Tests of perturbative QCD with hadronic final states in $e p$ collisions

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H1 and ZEUS Collaborations
Kinematics of Neutral Current Deep Inelastic Scattering

For a given $ep$ centre-of-mass energy, $\sqrt{s}$, the (fully) inclusive cross section for $ep \rightarrow e + X$ can be described by two independent kinematic variables, e.g.

$$Q^2 = -(k - k')^2$$

$$x_{Bj} = Q^2 / (2P \cdot q)$$

$\rightarrow$ Inelasticity variable

$$y = Q^2 / (x_{Bj} s)$$
Jet Production in Neutral Current Deep Inelastic Scattering

- Jet production in neutral current deep inelastic scattering up to $\mathcal{O}(\alpha_s)$:

Quark-Parton Model

Boson-Gluon Fusion

QCD Compton

- Perturbative QCD calculations of jet cross sections:

$$d\sigma_{jet} = \sum_{a=q,\bar{q},g} \int dx \ f_a(x, \mu_F^2) \ d\hat{\sigma}_a(x, \alpha_s(\mu_R), \mu_R^2, \mu_F^2)$$

- $f_a$: parton $a$ density in the proton, determined from experiment; long-distance structure of the target

- $\hat{\sigma}_a$: subprocess cross section, calculable in pQCD; short-distance structure of the interaction
Jet Production in Neutral Current Deep Inelastic Scattering

- In the region where the wealth of data from fixed-target and collider experiments has allowed an accurate determination of the proton PDFs, measurements of jet production in NC DIS provide
  - a sensitive test of the pQCD predictions of the short-distance structure
  - a determination of the strong coupling constant $\alpha_s$

- To perform a stringent test of the pQCD predictions and a precise determination of $\alpha_s$:
  - Observables for which the predictions are directly proportional to $\alpha_s$
    - Jet cross sections in the Breit frame
  - Small experimental uncertainties $\rightarrow$ Jets with relatively high transverse energy
  - Small theoretical uncertainties $\rightarrow$ NLO QCD calculations
    - Jet algorithm: longitudinally invariant $k_T$ cluster algorithm (Catani et al)
      (small parton-to-hadron effects, infrared safe, suppression of beam-remnant jet)
  - Jet selection criteria

- Exploration of the parton evolution at low $x$ $\Rightarrow$ footprints of BFKL effects?
In the Breit frame the virtual boson collides head-on with the proton

High-$E_T$ jet production in the Breit frame

→ suppression of the Born contribution (struck quark has zero $E_T$)
→ suppression of the beam-remnant jet (zero $E_T$)
→ lowest-order non-trivial contributions from $\gamma^* g \rightarrow q\bar{q}$ and $\gamma^* q \rightarrow qg$
⇒ directly sensitive to hard QCD processes ($\alpha_s$)
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Inclusive Jet Cross Sections in NC DIS at $Q^2 > 125$ GeV$^2$

- **New** measurement of inclusive jet cross sections in the kinematic region defined by $Q^2 > 125$ GeV$^2$ and $|\cos \gamma| < 0.65$ for jets with $E_{T,jet}^B > 8$ GeV and $-2 < \eta_{jet}^B < 1.5$ using $L = 81.7$ pb$^{-1}$
  - no cut is applied in the laboratory frame
- **Advantages:**
  - infrared insensitivity (no dijet cuts!)
  - suited to test resummed calculations
  - smaller theoretical uncertainties than for dijet
- **Small** experimental uncertainties:
  - jet energy scale (1\% for $E_{T,jet} > 10$ GeV)
    - $\Rightarrow \sim \pm 5\%$ on the cross sections
- **Small** parton-to-hadron corrections ($C_{h,a,d}$): < 10\%
- **NLO** QCD calculations (\mathcal{O}(\alpha_s^2)) using $\mu_R = E_{T,jet}^B$, $\mu_F = Q$ and the MRST99 parametrisations of the proton PDFs describe the measurements well
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Inclusive Jet Cross Sections in NC DIS at $Q^2 > 125$ GeV$^2$

- New measurement of the inclusive jet cross section $d\sigma/dE_{T,jet}^B$ in the kinematic region defined by $Q^2 > 125$ GeV$^2$ and $|\cos \gamma| < 0.65$ for jets with $E_{T,jet}^B > 8$ GeV and $-2 < \eta_{jet}^B < 1.5$

