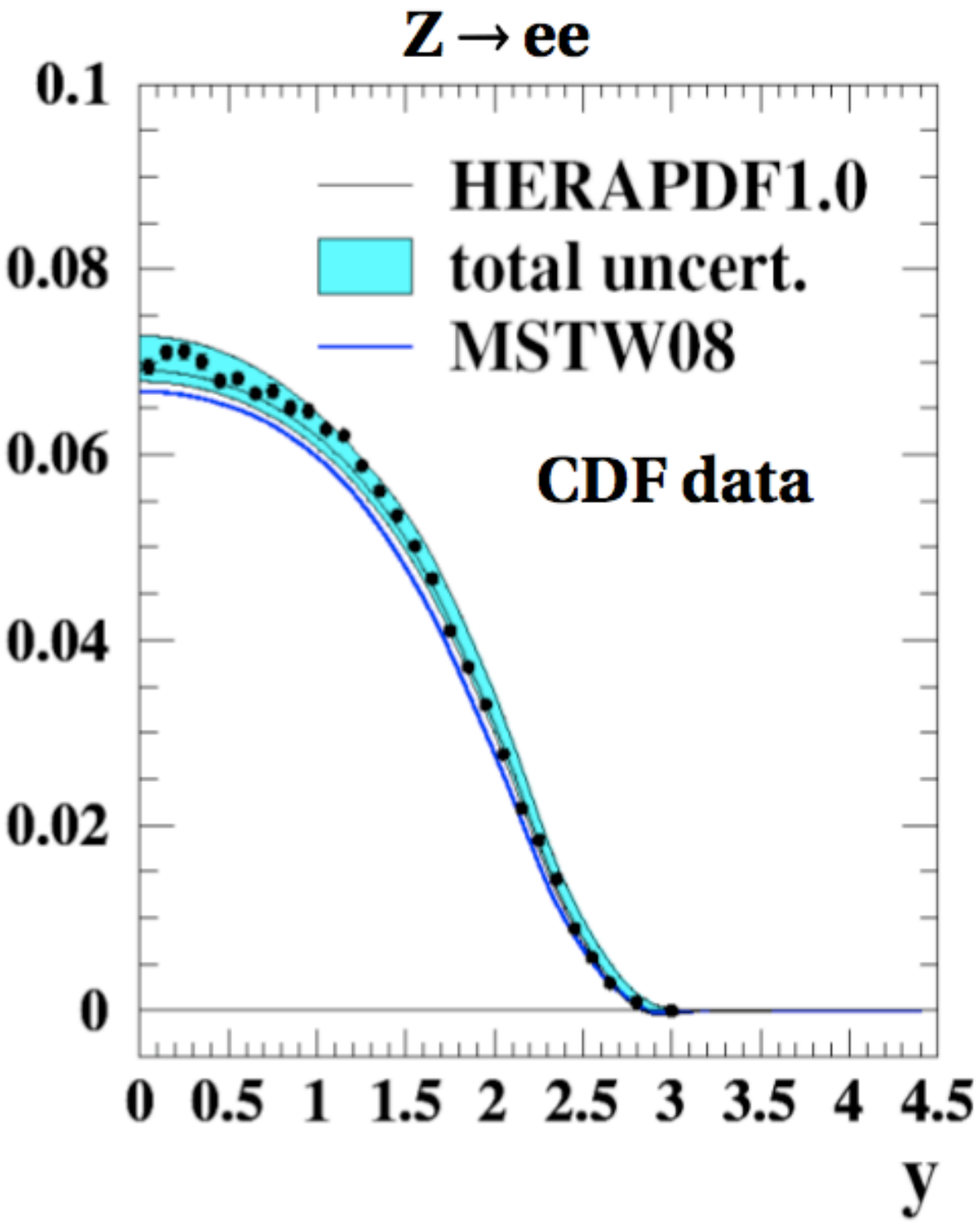
HERAPDF fits versus D0 jets

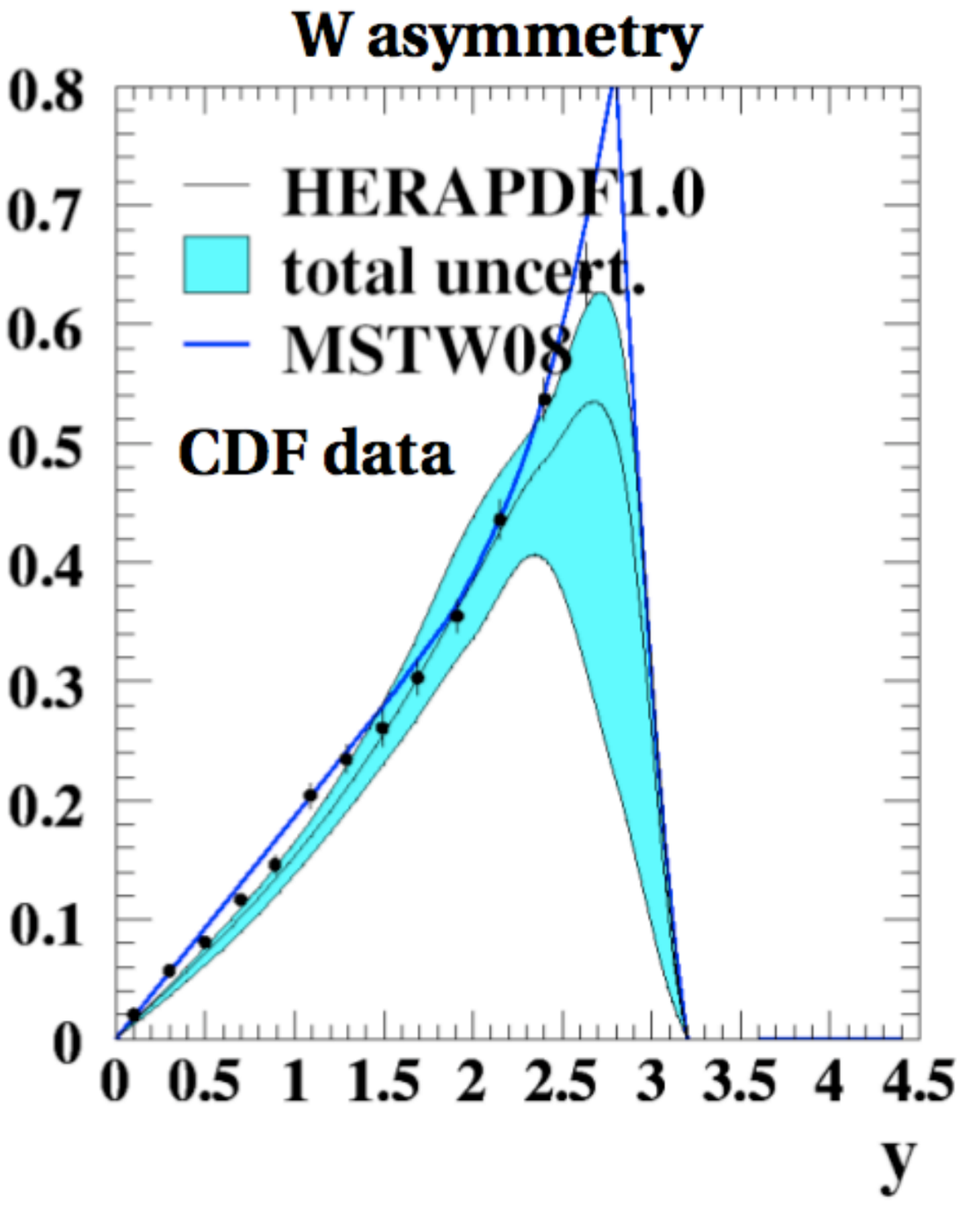


- Predictions for high-E_T jet cross-sections with full uncertainties compared to the D0 data
- DIS data predicts Tevatron jets production from ppbar process.

HERAPDF1.0 and Tevatron Z, W

Z and W at Tevatron are well predicted by HERAPDF1.0





some remarks

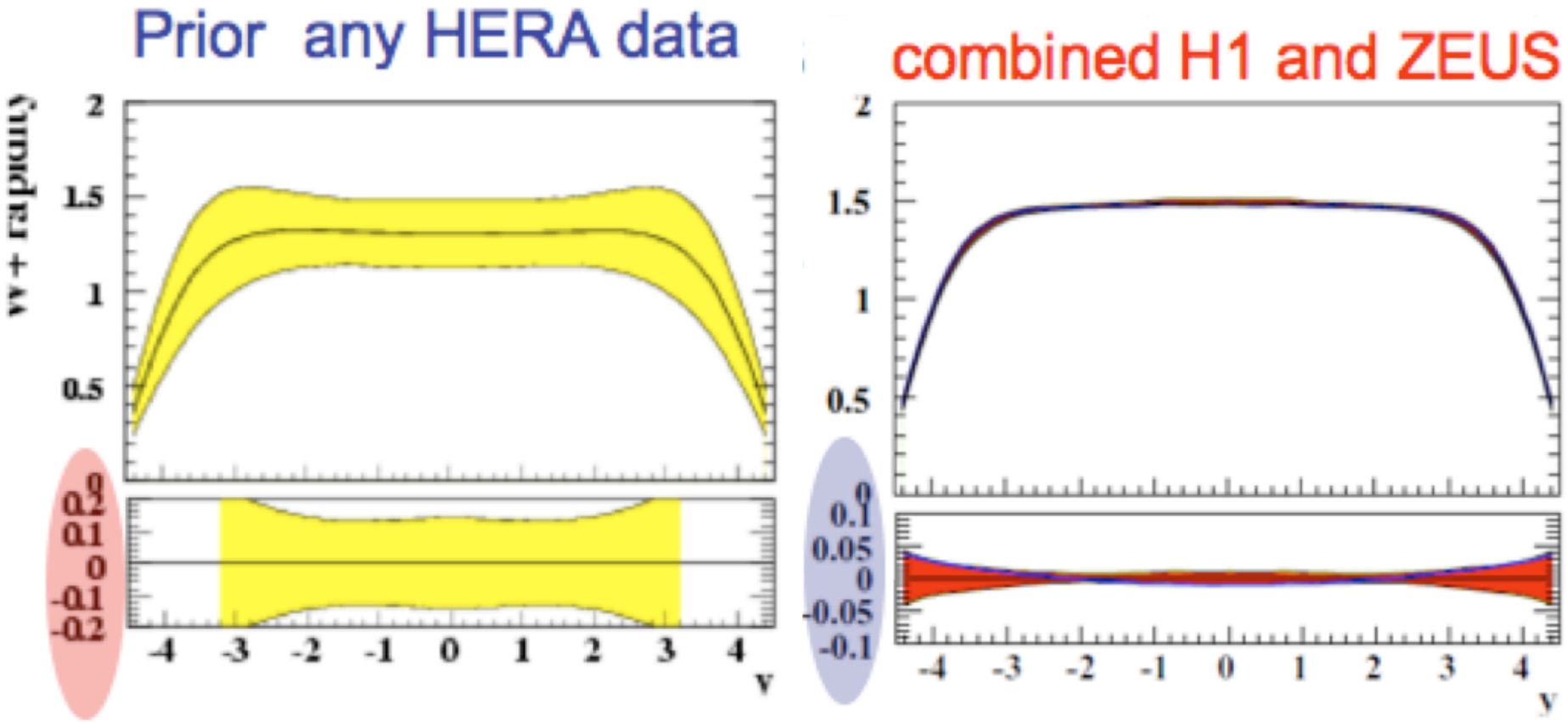
- Good agreement has been found between predictions using PDFs determined from DIS at HERA with measurements from the following processes:
  - Fixed Target (lower Q^2)
  - Drell Yan processes from D0
  - Zee from CDF
  - W+/- from CDF

- Hence, there is a universal description of partonic processes and all can be described with: HERA input, SM couplings and pQCD evolution!

- Still to do:
  - higher precision needed
  - low x deviations from DGLAP to be discovered
  - subtle fit questions: parametrisations, symmetry assumptions
  - heavy quarks: precision measurements
  - large x far from being accurately known ...

