Introduction to Small-\(x\) and Diffraction at HERA and LHC

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DESY

Photon 2005
Warszawa
1\textsuperscript{st} of September
HERA – ep Collider

**ZEUS**

Liquid Argon Calorimeter

Uranium-Scintillator Calorimeter

- $Q^2$ - virtuality of the incoming photon
- $W$ - CMS energy of the incoming photon-proton system
- $x$ - Fraction of the proton momentum carried by the struck quark

$x \sim Q^2/W^2$
\[
\frac{d^2 \sigma^{ep}}{dxdQ^2} = \frac{2 \pi \alpha_{em}^2}{xQ^4} \cdot [Y_F F_2(x, Q^2) - Y_x F_3(x, Q^2) - y^2 F_L(x, Q^2)]
\]

\[Y_\perp = 1 \pm (1 - y)^2\]

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**Infinite momentum frame**

Proton looks like a cloud of non-interacting quarks and gluons

\[F_2\text{ measures parton density in the proton at a scale } Q^2\]

\[Q^2 = sxy\]
Gluon density dominates $F_2$ for $x < 0.01$
Diffractive Scattering

Non-Diffractive Event
ZEUS detector

Diffractive Event

\[ M_X \] - invariant mass of all particles seen in the central detector
\[ t \] - momentum transfer to the diffractively scattered proton
\[ t \] - conjugate variable to the impact parameter
Dipole description of DIS
equivalent to Parton Picture in perturbative region

\[ \sigma_{tot}^{\gamma^* p} = \int d^2 \hat{r} \int_0^1 dz \Psi^* \sigma_{q\bar{q}} (x, r^2) \Psi \]

GBW - first Dipole Saturation Mode (rudimentary evolution)
Golec-Biernat, Wuesthoff

\[ \sigma_{q\bar{q}} (x, r) \approx \sigma_0 (1 - \exp (- \frac{r^2}{R_0^2})); \quad R_0^2 = \frac{1}{GeV^2} \left( \frac{x}{x_0} \right)^{\lambda_{GBW}} \]

BGBK - DSM with DGLAP
Bartels, Golec-Biernat, Kowalski

\[ \sigma_{q\bar{q}} (x, r) \approx \sigma_0 (1 - \exp (- \frac{\pi^2}{3\sigma_0} r^2 \alpha_s x g(x, \mu^2 = C / r^2 + \mu_0^2))) \]

Iancu, Itakura, Mounier
BFKL-CGC motivated ansatz
Impact Parameter Dipole Saturation Model

Glauber-Mueller, Levin, Capella, Kaidalov...

$$\frac{d\sigma_{qq}(x, r)}{dt^2} = 2 \cdot \left\{ 1 - \exp\left( -\frac{\pi^2}{2 \cdot 3} r^2 \alpha_s(\mu^2)x g(x, \mu^2) T(b) \right) \right\}$$

\[ T(b) \text{ - proton shape} \]

$$\frac{d\sigma_{\text{diff}}}{dt} \sim \exp(B \cdot t) \Rightarrow T(b) \sim \exp(-\bar{b}^2 / 2B)$$
Total $\gamma^* p$ cross-section

$\sigma_{tot}^{\gamma^*p} \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{2_{tot}}$

universal rate of rise of all hadronic cross-sections
2005 measurements:
\( \sigma_{\text{diff}} / \sigma_{\text{tot}} \)
integrated over all masses
16\% at \( Q^2 = 4 \text{ GeV}^2 \)
10\% at \( Q^2 = 27 \text{ GeV}^2 \)

logarithmic drop of diffractive contribution with \( Q^2 \)
in agreement with DSP

from fit to \( \sigma_{\text{tot}} \) predicts \( \sigma_{\text{diff}} \)
GBW, BGBK ...
Dipole cross section determined by fit to $F_2$.
Simultaneous description of many reactions.