- Small theoretical uncertainties:
  - higher-order terms (> NLO); varying $\mu_R$ between $\frac{1}{2} \cdot E_{T,jet}^B$ and $2 \cdot E_{T,jet}^B \Rightarrow \pm 5\%$
  - uncertainty on $\alpha_s(M_Z)$ ($\pm 0.0027$); $\Rightarrow \pm 4\%$
  - uncertainties on the proton PDFs; $\Rightarrow \pm 3\%$

- NLO QCD calculations ($\mathcal{O}(\alpha_s^2)$) using $\mu_R = E_{T,jet}^B$, $\mu_F = Q$ and the MRST99 parametrisations of the proton PDFs describe the measurements well
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Inclusive Jet Cross Sections in NC DIS at $Q^2 > 125 \text{ GeV}^2$

- New measurement of the inclusive jet cross section $d\sigma / dE_{T,jet}^B$ in different regions of $Q^2$ for jets with $E_{T,jet}^B > 8 \text{ GeV}$ and $-2 < \eta_{jet}^B < 1.5$

- NLO QCD calculations provide a good description of the data → validity of the description of the dynamics of inclusive jet production by pQCD at $O(\alpha_s^2)$

- Inclusive jet cross sections in NC DIS in the Breit frame provide direct sensitivity to $\alpha_s$ and the gluon density in the proton with small experimental and theoretical uncertainties
Inclusive Jet Cross Sections and extraction of $\alpha_s$:

- The inclusive jet cross section $d\sigma/dQ^2$ at $Q^2 > 500$ GeV$^2$ has been used to extract $\alpha_s(M_Z)$:
  
  $\alpha_s(M_Z) = 0.1196 \pm 0.0011$ (stat.)
  $\pm 0.0025$ (exp.) $\pm 0.0017$ (th.)

- Experimental uncertainties:
  
  → jet energy scale (1% for $E_{T,jet} > 10$ GeV)

- Theoretical uncertainties:
  
  → terms beyond NLO $\Delta \alpha_s(M_Z) = 1 - 2$
  → uncertainties proton PDFs $\Delta \alpha_s(M_Z) = 1$

- Consistent with other determinations of $\alpha_s$

- Very precise determination of $\alpha_s(M_Z)$!

- Study of the scale dependence of $\alpha_s(E_{T,jet}^B)$: from the measured $d\sigma/dE_{T,jet}^B$ in each $E_{T,jet}^B$ region $\rightarrow \alpha_s(< E_{T,jet}^B >)$ is extracted

- The measurements are consistent with the running of $\alpha_s$ predicted by perturbative QCD
Dijet and Trijet Cross Sections in NC DIS (150 < \(Q^2\) < 15000 GeV\(^2\))

- **New measurement** of dijet and trijet cross sections over a wide range in \(Q^2\) → 150 < \(Q^2\) < 15000 GeV\(^2\) and 0.2 < \(y\) < 0.6 for jets with 
  
  \[E_T^{jet} (\text{Breit}) > 5 \text{ GeV}, -1 < \eta^{jet} (\text{Lab}) < 2.5, M_{jj} > 25 \text{ GeV} (M_{jjj} > 25 \text{ GeV})\]
  
  using \(\mathcal{L} = 65.4\) pb\(^{-1}\)

- **Trijet cross sections test** QCD beyond LO directly → \(\sigma_{3\text{jet}} \propto \alpha_s^2\)

- **Comparison with NLO QCD calculations:**
  
  → \(\mu_R = \mu_F = Q\)
  
  → CTEQ5M set of proton PDFs
  
  → parton-to-hadron corrections applied

- **NLO QCD gives a good description of the data** over a wide range in \(Q^2\)
Dijet and Trijet Cross Sections in NC DIS (150 < Q^2 < 15000 GeV^2)

- New measurement of dijet and trijet cross sections over a wide range in Q^2 → 150 < Q^2 < 15000 GeV^2 and 0.2 < y < 0.6 for jets with 
  \[ E_T^{jet} (\text{Breit}) > 5 \text{ GeV}, \ -1 < \eta^{jet} (\text{Lab}) < 2.5, \ M_{jj} > 25 \text{ GeV} (M_{jjj} > 25 \text{ GeV}) \]

using \( L = 65.4 \text{ pb}^{-1} \)