Impact of HERA on the LHC predictions

- Impressive precision of HERAPDF sea and gluon is relevant for W, Z production at the LHC:



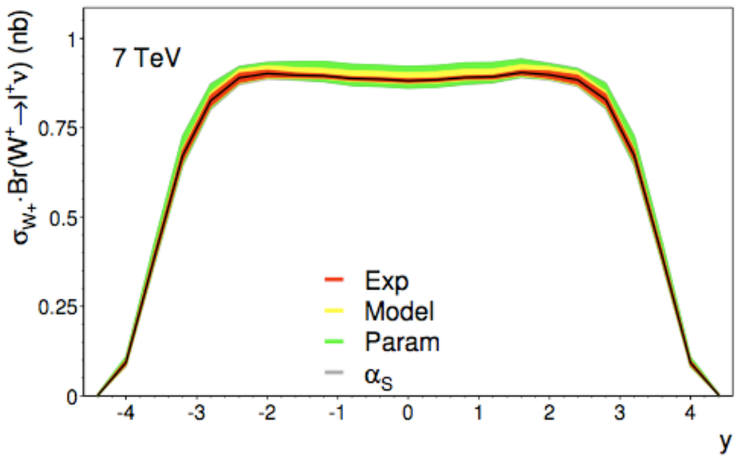
exp errors only

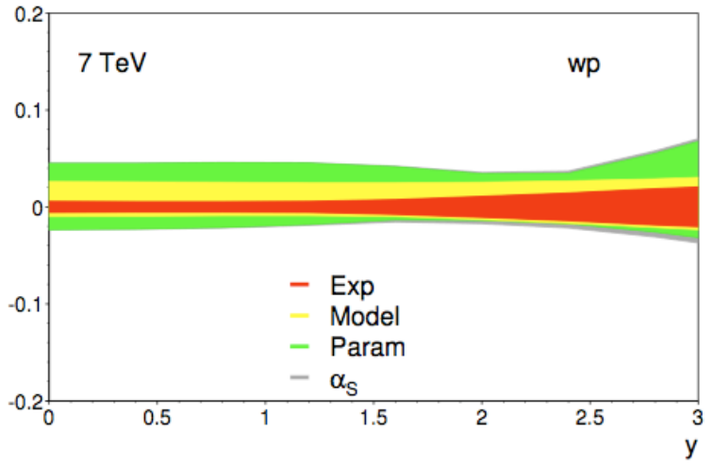
➡ this assumes that DGLAP worked which was confirmed at HERA

- Inclusion of HERA data shows tremendous improvement on the predictions for W and Z production at the central rapidity:
  - such an improvement is due to improvement in the low-x gluon and sea:
    - remember that at the LHC the W, Z bosons are made mostly of sea-sea partons at low x and the the scale of M_Z^2 the sea is driven by the gluon.

LHC predictions based on HERAPDF1.0

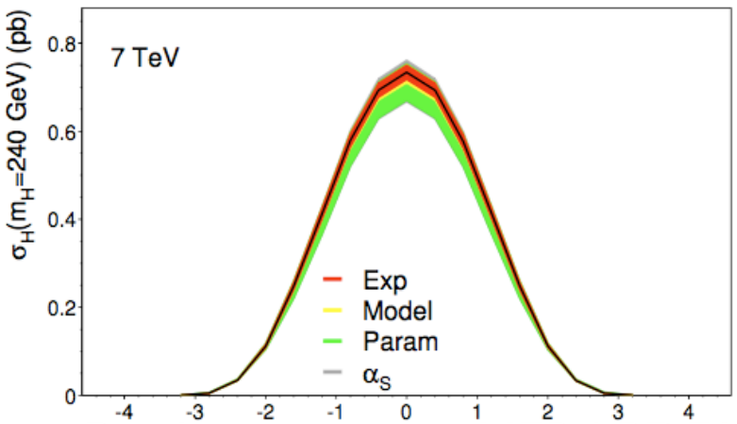
- Predictions using HERAPDF1.0 for W, Z cross sections .

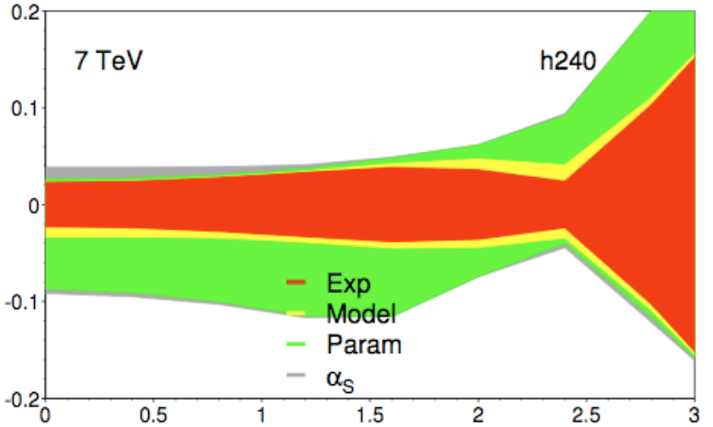




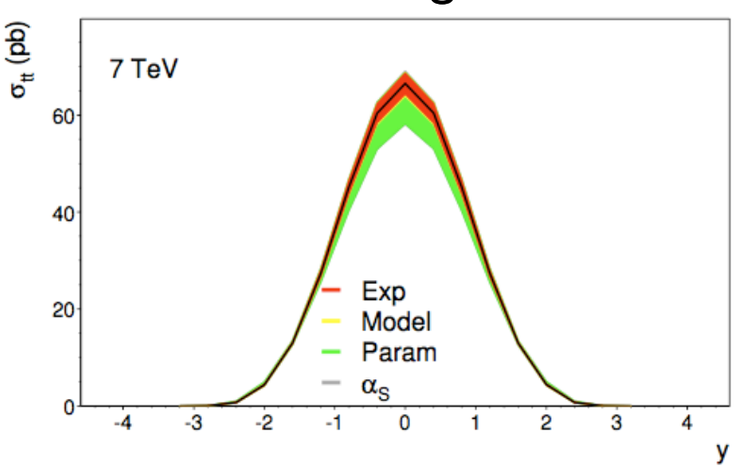
Separate experimental, model, parametrisation uncertainties and the uncertainty from the variation of alphas by ±0.002

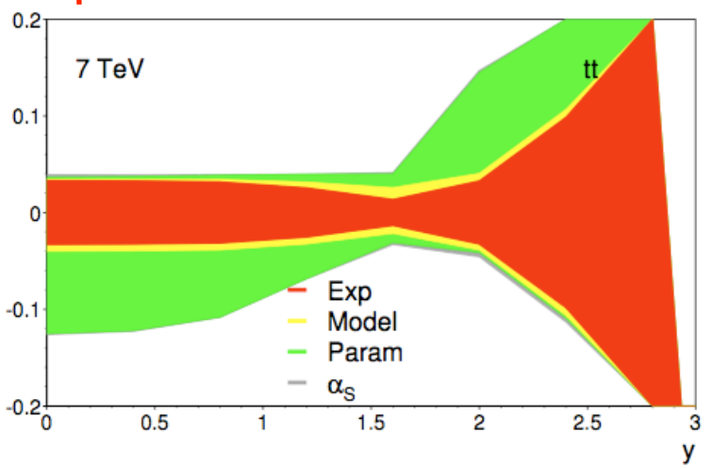
- Predictions using HERAPDF1.0 for Higgs cross-sections [Higgs Masses = 240 GeV, other scenarios in backup].





- Predictions using HERAPDF1.0 for top cross-sections.





Exciting new times ahead to actually compare the predictions to real measurements from the LHC!

Summary

- Tremendous progress since 40 years ago in understanding the proton structure
  - established QCD as the theory of strong interactions

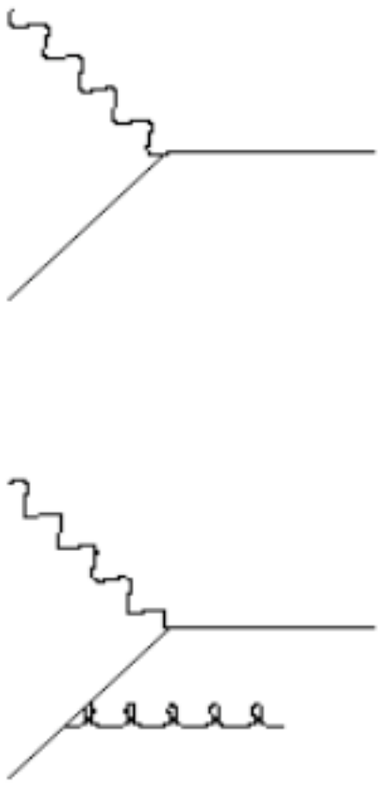
- LHC cross-sections and its discovery potential relies on the precision of proton structure:
  - HERA provides accurate determinations of the proton structure and can predict related Standard Model processes
  - New measurements from HERA II time period are becoming available and ready to be used in the QCD analyses

- Using HERA information we have precise predictions for the LHC and the time has come to confront them with the data!

EXTRA

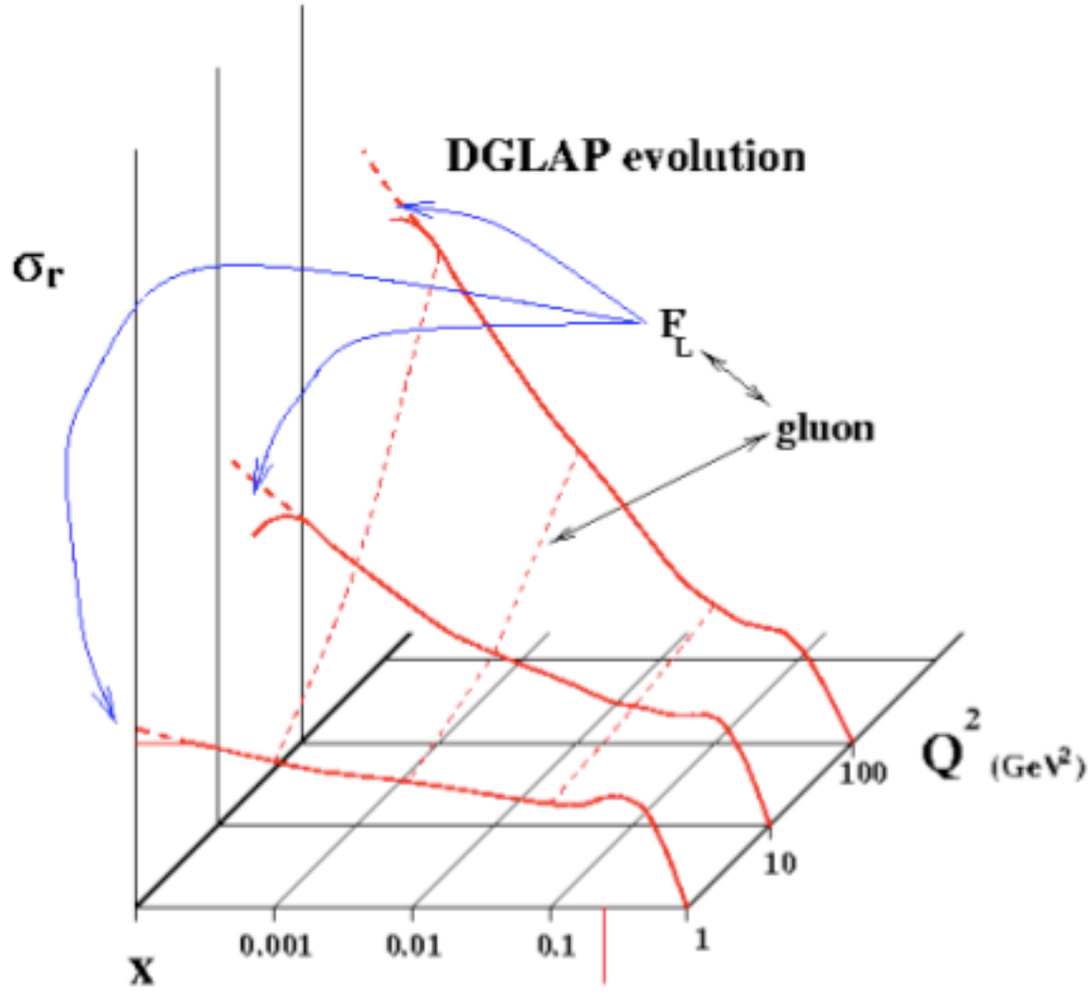
Latest Additions to HERA fits from HERA II