Gluon density test?  
Teubner

$\gamma^* p \rightarrow J/\psi p$

IP-Dipole Model

$F_2^C$

$\gamma^* p \rightarrow J/\psi p$

$\langle Q^2 \rangle = 0.05 \text{ GeV}^2$
Saturation and Absorptive corrections

\[ F_2 \sim \begin{cases} 
\text{Single inclusive} & \text{pure DGLAP} \\
\text{Diffraction} & 
\end{cases} \]

Example in Dipole Model

\[ \frac{d\sigma}{d^2b} = 2(1 - \exp(-\Omega/2)) = \Omega - \Omega^2/4 + \ldots \]
**GBW Model**

\[ \sigma_{qq}(x, r) = \sigma_0 \left[ 1 - \exp\left( - \frac{r^2}{4 R_0^2} \right) \right] \]

\[ R_0^2(x) = \frac{1}{\text{GeV}^2} \left( \frac{x}{x_0} \right)^4 \]

**IP Dipole Model**

\[ \frac{d\sigma_{qq}(x, r)}{d^2b} = 2 \left\{ 1 - \exp\left( - \frac{\pi^2}{2 \cdot 3} r^2 \alpha_s(x, C / r^2 + Q_0^2) T(b) \right) \right\} \]

\[ x = 10^{-5} \]

\[ x = 10^{-2} \]

**strong saturation**

**less saturation** (due to IP and charm)
Saturation scale
(a measure of gluon density)

\[ Q_s^2 = \frac{2}{r_s^2} \]

\[ (Q_s^2)_g = \frac{N_C}{C_F} (Q_s^2)_q = \frac{9}{4} (Q_s^2)_q \]

\[ Q_s^2 = \frac{4 \pi^2 \alpha_s N_C}{N_C^2 - 1} \frac{1}{\pi R^2} \frac{dN}{dy} \]

\[ \frac{dN}{dy} \approx 1000 \quad R \approx 7 \text{ fm} \]

\[ Q_s^2 \approx 1 \text{ GeV}^2 \]

\[ Q_{s \text{RHIC}} \sim Q_{s \text{HERA}} \]
Geometrical Scaling
A. Stasto & Golec-Biernat
J. Kwiecinski

\[ \tau = Q^2 R_0^2(x) \]

\[ R_0^2(x) = \frac{1}{GeV^2} \left( \frac{x}{x_0} \right)^2 \]
Geometrical Scaling can be derived from traveling wave solutions of non-linear QCD evolution equations.

⇒ Velocity of the wave front gives the energy dependence of the saturation scale

Munier, Peschansk
L. McLerran +...
Al Mueller +..

Question:
Is GS an intrinsic (GBW) or effective (KT) property of HERA data?
2-Pomeron exchange in QCD

Final States (naïve picture)

0-cut

1-cut

2-cut

Diffraction

\[ \langle n \rangle \]

\[ \langle 2n \rangle \]
AGK rules in the Dipole Model

\[
\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!} \exp(-\Omega)
\]

\[
\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)
\]

Note: AGK rules underestimate the amount of diffraction in DIS
\[
\frac{d\sigma_{qq}}{d^2b} = 2 \cdot \left\{ 1 - \exp\left(-\frac{\Omega}{2}\right) \right\}
\]

\[
\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!} \exp(-\Omega)
\]

\[
\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)
\]
**HERA Result**

**Unintegrated Gluon Density**

\[
\frac{\alpha_s f(x, l^2)}{l^4} = \frac{3}{8\pi^2} \int_0^\infty dr \, r \, J_0(l r) \left\{ \hat{\sigma}_\infty(x) - \hat{\sigma}(x, r) \right\}
\]

\[
\alpha_s(Q^2) x g(x, Q^2) = \int_{Q^2}^{\infty} \frac{d^21}{\pi l^2} \alpha_s f(x, l^2)
\]

**Dipole Model**

Another approach (KMR)

\[
f_g(x, \mu^2) = \beta(t) \cdot \frac{\partial}{\partial \ln Q_t^2} \left[ \sqrt{T(Q_t, \mu) \cdot x g(x, Q_t^2)} \right]
\]

\[
T(Q_t, \mu) = \exp \left( -\int_{Q_t^2}^{\infty} \frac{\alpha_s(k_t^2)}{2\pi} \frac{dk_t^2}{k_t^2} \int_0^{k_t/(\mu + k_t)} z P_{gg}(z) dz \right)
\]

\[
f_g(x, t, Q_t, \mu) = \beta(t) f_g(x, t = 0, Q_t, \mu) \quad b(t) = \exp(Bt / 2)
\]

**Active field of study at HERA:**

UGD in heavy quark production, new result expected from high luminosity running in 2005, 2006, 2007
Exclusive Double Diffractive Reactions at LHC