- NLO QCD gives a good description of the data over a wide range in Q^2 
  (Z^0-exchange effects not included in NLO; significant only for the highest Q^2 point)
Tests of perturbative QCD with hadronic final states in \( ep \) collisions

Dijet and Trijet Cross Sections in NC DIS \((150 < Q^2 < 15000 \text{ GeV}^2)\)

- New measurement of the ratio of the trijet to dijet cross section over a wide range in \( Q^2 \)

\[ R_{3/2} \equiv \frac{\sigma_{\text{trijet}}(Q^2)}{\sigma_{\text{dijet}}(Q^2)} \]

- Small experimental uncertainties.
- Small theoretical uncertainties:
  - uncertainties on the proton PDFs
  - higher-order terms (> NLO)
    (reduced to \( \sim 5\% \))
- Since \( R_{3/2} \propto \alpha_s \) at LO it allows
  a determination of the strong coupling constant with small theoretical uncertainties
- The measured values of \( R_{3/2} \) have been fitted with NLO QCD calculations to
  extract a combined value of \( \alpha_s(M_Z) \):

\[
\alpha_s(M_Z) = 0.1175 \pm 0.0017 \text{ (stat.)} \pm 0.0050 \text{ (syst.)}^{+0.0054}_{-0.0068} \text{ (th.)}
\]
Dijet and Trijet Cross Sections in NC DIS (150 < \(Q^2\) < 15000 GeV\(^2\))

- The observed dependence of \(\alpha_s\) with \(Q^2\) is consistent with the prediction of QCD →

- Determination of \(\alpha_s\) as a function of the energy scale using the measured values of \(R_{3/2}\)

\[ R_{3/2} = \frac{\sigma_{\text{dijet}}}{\sigma_{\text{trijet}}} = \frac{3}{2} R \quad 0.15 < R < 0.55 \]

\[ 0.1 < R_{3/2} < 0.22 \]

H1 Preliminary 99-00

\(\alpha_s(Q^2)\)

Averaged \(\alpha_s(M_Z)\)

World Average (PDG):

\(\alpha_s(M_Z) = 0.1187 \pm 0.0020\)
Parton evolution at low $x$

- DGLAP equations sum the leading powers of $\alpha_s \log Q^2$ in the region of strongly-ordered transverse momenta
  $$Q^2 \gg k_{Tn}^2 \gg \ldots \gg k_{T2}^2 \gg k_{T1}^2$$
- When $\log Q^2 \ll \log 1/x$ terms proportional to $\alpha_s \log 1/x$ become important and need to be summed
  the BFKL equation accomplishes that; the integration is taken over the full $k_T$ phase space of the gluons
  $\Rightarrow$ no $k_T$ ordering
- Mueller and Navelet’s proposal:
  forward (proton’s direction) jet production with $x_1/x$ as large as possible
  and $k_{T1} \sim Q$
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Measurement of Forward Jet Production at low $x$

- **Measurement of the differential cross section** $d\sigma/dx$
  
  for jet production with $p_{t,\text{jet}} > 3.5$ GeV, $7^\circ < \theta_{\text{jet}} < 20^\circ$, $0.5 < p_{t,\text{jet}}^2/Q^2 < 2$ and $x_{\text{jet}} = E_{\text{jet}}/E_p > 0.035$

  in the kinematic region

  $10^{-4} < x < 4 \cdot 10^{-3}$ and $5 < Q^2 < 85$ GeV$^2$

- **Strong rise towards low $x$ is observed**

- **Comparison to calculations**

  $\rightarrow$ NLO QCD (DGLAP) lies well below the data at low $x$

  $\rightarrow$ MC models with extra parton radiation provide an improved description of the data

  $\rightarrow$ inclusion of a resolved-photon component (RG-DIR+RES)

  $\rightarrow$ parton emissions not ordered in $k_T$ (CDM)
Measurement of Forward Jet Production at low $x$

- Measurement of the triply-differential cross section $d^3\sigma/dxdQ^2dp_t^2$

  $\rightarrow$ NLO QCD fails at low $Q^2$, $x$, $p_t$  
  $\rightarrow$ RG-DIR+RES best overall
Photoproduction of Jets