Longitudinal Structure Function F_L



LO: $k_T = 0$, $F_L = 0$

NLO: $k_T \neq 0$, $F_L \neq 0$

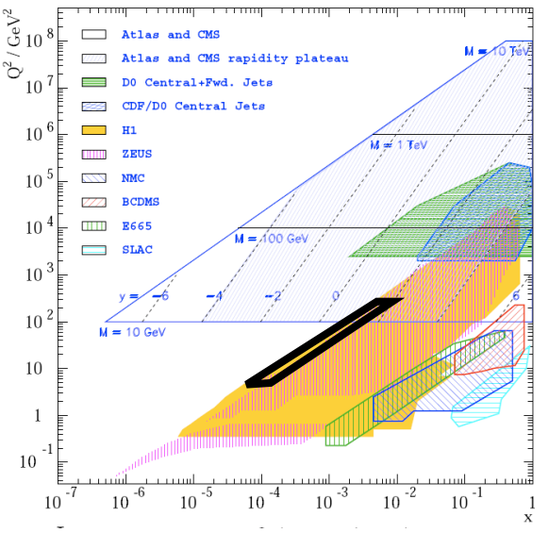


Measurement of the Structure Function F_L

- Direct Measurement of F_L requires measurement of the differential cross sections at same x and Q2 but different y
  - larger difference in y, better sensitivity to F_L
  - $Q^2=xys$, so that different y means different s(CME) means different beam energies

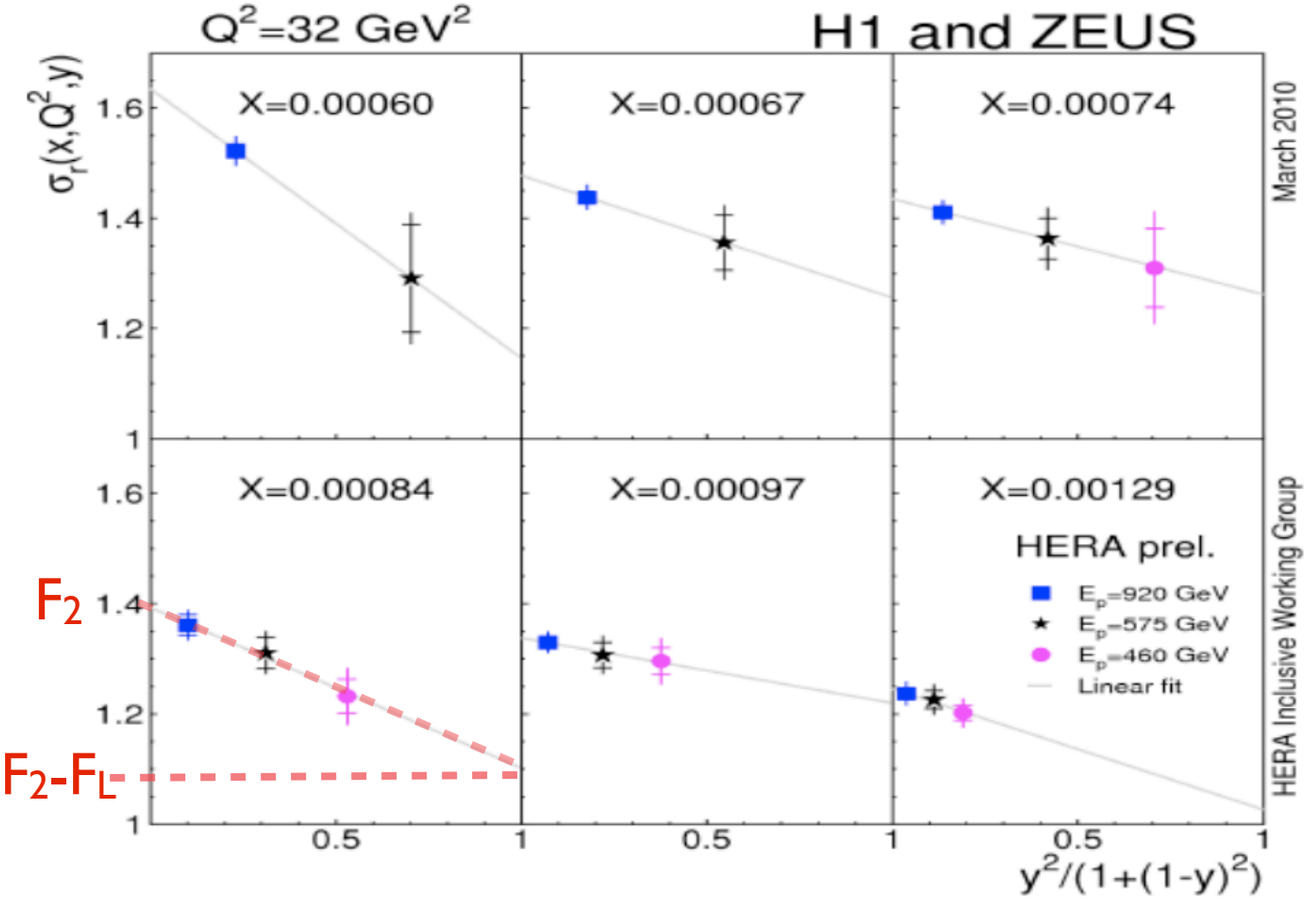
$$\frac{d^2\sigma_{NC}^{e^\pm p}}{dxdQ^2} = \frac{2\pi\alpha^2 Y_+}{xQ^4}\left[F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\right]$$

- Reduced proton beam energy runs at the end of HERA operation were dedicated to measure F_L.



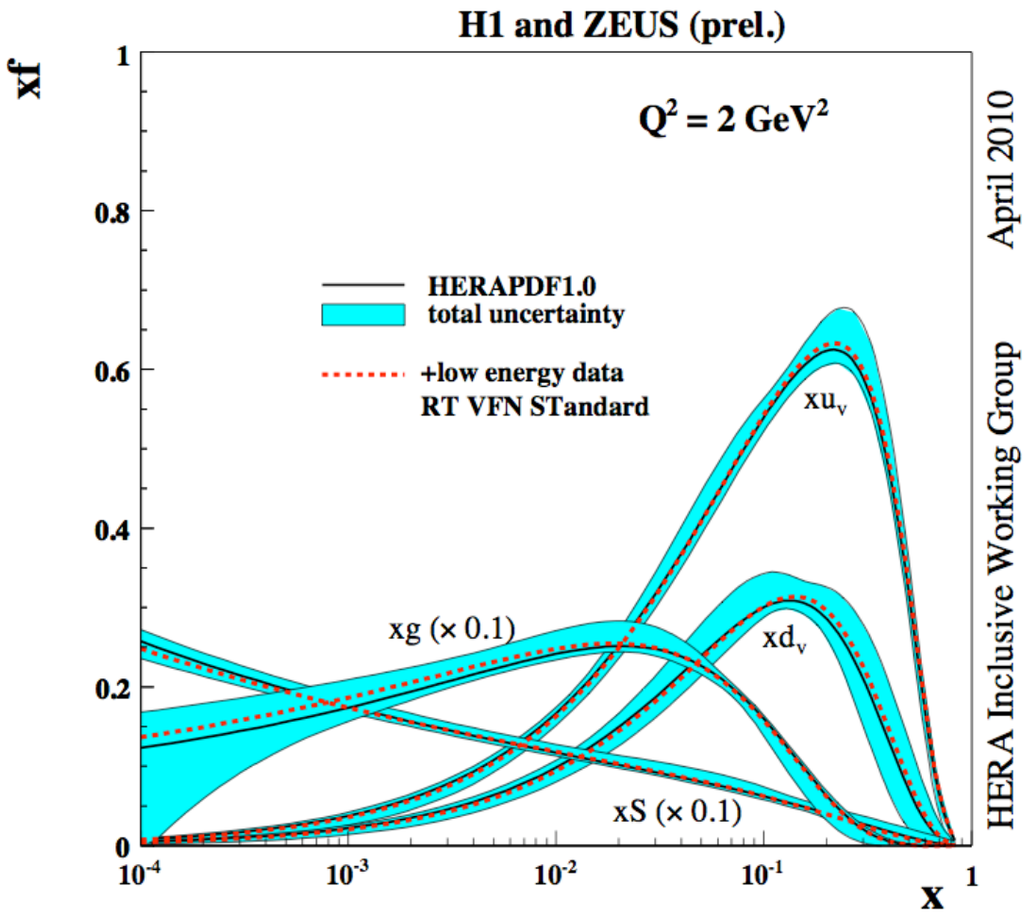
H1: $2.5<Q^2<800$ GeV^2
ZEUS: $24<Q^2<110$ GeV^2
with Ep=920, 460 575 GeV

At given x and Q^2:
→ F_2 is the intercept at y-axis
→ F_L is the negative slope

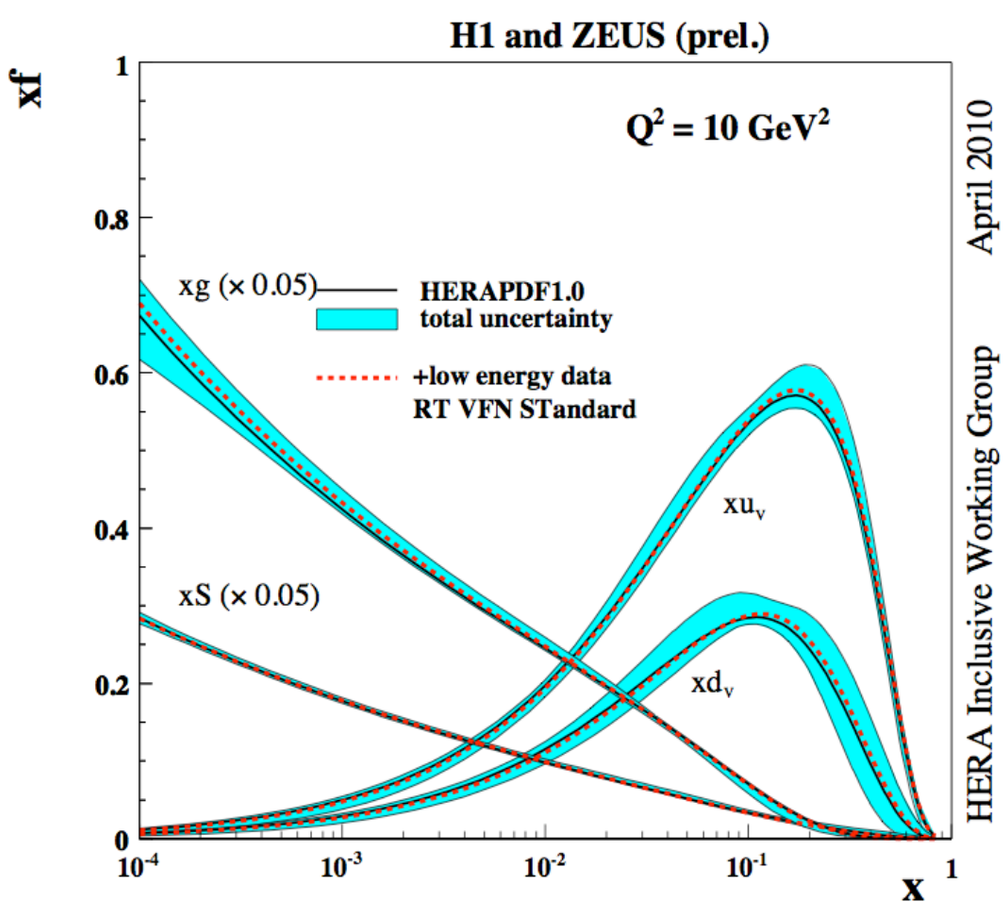


HERAPDF including Low Energy Data

- Include low energy data
  - Study impact of those data on PDFs and investigate the low Q^2 region;
  - Test sensitivity to different heavy flavour treatments;
  - Compare fit results and measured structure function F_L.