$x_{IP} = \Delta p/p, \ p_T \ x_{IP} \sim 0.2\text{-}1.5\%$

Leading proton detector

1 event/sec

High momentum measurement precision

$\sigma^{\text{Diff}} = \hat{\sigma} \cdot L$

$M^2 \frac{\partial L}{\partial v \partial M^2} = S^2 O$

$O^{\text{exclusive}} = \left( \frac{\pi}{(N_c^2 - 1)b} \int \frac{dQ_t^2}{Q_t^4} f_g(x_1, Q_t) f_g(x_2, Q_t) \right)^2$

low $x$ QCD reactions:

$pp \Rightarrow pp + g_{\text{Jet}} g_{\text{Jet}} \quad \sigma \sim 1 \text{ nb} \quad \text{for} \quad M(jj) \sim 50 \text{ GeV}$

$\sigma \sim 0.5 \text{ pb} \quad \text{for} \quad M(jj) \sim 200 \text{ GeV}$

$|\eta_{\text{Jet}}| < 1$

$p_1$ $x_1'$ $x_1$

$p_2$ $x_2' x_2$

$Q$

$M$

$p$ $p$

$\sigma^{\text{exclusive}} = \int dQ_t^2 f_g(x_1, Q_t) f_g(x_2, Q_t)$

$p_1$ $x_1'$ $x_1$

$p_2$ $x_2' x_2$

$Q$

$M$

$p$ $p$

$\sigma^{\text{exclusive}} = \int dQ_t^2 f_g(x_1, Q_t) f_g(x_2, Q_t)$

$pp \Rightarrow pp + \text{Higgs} \quad \sigma \sim \mathcal{O}(3) \text{ fb} \quad \text{SM}$

$\sim \mathcal{O}(100) \text{ fb} \quad \text{MSSM}$
$t$ – distributions at LHC

with the cross-sections of the $O(1)$ nb and $L \sim 1$ nb$^{-1}$ s$^{-1}$ =>
$O(10^7)$ events/year are expected.

For hard diffraction this allows to follow the $t$ – distribution to
$t_{max} \sim 4$ GeV$^2$

For soft diffraction $t_{max} \sim 2$ GeV$^2$

$t$-distribution of hard processes should be sensitive to the evolution and/or saturation effects

see:
Al Mueller dipole evolution, BK equation, and the impact parameter saturation model for HERA data
Survival Probability $S^2$

Effects of soft proton absorption modulate the hard $t$-distributions

$t$-measurement will allow to disentangle the effects of soft absorption from hard behavior.
\[ F_2 \quad \rightarrow \quad \text{Gluon Luminosity} \]

**Dipole Model**

\[ \sigma_{\text{Diff}} = \hat{\sigma} \cdot L \]

\[ M^2 \frac{\partial L}{\partial y \partial M^2} = S^2 O \]

**Exclusive Double Diffraction**

\[ O^{\text{exclusive}} = \left( \frac{\pi}{(N_c - 1)b} \right) \left( \frac{dQ_t^2}{Q_t^4} \right) f_g(x_1, x_1', t, Q_t, \mu) f_g(x_2, x_2', t, Q_t, \mu) \]

\( f_g \) – unintegrated gluon densities
Conclusions

We are developing a very good understanding of inclusive and diffractive $g*p$ interactions: $F_2$, $F_2^{D(3)}$, $F_2^c$, Vector Mesons (J/Psi)...

Observation of diffraction indicates multi-gluon interaction effects at HERA

HERA measurements suggests presence of Saturation phenomena
Saturation scale determined at HERA agrees with RHIC

HERA determined properties of the Gluon Cloud

Diffractive LHC ~ pure Gluon Collider
=> investigations of properties of the gluon cloud in the new region

*Gluon Cloud is a fundamental QCD object - SOLVE QCD!!!!*
Higher symmetries (e.g. Supersymmetry) lead to existence of several scalar, neutral, Higgs states, $H, h, A$ . . . . Higgs Hunter Guide, Gunnion, Haber, Kane, Dawson 1990

In MSSM Higgs $x$-section are likely to be much enhanced as compared to Standard Model (tan$\beta$ large because $M_{\text{Higgs}} > 115$ GeV)

CP violation is highly probable in MSSM all three neutral Higgs bosons have similar masses $\sim 120$ GeV can ONLY be RESOLVED in DIFFRACTION


Correlation between transverse momenta of the tagged protons give a handle on the CP-violation in the Higgs sector

Khoze, Martin, Ryskin, hep-ph 040178