- Production of jets in $\gamma p$ collisions has been measured via $ep$ scattering at $Q^2 \approx 0$
- At lowest order QCD, two hard scattering processes contribute to jet production $\Rightarrow$
- pQCD calculations of jet cross sections

$$d\sigma_{jet} = \sum_{a,b} \int_0^1 dy \ f_{\gamma/e}(y) \int_0^1 dx_{\gamma} \ f_{a/\gamma}(x_{\gamma}, \mu^2_{F\gamma}) \int_0^1 dx_p \ f_{b/p}(x_p, \mu^2_{Fp}) \ d\hat{\sigma}_{ab \rightarrow jj}$$

longitudinal momentum fraction of $\gamma/e^+$ $(y)$, parton $a/\gamma$ $(x_{\gamma})$, parton $b/proton$ $(x_p)$

$\rightarrow f_{\gamma/e}(y) = \text{flux of photons in the positron (WW approximation)}$

$\rightarrow f_{a/\gamma}(x_{\gamma}, \mu^2_{F\gamma}) = \text{parton densities in the photon (for direct processes } \delta(1 - x_{\gamma}))$

$\rightarrow f_{b/p}(x_p, \mu^2_{Fp}) = \text{parton densities in the proton}$

$\rightarrow \sigma_{ab \rightarrow jj} \text{ subprocess cross section; short-distance structure of the interaction}$
Photoproduction of Jets

- Measurements of jet photoproduction provide
  - Test of NLO QCD predictions based on current parametrisations of the proton and photon PDFs
  - Dynamics of resolved and direct processes
  - Photon structure: information on quark densities from $F_2^\gamma$ in $e^+e^-$; gluon density poorly constrained.

Jet cross sections in photoproduction are sensitive to both the quark and gluon densities in the photon at larger scales $\mu^2_{F\gamma} \sim E^2_{T,jet}$ ($200 - 10^4$ GeV$^2$)

- Proton structure: well constrained by DIS except for the gluon density at high $x$. Jet cross sections in $\gamma p$ are sensitive to parton densities at $x_p$ up to $\sim 0.6$
  - Observable to separate the contributions: the fraction of the photon’s energy participating in the production of the dijet system

$$x^{OBS}_\gamma = \frac{1}{2E_\gamma} \sum_{i=1}^{2} E_{T,jet_i} e^{-\eta_{jet_i}}$$
### Dijet Photoproduction: photon structure

- **New measurement of the dijet cross sections** $d\sigma/dx_\gamma$ and $d\sigma/dx_p$ for dijet events with $E_{T,max} > 25$ GeV, $E_{T,second} > 15$ GeV and $-0.5 < \eta^{jet} < 2.75$ (both jets) in the kinematic region $Q^2 < 1$ GeV$^2$ and $0.1 < y < 0.9$

- **$x_p$ variable:** $x_p = \frac{1}{2E_p} \sum_{i=1}^2 E_T^{jet_i} e^{\eta^{jet_i}}$

- **Measurements of** $d\sigma/dx_\gamma$ for $x_p < 0.1$ ($g_p$ processes) and $x_p > 0.1$ ($q_p$ processes)

- **Comparison to NLO calculations using** CTEQ6M (proton) and GRV-HO (photon) PDFs → Good description of the data along the entire range of $x_\gamma$ for both ranges in $x_p$

- **Consistent with QCD-evolved photon PDFs** determined from measurements in $\gamma\gamma$ at lower scales and proton PDFs as determined from HERA and fixed-target experiments

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<th>$x_p$</th>
<th>$d\sigma/dx_\gamma$ [pb]</th>
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<tr>
<td>&gt; 0.1</td>
<td><img src="image2.png" alt="Graph" /></td>
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Dijet Photoproduction: proton structure

- New measurements of $d\sigma/dx_p$ for $x_\gamma < 0.8$ (resolved processes) and $x_\gamma > 0.8$ (direct processes)
- Comparison to NLO calculations using CTEQ6M (proton) and GRV-HO (photon) PDFs
- Theoretical uncertainties:
  - terms beyond NLO; estimated by varying $\mu_R$ up and down by a factor of 2
    $\Rightarrow$ dominant at low $x_p$
  - uncertainties of the proton PDFs; $\Rightarrow$ increasingly important as $x_p$ increases
- The NLO QCD calculations agree with the data in the low $x_p$ range
- At high $x_p$ ($x_p > 0.32$) there are discrepancies between data and NLO
  $\Rightarrow$ Measurements at high $x_\gamma$ provide further constraints on the proton PDFs, particular the gluon density, free from the photon PDFs
Dijet Photoproduction: proton structure