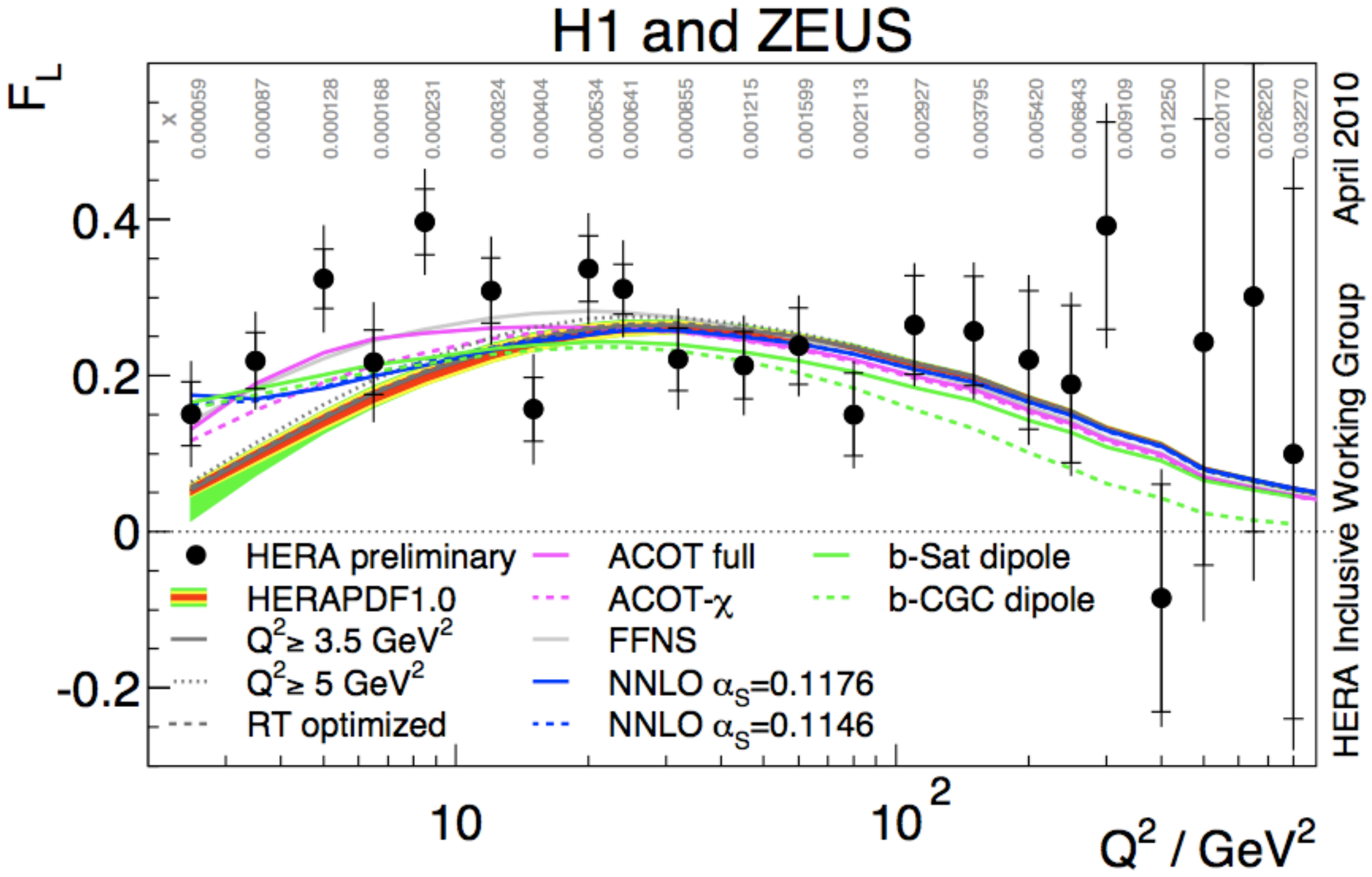


Q^2 evolution



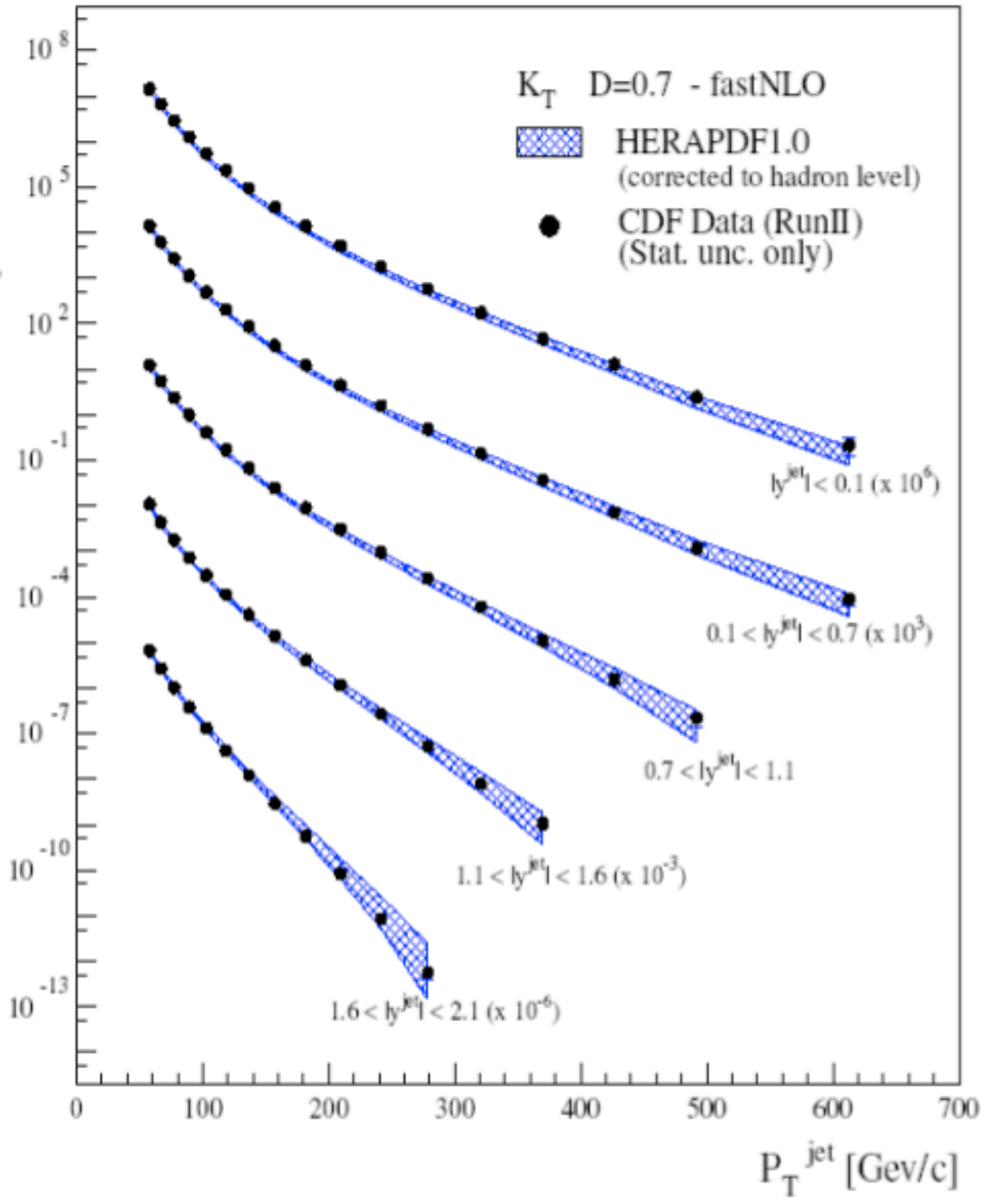
HERA F_L data vs F_L prediction

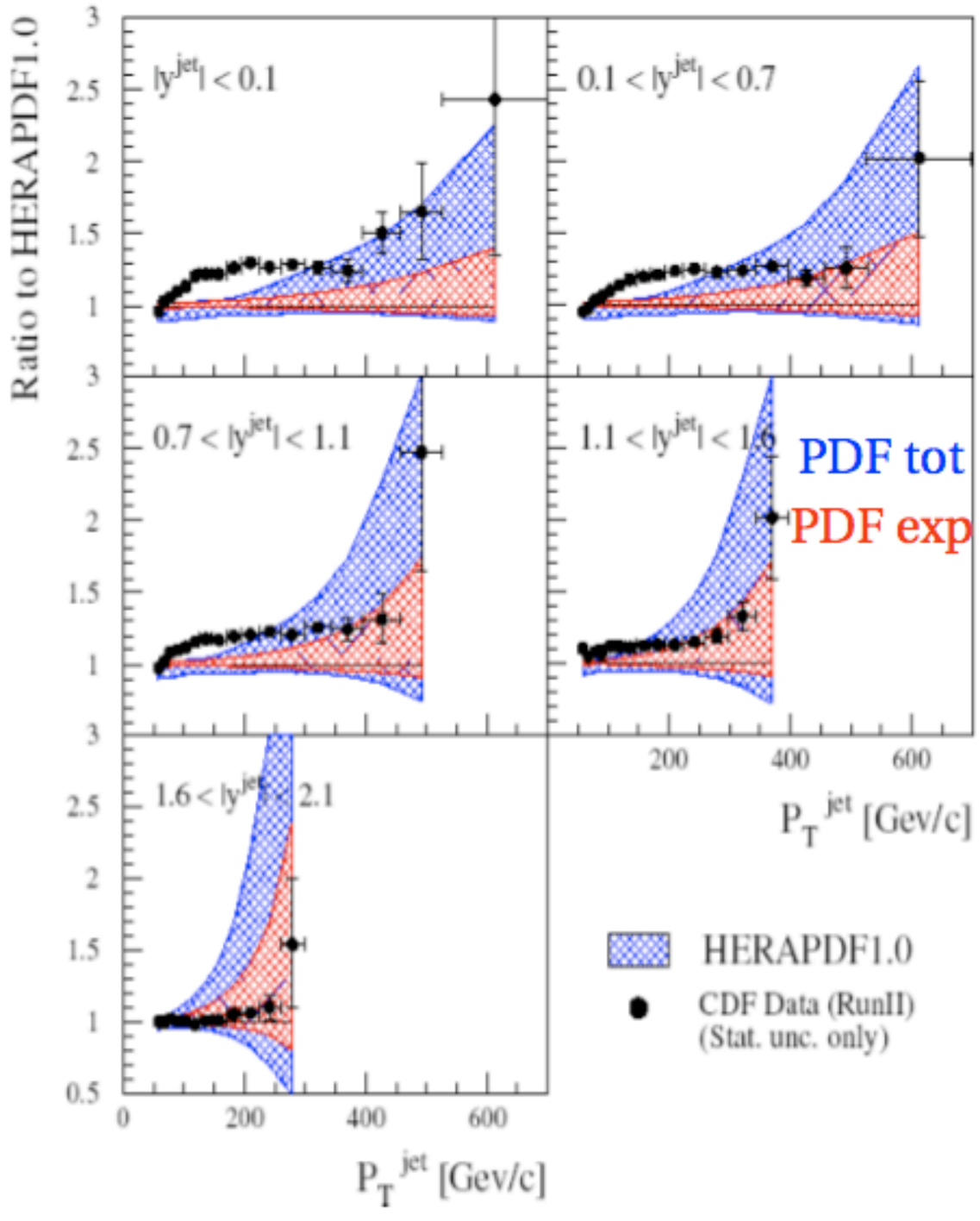
- The lines are F_L predictions based on combined HERA I and low energy data



BACK UP

HERAPDF fits .vs. CDF jets





Experimental Uncertainties: Monte Carlo Technique

- Method consists in preparing replicas of data sets allowing the central values of the cross sections to fluctuate within their systematic and statistical uncertainties taking into account all point to point correlations [A.Glazov and VR, HERA-LHC proceedings, arXiV:0901.2504, page 41-42]

  - Shift central values randomly within the uncorrelated errors assuming Gauss distribution of the errors:

$$\sigma_i = \sigma_i(1 + \delta_i^{uncorr} RAND_i)$$

  - Shift central values with the same probability of the corresponding correlated systematic shift assuming Gauss distribution of the errors:

$$\sigma_i = \sigma_i(1 + \delta_i^{uncorr} RAND_i + \sum_j^{N_{sys}} \delta_{ij}^{corr} RAND_j)$$