Measurements at $p_{T,max} > 0.8$ ⇒ useful constrain in a global determination of proton PDFs
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Improving the Determination of the Proton PDFs

- **Observables used in the fits to determine the proton PDFs:**
  - Inclusive measurements of deep inelastic $lN$ scattering

- **Advantages:**
  - Inclusive (only final-state lepton is tagged)

  $\Rightarrow$ no QCD corrections associated to the final-state lepton

- **Disadvantages:** the gluon distribution contributes indirectly

- **Observables based on jets have hardly been used**
  - large QCD corrections and hadronisation corrections

- **Fixed-target DIS:** higher twists, heavy-target corrections and isospin-symm. assumptions

- **That’s the past! NOW there are measurements of jet cross sections at HERA**
  - directly sensitive to the gluon density with small experimental+theoretical uncertainties!

- **Sufficient sensitivity to determine the proton PDFs within a single ($ep$) experiment**

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Determination of PDFs using structure function and jet data from ZEUS

- **Determination of the proton PDFs using SF data**

  \[ 6.3 \cdot 10^{-5} < x < 0.65, \quad 2.7 < Q^2 < 30000 \text{ GeV}^2 \]

  and jet data in NC DIS and $\gamma p$ collisions from ZEUS only (!)

- **Sufficient sensitivity to determine the proton PDFs within a single experiment**
Tests of perturbative QCD with hadronic final states in $e p$ collisions

Determination of PDFs using structure function and jet data from ZEUS

- Data sets used in the fit (577 data points):
  - **Structure function measurements:** reduced double differential cross sections in $x$ and $Q^2$
    - neutral current DIS $e^+ p$ and $e^- p$
    - charged current DIS $e^+ p$ and $e^- p$
  - **Jet cross section measurements:**
    - inclusive jet production in NC DIS
    - dijet production in $\gamma p$ collisions
- Evolution of the PDFs with the energy scale:
  - DGLAP equations at NLO ($\overline{MS}$ scheme);
  - 11 free parameters ($+\alpha_s$ when free)
- Full account of correlated experimental uncertainties using the offset method
- A good description of the data is obtained:
  $\chi^2 = 470$ for 577 data points

\[ Q^2 = 10 \text{ GeV}^2 \]
Tests of perturbative QCD with hadronic final states in $ep$ collisions

Comparison of proton PDFs

- Compatible with MRST2001 and CTEQ6.1M
- Compatible with H1 analysis

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July 9th, 2005
Improving the gluon distribution: jet data

- Comparison of gluon distributions from fits with and without jet data
  - no significant change of the shape: no tension between jet and inclusive data
  - the jet cross sections constrain the gluon density in the range $0.01 \rightarrow 0.4$
  - **Sizeable reduction of the gluon uncertainty**
    - e.g. from 17% to 10% at $x = 0.06$ and $Q^2 = 7$ GeV$^2$
    - similar reduction by a factor of two in the mid-$x$ region over the full $Q^2$ range
Determination of $\alpha_s(M_Z)$

- Simultaneous determination of the proton PDFs and $\alpha_s(M_Z)$
- Jet cross sections are directly sensitive to $\alpha_s(M_Z)$ via $\gamma^*(g \rightarrow q\bar{q}$ (coupled to gluon density) and $\gamma^*(q \rightarrow qg$ (NOT coupled to gluon density)

$\Rightarrow$ The inclusion of the jet cross sections allows an extraction of $\alpha_s(M_Z)$ that is NOT strongly correlated to the gluon density

- Determination of $\alpha_s(M_Z)$ from the ZEUS-JETS-$\alpha_s$ fit:

$$\alpha_s(M_Z) = 0.1183 \pm 0.0007 \text{ (uncorr.)} \pm 0.0022 \text{ (corr.)} \pm 0.0016 \text{ (norm.)} \pm 0.0008 \text{ (model)}$$

$\Rightarrow$ Precise determination $\alpha_s(M_Z) = 0.1183 \pm 0.0058$ from ZEUS data alone

$\Delta \alpha_s(M_Z) = \pm 0.0050$
Photoproduction of Three-Jet Events: Colour Factors

- **Direct processes** provide a clean way to study the effects of the different color configurations.

\[
\sigma_{ep\rightarrow3\text{jets}} = C^2_F \cdot \sigma_A + C_F C_A \cdot \sigma_B + C_F T_F \cdot \sigma_C + T_F C_A \cdot \sigma_D
\]
Photoproduction of Three-Jet Events: Colour Factors

- **Variables** to highlight the contributions from the different color configurations
  - ⇒ angular correlations between the jets
    - \( \theta_H \) (Muñoz-Tapia, Stirling);
    - \( \alpha_{23} \)
  - \( \cos \beta_{KS} = \cos \frac{1}{2} [\angle ((\vec{p}_1 \times \vec{p}_3), (\vec{p}_2 \times \vec{p}_B)) + \angle ((\vec{p}_1 \times \vec{p}_B), (\vec{p}_2 \times \vec{p}_3))] \),
    where \( \vec{p}_i \) is the momentum of jet \( i \) (ordered according to decreasing \( E_{\text{jet}}^i \)) and \( \vec{p}_B \) is a unit vector in the direction of the proton beam;

- **Fixed-order** (\( \mathcal{O}(\alpha_s^2) \)) calculations using direct processes
  - ⇒ separation of the different colour components
  - \( \sigma_{ep \rightarrow 3\text{jets}} = C_F^2 \cdot \sigma_A + C_F C_A \cdot \sigma_B + C_F T_F \cdot \sigma_C + T_F C_A \cdot \sigma_D \)
  - The predicted relative contributions for \( SU(3) \):
    - \( A \rightarrow 13\% \), \( B \rightarrow 10\% \), \( C \rightarrow 45\% \), \( D \rightarrow 32\% \)

- **Example**: the distribution in \( \theta_H \) for the contribution \( \sigma_B \) is particularly distinct from the other color configurations
Photoproduction of Three-Jet Events: Colour Factors

- Measurement of the normalised cross section $1/\sigma \frac{d\sigma}{d\theta_H}$ for the production of events with three jets satisfying $E_T^{\text{jet}} > 14 \text{ GeV}, -1 < \eta^{\text{jet}} < 2.5$ and $x^{\text{obs}}(3\text{jets}) > 0.7$

  in the kinematic region $Q^2 < 1 \text{ GeV}^2$ and $0.2 < y < 0.85$

- Comparison to the predictions of PYTHIA and HERWIG (leading-logarithm parton-shower calculations based on SU(3))

  $\rightarrow$ PYTHIA reproduces reasonably well the measured distribution

  $\rightarrow$ The distributions for direct and resolved processes ($\sim 34\%$) are very similar
Photoproduction of Three-Jet Events: Colour Factors

- Comparison to fixed-order calculations based on different symmetry groups
  \(SU(3), SU(N)\) with \(N \gg 1\), \(U(1)^3\) and the extreme choice \(C_F = 0\)
- \(U(1)^3\) vs \(SU(3)\): similar shapes due to the smallness of \(\sigma_B\)
- The data disfavour \(T_F/C_F \approx 0\) (\(SU(N) \gg 1\)) or \(C_F = 0\)
- The predictions of \(SU(3)\) describe reasonably well the data
Jet Substructure

- At sufficiently high $E_T^{jet}$, where fragmentation effects become negligible, the jet substructure is expected to be calculable by pQCD.

- Measurement of jet substructure allows investigations on
  → the differences between quark- and gluon-originated jets and
  → the dynamics of the different partonic final states,
  → as well as determinations of $\alpha_s$.

- Integrated jet shape: $\langle \Psi(r) \rangle = \frac{1}{N_{jets}} \sum_{jets} \frac{E_T(r)}{E_T(r = 1)}$

- Jet substructure in photoproduction:
  → Resolved processes give rise to quark and gluon jets through $q\gamma g p \rightarrow qg, g\gamma g p \rightarrow gg, \ldots$
  → Direct processes give rise mostly to quark jets through $\gamma g \rightarrow q\bar{q}$

$\Rightarrow$ The $\eta^{jet}$ dependence of the jet substructure should show:
  quark-like jets for $\eta^{jet} < 0$ and gluon-like jets in the forward direction.
Jet Substructure in Photoproduction