- Preparation of the data is repeated for N times (N>100)
  - For each MC replica, NLO QCD fit is performed to extract the N PDF sets

- Errors on the PDFs are estimated from the RMS of the spread of the N curves corresponding to the N individual extracted PDFs

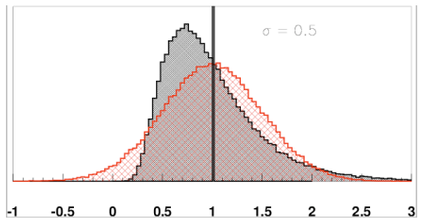
Results of the MC method

- Standard error estimation of PDFs relies on the assumption that all errors follow Gauss statistics

- MC method can provide an independent cross check of it

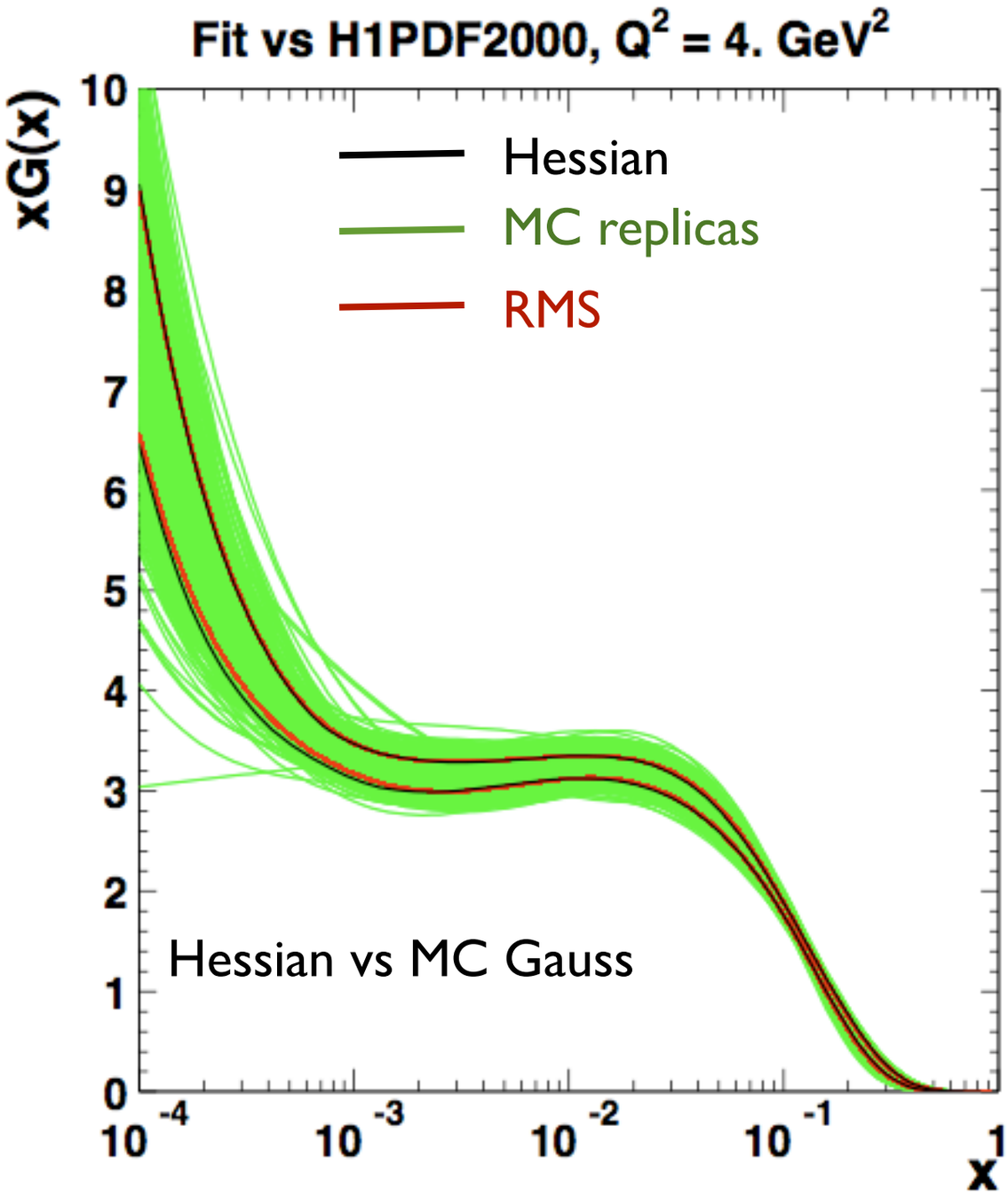
  - Hessian Method and MC method give the same results in the linear error propagation approximation

- MC method allows to test various assumptions for error distributions
  - some systematic uncertainties follow Log-Normal distribution (i.e., lumi, detector acceptance, ...)



  - does this affect the PDF uncertainty?

    - Similar results to Gauss distributions when using Log-Normal assumptions



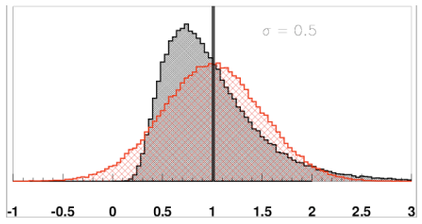
Results of the MC method

- Standard error estimation of PDFs relies on the assumption that all errors follow Gauss statistics

- MC method can provide an independent cross check of it

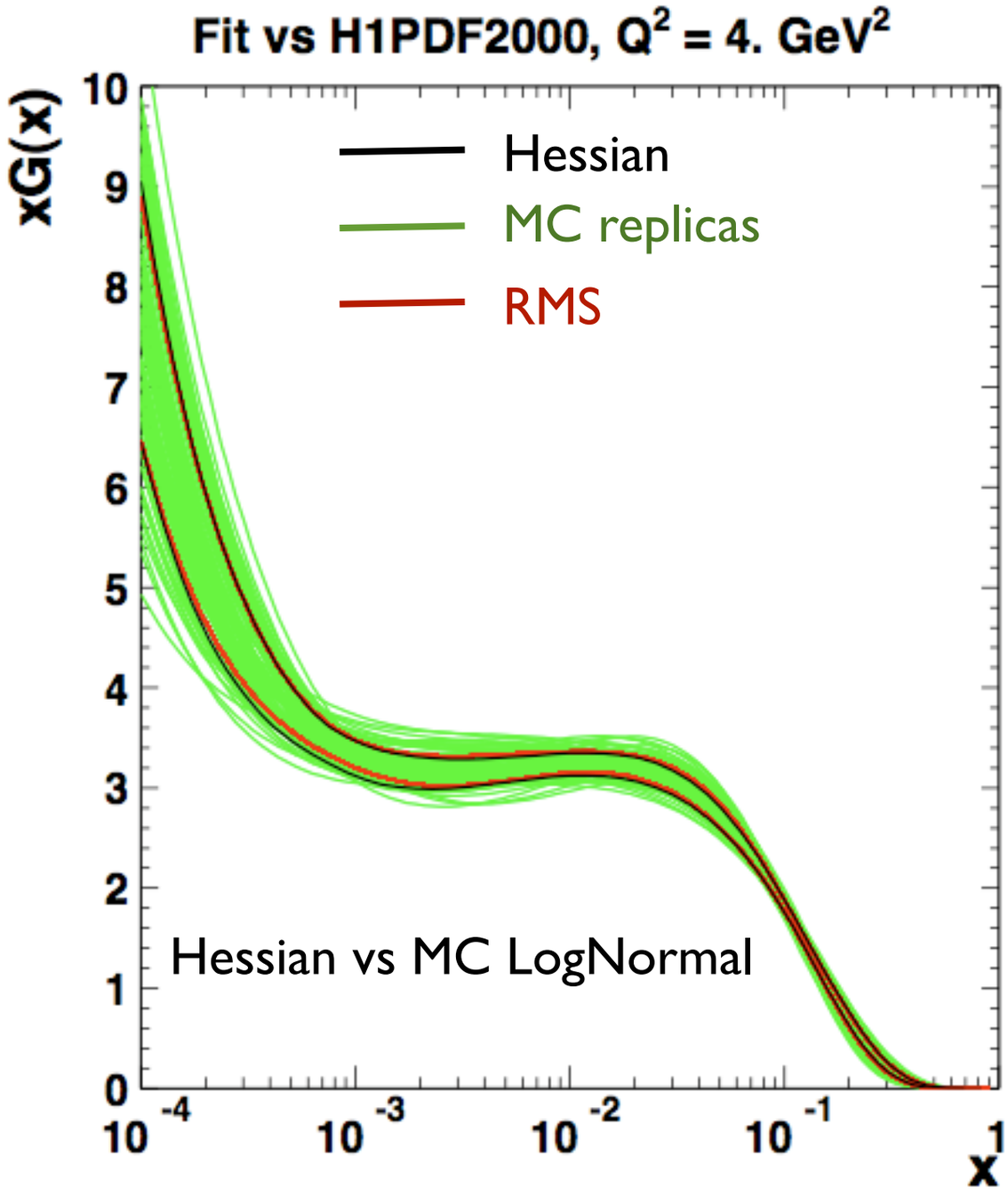
  - Hessian Method and MC method give the same results in the linear error propagation approximation

- MC method allows to test various assumptions for error distributions
  - some systematic uncertainties follow Log-Normal distribution (i.e., lumi, detector acceptance, ...)



  - does this affect the PDF uncertainty?

    - Similar results to Gauss distributions when using Log-Normal assumptions



Heavy Flavour Schemes

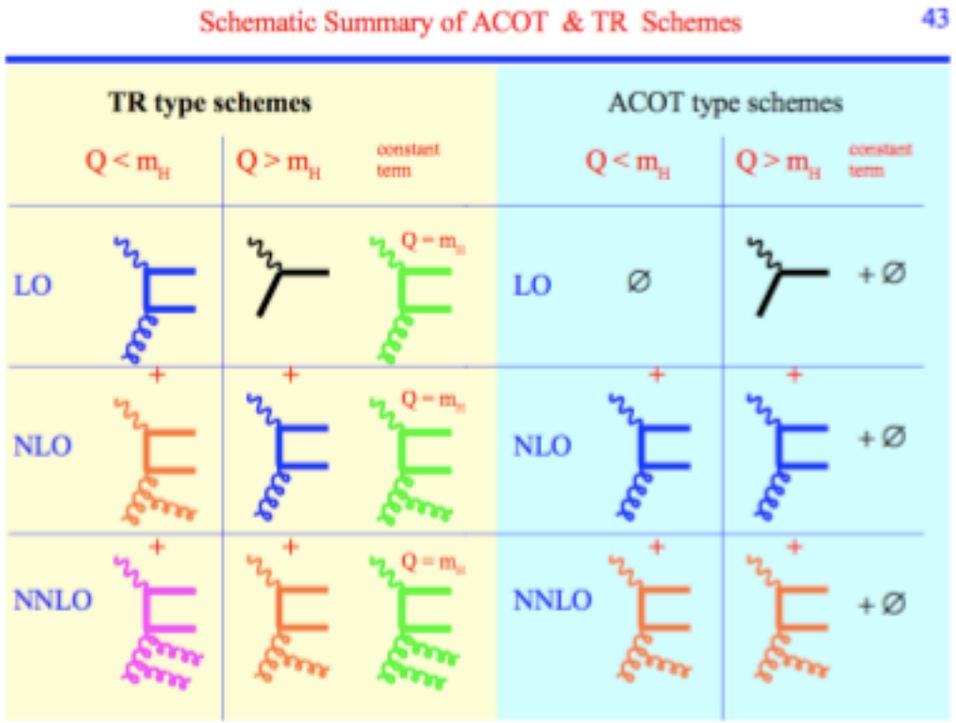
- Perturbative calculations are performed in context of the factorisation theorem:
  - extended to the case of heavy quarks [Collins 1998]
- Heavy Quarks introduce an additional scale: Q^2, M_H

- 3 regions of interests are observed:
  - $Q^2 << M_H^2 \longrightarrow$ decoupling
  - $Q^2 \sim M_H^2 \longrightarrow$ transition region
  - $Q^2 >> M_H^2 \longrightarrow$ heavy quarks treated as light quarks

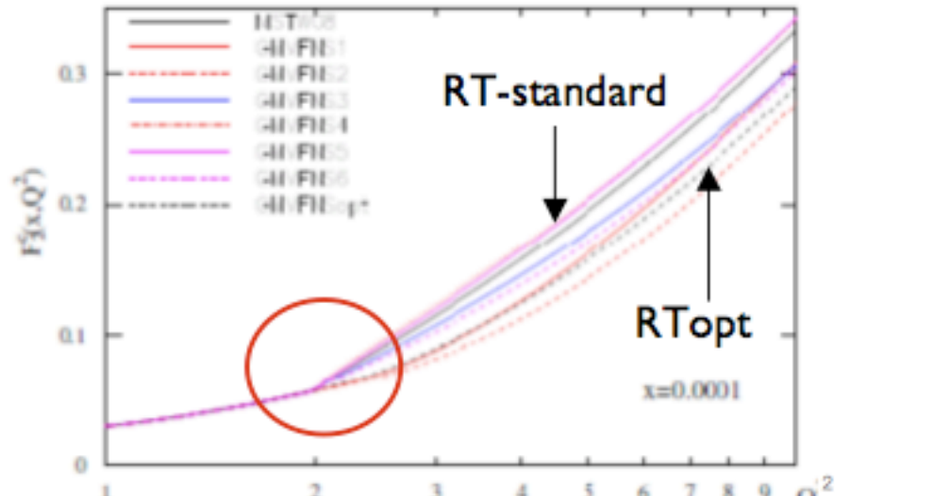
- Mass-retaining factorisation schemes:
  - equivalent if calculations done to all orders in strong coupling!
  - Fixed Flavour Number Scheme (FFNS)
    - it sums over fixed number of active parton flavours
    - heavy quarks are created from processes initiated by light quarks and gluons
    - Returns an efficient organisation of the perturbative series at $M_H^2 \sim Q^2$
      - at $Q^2 >> M_H^2$ large logarithms spoil the perturbative series
  - Variable Flavour Number Scheme (VFNS)
    - introduces a PDF for heavy quarks and the number of active flavours increases by one unit when a heavy quark threshold is crossed
    - ZM-VFNS: heavy quarks omitted entirely below M_H^2 and included as massless partons above thresholds
    - GM-VFNS: it's basically a series of FFNSs matched at matching scale of order M_H^2 for each of the heavy quark thresholds:
      - Thorne-Roberts (TR VFNS)
      - Aivaziz-Collins-Olness-Tung (ACOT scheme)

Hevay Flavour

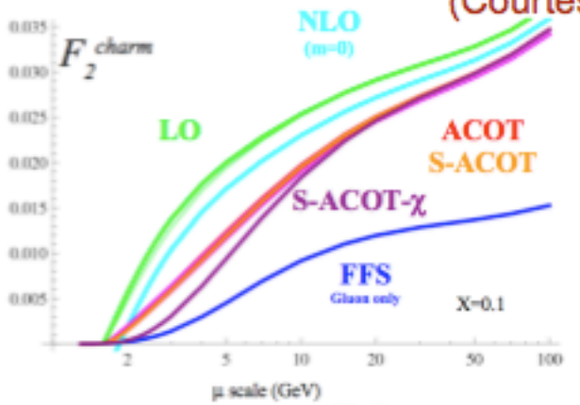
- Compare various schemes taking into account heavy quark production:
  - VFNS RT (standard [MSTW08] and optimal [R. Thorne's presentation])
  - VFNS ACOT (full [Phys.Rev.D50,1994] and χ [Phys.Rev.D62,2000])



(Courtesy of F. Olness)



(Courtesy of R. Thorne)



  - FFNS (from QCDNUM17v06 [M. Botje])
- We observe significant differences among these schemes → next slides.

Sources of Experimental Uncertainties

- Uncorrelated uncertainties:
  - statistical errors
  - point-to-point uncorrelated uncertainties added in quadrature to the statistical errors
    - e.g., statistical errors due to MC simulations

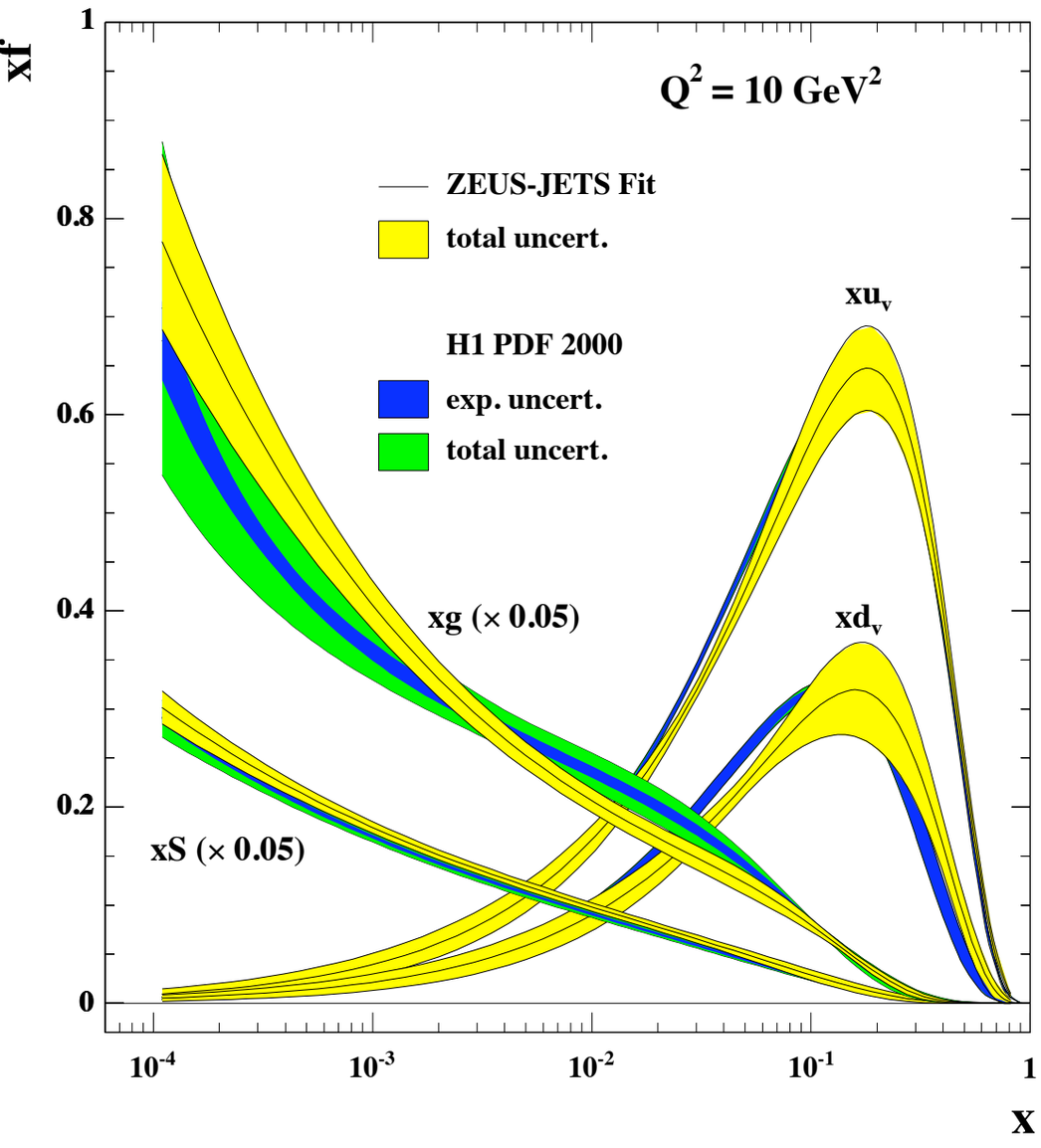
- Correlated uncertainties:
  - point-to-point correlated uncertainties
    - e.g., electromagnetic and hadronic energy scale calibration

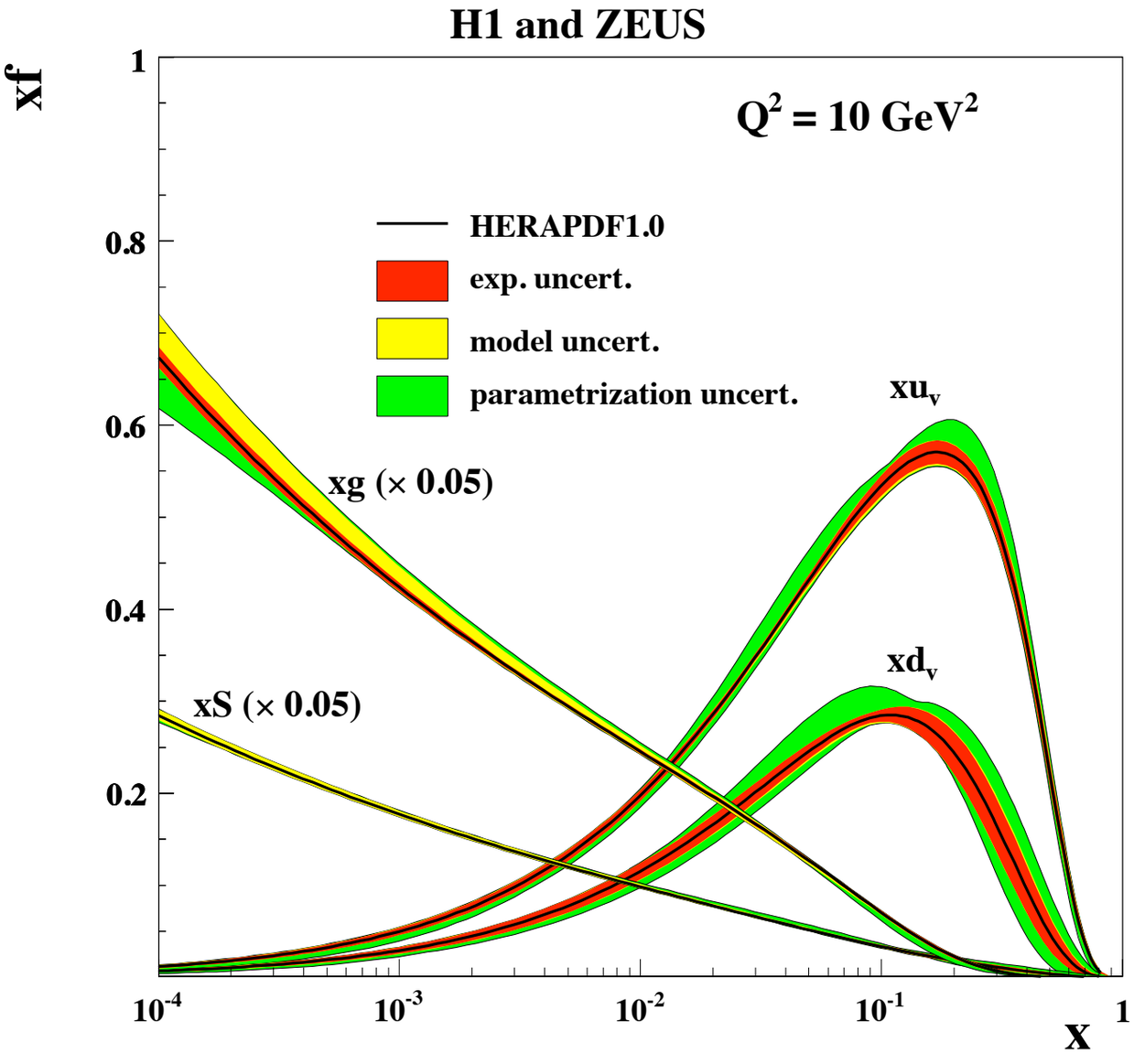
- Common H1 and ZEUS sources of errors:
  - H1 and ZEUS use similar analyses methods
    - background photo-production MC (typically below 0.5%, but higher at high y)
    - hadronic energy scales (typically below 0.5%, but significant only at low y)

- There are 110 systematic errors and 3 sources from the combination procedure

Comparison with old H1 and ZEUS fits separately

- Careful! H12000 and ZEUS-jets have not considered PDF param. uncertainty
- The variation of strong coupling is included in H12000 PDFs
- Combined HERA data includes a more precise low Q^2 data





Extra on PDF parametrisation

- Central Values of the HERA PDF Parameters:

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}, \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}
$$

	A	B	C	E
xg	6.8	0.22	9.0	
xu_v	3.7	0.67	4.7	9.7
xd_v	2.2	0.67	4.3	
$x\bar{U}$	0.113	–0.165	2.6	
$x\bar{D}$	0.163	–0.165	2.4	

- Forms tried for evaluation of PDF parametrisation uncertainty:

$$xf(x) = Ax^B(1-x)^C(1+\epsilon\sqrt{x}+Dx+Ex^2)$$

- For low x: subtract $A'_g x^{B'_g}(1-x)^{C'_g}$ with C'g fixed to 25, A'g and B'g left to vary

DIS Cross Sections and Structure Functions

- General double differential cross-section for eN scattering:

$$\sigma_r(x,Q^2) = \frac{d^2\sigma(e^{\pm}p)}{dxdQ^2}\frac{Q^4x}{2\pi\alpha^2 Y_+} = F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2) \mp \frac{Y_-}{Y_+}xF_3(x,Q^2)$$

$$Y_{\pm} = 1 \pm (1-y^2)$$

- F_2, F_L, xF_3 are structure functions (hadronic part) which are related to the momentum distributions of quarks within the nucleon: Parton Distribution Functions (PDFs):
  - probability density that interacting parton carries a fraction x of the hadrons longitudinal momentum
    - valence quarks: carriers of charge
    - sea quarks and gluons: evolved by complex dynamics

At Leading Order (LO):

$$F_2 = x\sum e_q^2\left(q(x)+\bar{q}(x)\right)$$
$$xF_3 = x\sum 2e_q a_q\left(q(x)-\bar{q}(x)\right)$$

- F_2 dominates
  - sensitive to all quarks
- xF_3
  - sensitive to valence quarks
- F_L
  - sensitive to gluons

- So, we can extract the structure functions experimentally by looking at the x,y,Q^2 dependence of the double differential cross-section.

Combination of Data Method

- Move 820 GeV data to 920 GeV p-beam energy
- Swim all points to a common x-Q^2 grid
- Calculate average values and uncertainties

- The combination of data uses the chisquare minimisation method

Additive error sources: $\chi^2_{\exp}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_j \Gamma^i_j b_j - \mu^i\right]^2}{\Delta_i^2} + \sum_j b_j^2.$

Multiplicative error sources:

small biases to lower cross sections values avoided by a modified ^{2} definotion

$$\chi^2_{\exp}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i b_j - \mu^i\right]^2}{\delta^2_{i,\mathrm{stat}}\left(m^i - \sum_j \gamma^i_j m^i b_j\right) + \left(\delta_{i,\mathrm{uncor}}\, m^i\right)^2} + \sum_j b_j^2.$$

Measured central values μ_i

Relative correlated systematic uncertainties $\gamma^i_j = \Gamma^i_j/\mu^i$

Relative statistical uncertainties $\delta_{i,\mathrm{stat}} = \Delta_{i,\mathrm{stat}}/\mu^i$

Relative uncorrelated systematic uncertainties $\delta_{i,\mathrm{uncor}} = \Delta_{i,\mathrm{uncor}}/\mu^i$

HERA QCD Analysis Settings

- DATA: Combined HERA I NC and CC e^+p and e^-p Data is a sole input in the fit:
  - Q^2_{min}>3.5 GeV^2

- THEORY:
  - PDFs are parametrised at a starting scale of Q_0^2=1.9 GeV^2<Mc^2
  - PDFs are evolved using NLO DGLAP equations in MS scheme
    - strong coupling α_S=0.1176 [PDG 2008]
  - Structure Functions are calculated using an improved theoretical treatment which takes into account quark masses (GM-VFNS)
  - Choices for heavy quark masses:
    - Mc=1.4 GeV and Mb=4.75 GeV

Scheme	TRVFNS
Evolution	QCDNUM17.02
Order	NLO
Q_0^2	1.9 GeV^2
$f_s = s/D$	0.31
Renorm. scale	Q^2
Factor. scale	Q^2
Q^2_{min}	3.5 GeV^2
$\alpha_S(M_Z)$	0.1176
M_c	1.4 GeV
M_b	4.75 GeV

Experimental Errors: Hessian Method

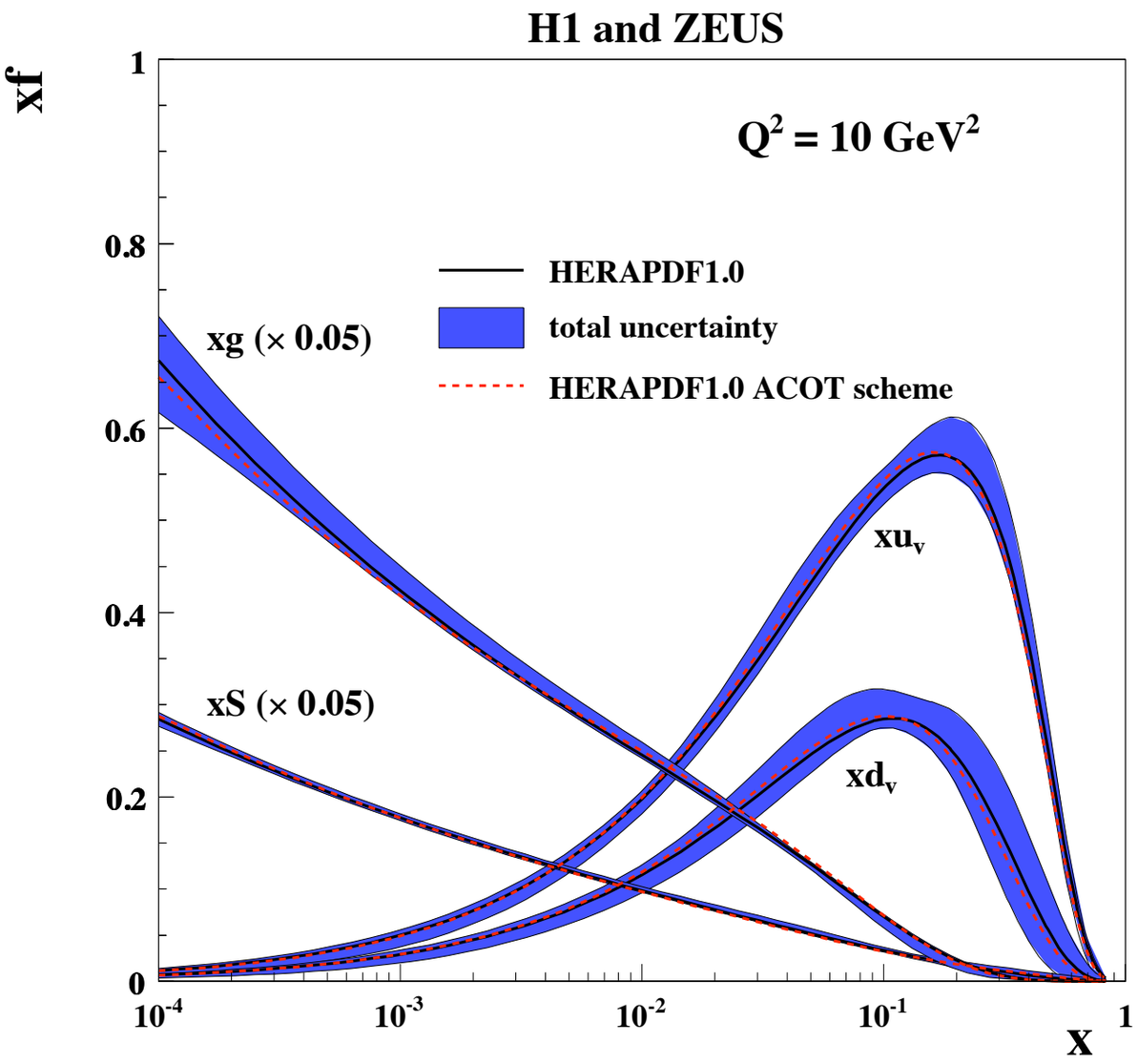
- The experimental errors are propagated to PDFs by Hessian method to calculate χ^2
  - It allows the systematic uncertainty to vary in the fit so that the model prediction is not fitted to the central values of the data, but it allows for the points to move collectively according to their correlated systematic error:

$$\chi^2 = \sum_i \frac{\left(\left(F_i^{NLO} + \sum_\lambda s_\lambda \Delta_{i\lambda}^{sys}\right) - F_i^{meas}\right)^2}{\sigma_i^2} + \sum_\lambda s_\lambda^2$$

- For HERA PDF fit:
  - errors extracted using the tolerance criteria: $\Delta\chi^2 = 1$
  - the experimental 110 systematic errors are combined in quadrature with the statistical errors and 3 sources of errors from the combination procedure are offset
    - Small effects are observed when errors are treated fully correlated vs uncorrelated

Model/Theory Uncertainties

- Model/Theory uncertainties are evaluated by varying the input assumptions:
  - Variation of the minimum Q^2 cut on data
    - Q^2_{min} = 3.5 GeV^2 → 2.5 - 5.0 GeV^2
  - Variation of the heavy quark thresholds:
    - Mc = 1.40 GeV → 1.35 - 1.65 GeV
      - low bound varied with Q_0^2 = 1.8 GeV^2
    - Mb = 4.75 GeV → 4.30 - 5.00 GeV
  - Variation of the sea fractions:
    - fs = s/$\overline{D}$ = 0.31 → 0.23 - 0.38



- A cross check of the sensitivity to the heavy flavour treatment is performed by comparing the TR-VFNS with the ACOT Scheme

PDF Parametrisation Uncertainty: Standard Method

- Scanning the parameter space using PDF parametrisation of form:

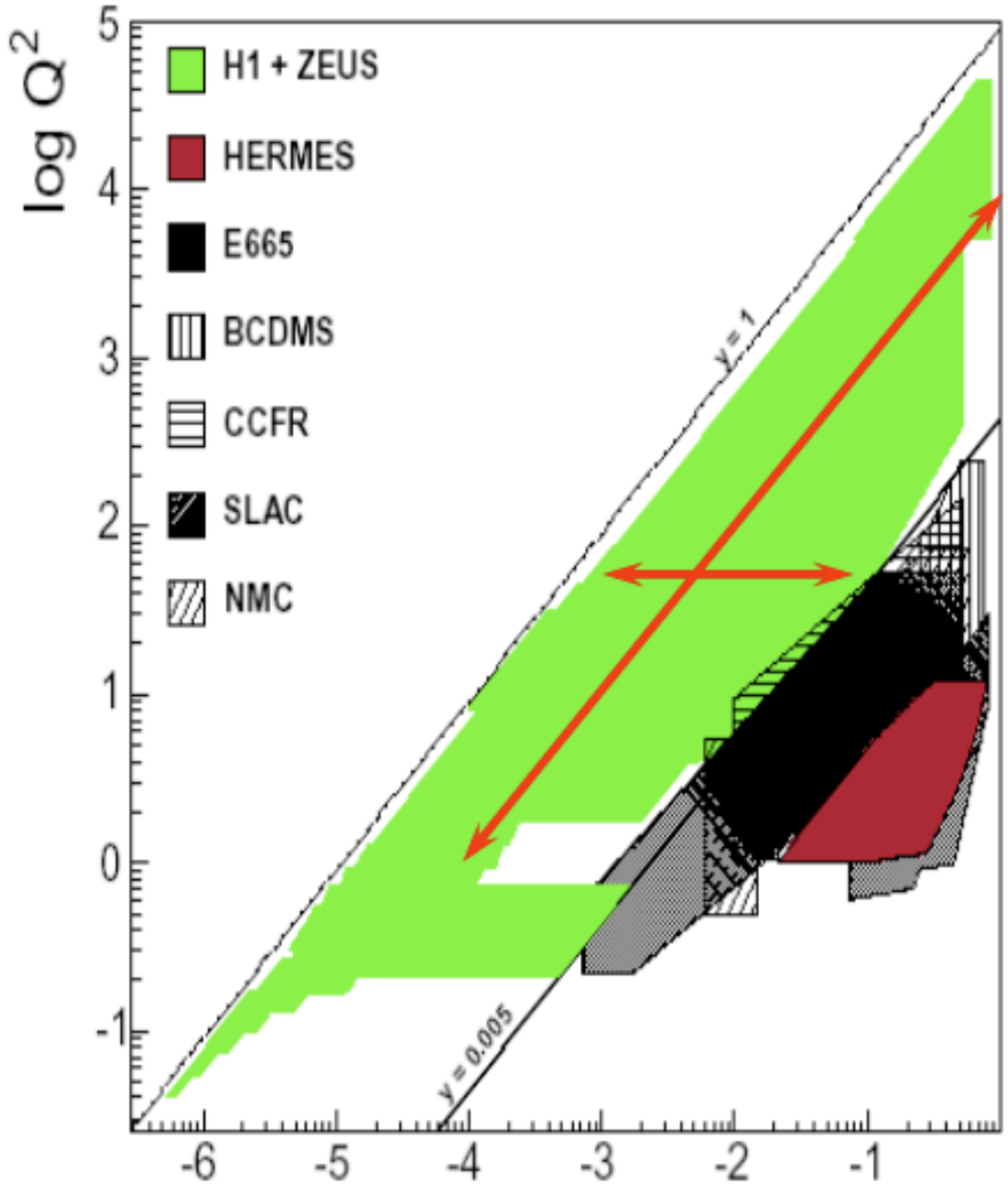
$$xf(x) = Ax^B(1-x)^C(1+\epsilon\sqrt{x}+Dx+Ex^2)$$

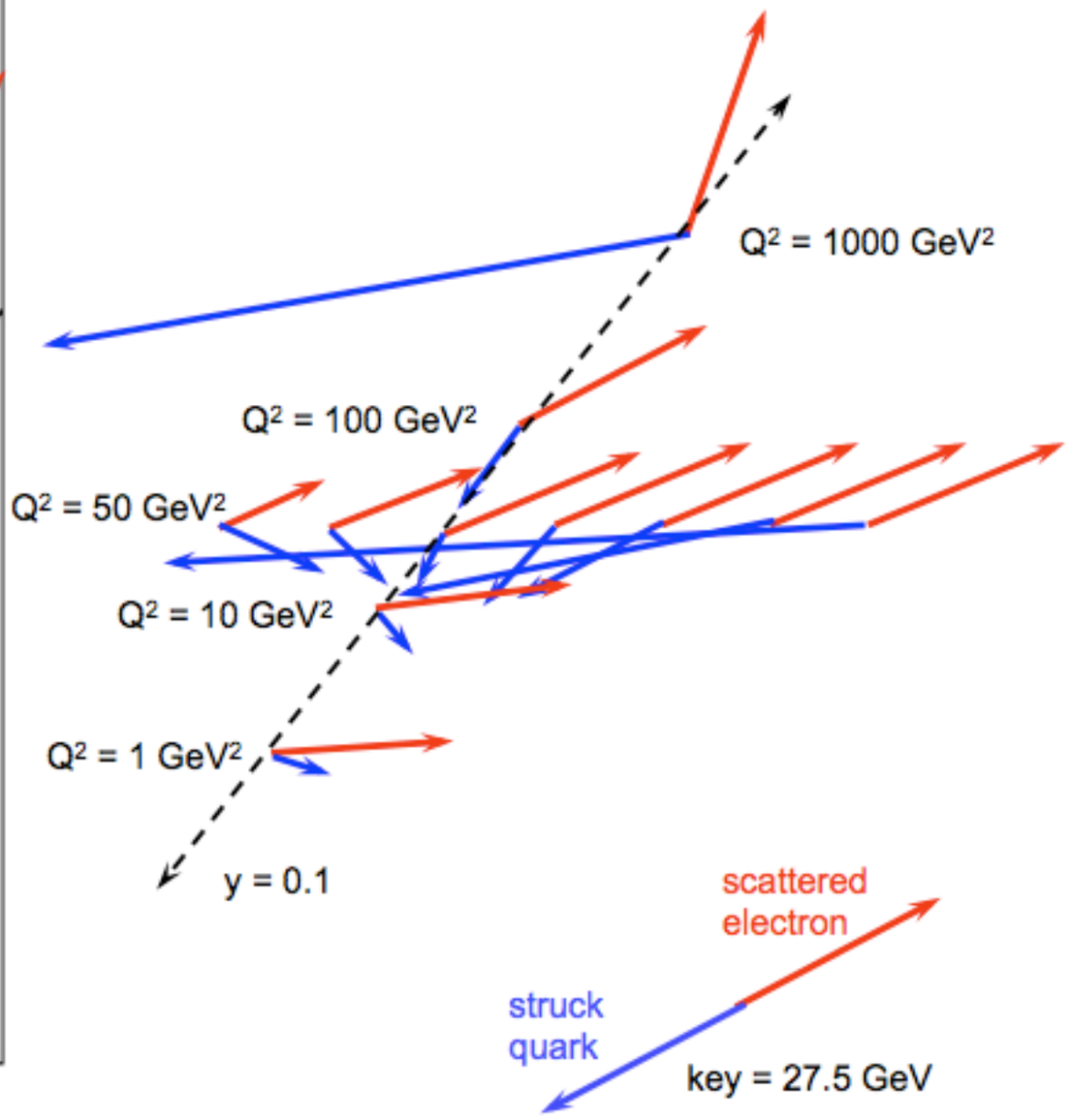
  - variation of number of terms for each fitted parton distribution:
    - all of the 11 parameter fits which have Euv ≠ 0 and one of the D, E, ε ≠0 are considered
  - an envelope is formed using these variants:
    - mostly affects the high x region

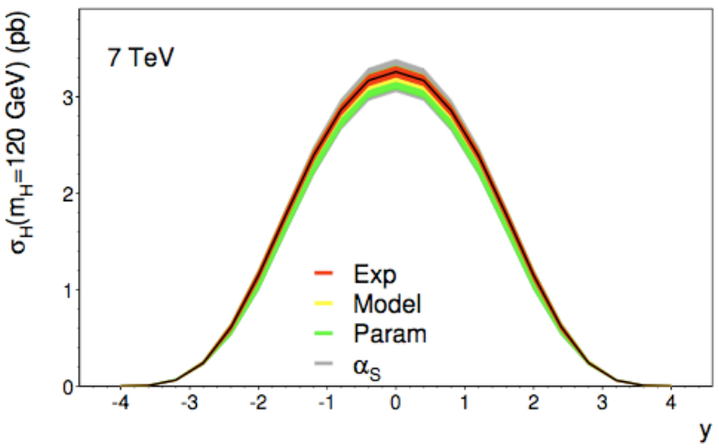
- Variation of Q_0^2 is regarded as a parametrisation uncertainty
  - Q_0^2 = 1.9 GeV^2 → 1.5 - 2.5 GeV^2
    - low bound varied with fs = 0.29 and upper bound fs = 0.34 and Mc = 1.6 GeV
  - when lowering the starting point use a parametrisation form that explicitly allows for negative gluon contribution at low x:
    - subtract $A'_g x^{B'_g}(1-x)^{C'_g}$ from the standard for with C'g =25 and A'g and B'g let to vary
  - mostly affects the low x region

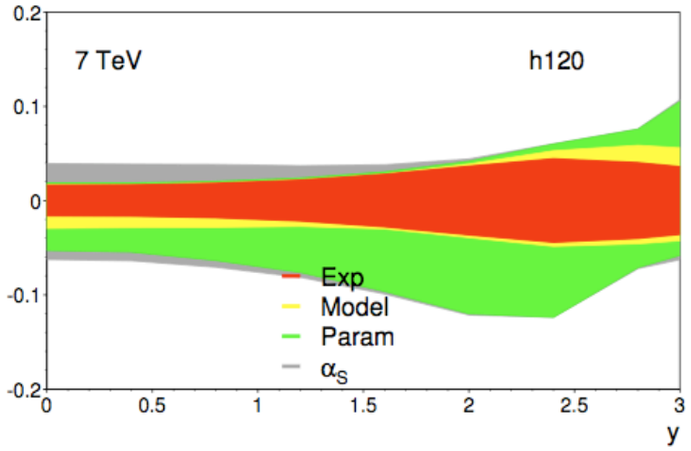
- Allowing Buv ≠ Bdv increases the uncertainty on the valence quarks at low x

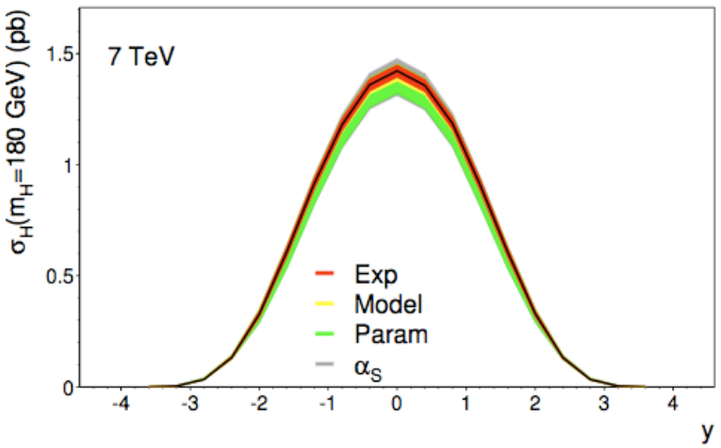
Event Topology at the HERA Collider

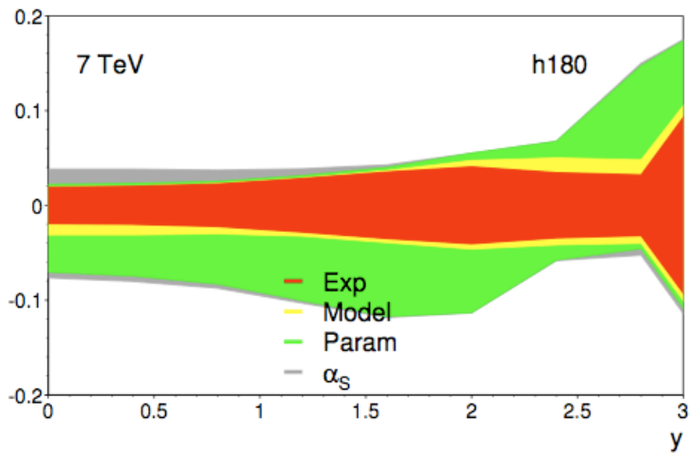





7 TeV
$\sigma_H(m_H=120\ \text{GeV})$ (pb)
Exp
Model
Param
α_S
y


7 TeV
h120
Exp
Model
Param
α_S
y


7 TeV
$\sigma_H(m_H=180\ \text{GeV})$ (pb)
Exp
Model
Param
α_S
y


7 TeV
h180
Exp
Model
Param
α_S
y

DIS Cross Sections and Structure Functions

- General double differential cross-section for eN scattering:

NC $e^{\pm}p \to e^{\pm}X$

$$\frac{\mathrm{d}^2\sigma^{\pm}_{NC}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+\tilde{F}_2 \mp Y_- x\tilde{F}_3 - y^2\tilde{F}_L\right] \equiv \frac{2\pi\alpha^2}{xQ^4}\,Y_+\tilde{\sigma}^{\pm}_{NC}$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

CC $e^{\pm}p \to \nu X$

$$\frac{\mathrm{d}^2\sigma^{\pm}_{CC}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{G_F^2}{4\pi x}\left[\frac{m_W^2}{Q^2+m_W^2}\right]^2\left[Y_+\tilde{W}_2 \mp Y_- x\tilde{W}_3 - y^2\tilde{W}_L\right]$$

At Leading Order (LO):

$$F_2 = x\sum e_q^2\,(q(x)+\bar{q}(x))$$
$$xF_3 = x\sum 2e_q a_q\,(q(x)-\bar{q}(x))$$

- F_2, F_L, xF_3 are structure functions (hadronic part) which are related to the momentum distributions of quarks within the nucleon: Parton Distribution Functions (PDFs):
  - probability density that interacting parton carries a fraction x of the hadrons longitudinal momentum
    - valence quarks: carriers of charge
    - sea quarks and gluons: evolved by complex dynamics

- F_2 dominates
  - sensitive to all quarks
- xF_3
  - sensitive to valence quarks
- F_L
  - sensitive to gluons

- So, we can extract the structure functions experimentally by looking at the x,y,Q^2 dependence of t double differential cross-section.