- New measurements of the integrated jet shape $\langle \Psi(r) \rangle$ for a sample of dijet events
  - $E_{T}^{jet,1}(Lab) > 7$ GeV
  - $E_{T}^{jet,2}(Lab) > 6$ GeV
  - $20^\circ < \theta^{jet}(Lab) < 160^\circ$

  in photoproduction ($Q^2 < 0.01$ GeV$^2$) with $0.3 < y < 0.65$

- Measurements in two regions of $x_\gamma$:
  - $x_\gamma < 0.75$ (resolved) and $x_\gamma > 0.75$ (direct)

- Comparison with model predictions for resolved and direct processes including initial- and final-state QCD radiation (PYTHIA): good description of the data

- The jets in the region $x_\gamma < 0.75$ are broader than in the region $x_\gamma > 0.75$, consistent with a larger fraction of gluon (broad) jets in resolved-photon events than in direct-photon events
Jet Substructure in Photoproduction

- The dependence of $\langle \psi(r = 0.5) \rangle$ on $\eta^{\text{jet}}, E_T^{\text{jet}}, E^{\text{jet}}$ and $x_\gamma$:
  - the jet becomes broader as $\eta^{\text{jet}}$ increases (proton direction)
  - the jet becomes narrower as $E_T^{\text{jet}}$ or $x_\gamma$ increases (direct processes)
- The measured dependencies of $\langle \psi(r = 0.5) \rangle$ on the different variables are well described by the predictions of PYTHIA
- The observed decrease in $\langle \psi(r = 0.5) \rangle$ as $\eta^{\text{jet}}$ increases is consistent with the increase in the fraction of gluon jets as predicted by PYTHIA
- Dominant hard subprocesses:
  - $\gamma g \rightarrow q\bar{q}$ in direct processes
  - $q_\gamma g_p \rightarrow qg$ in resolved processes
  - with a large and increasing fraction of gluon jets as $\eta^{\text{jet}}$ increases (\hat{t}-pole)
Jet Substructure in Photoproduction: Charm jets

- Investigation of the different processes contributing to charm photoproduction by selecting dijet events with one jet tagged as the charmed jet and measuring the jet shape of the “other jet” in the event.
- At high $x_\gamma$ the major process contributing to the sample is boson-gluon fusion ⇒ the “other jet” in an event is also a charm quark.
- At low $x_\gamma$ there are several contributing processes:
  - gluon-gluon fusion; the “other jet” is a charm quark
  - charm-excitation processes; the “other jet” is a gluon or a quark.
Sample of dijet events with a $\mu$-tagged jet: measurement of $\langle \Psi(r) \rangle$ for the "other" jet (purity of the tagged jet: 71 – 73%; the background is statistically subtracted)

The predictions of PYTHIA (including charm-excitation) describes well the data in the region $x_\gamma > 0.75$; differences are observed in the region $x_\gamma < 0.75$

$\Rightarrow$ the data suggest a smaller fraction of gluon jets at low $x_\gamma$ than predicted by PYTHIA
Comparing the measurements of $\langle \Psi(r) \rangle$ between inclusive dijet events and the “other jet” in $\mu$-tagged dijet events (“charm events”)
Jet Substructure in Neutral Current Deep Inelastic Scattering

- The lowest non-trivial-order contribution to the measurements is given by $\mathcal{O}(\alpha \alpha_s)$ pQCD calculations

\[
\langle 1 - \Psi(r) \rangle = \int \frac{dE_T}{E_T^{\text{jet}}} \frac{E_T^{\text{jet}} \sigma_{\text{jet}}(E_T^{\text{jet}})}{d\sigma(\text{ep} \rightarrow \text{2 partons})/dE_T}
\]

- NLO QCD calculations of jet substructure can be made in the laboratory frame since it is possible to have 3 partons in the same jet (not possible in the Breit frame)

- The dependence of the pQCD calculations on the knowledge of the proton PDFs is reduced

- Measurements of jet substructure in NC DIS provide:
  - a stringent test of pQCD calculations beyond LO
  - a determination of $\alpha_s$
Jet Substructure in Neutral Current Deep Inelastic Scattering

- Measurement of $\langle \Psi(r) \rangle$ for an inclusive sample of jets with $E_T^{\text{jet}}(\text{Lab}) > 17$ GeV and $-1 < \eta^{\text{jet}}(\text{Lab}) < 2.5$ in NC DIS at $Q^2 > 125$ GeV$^2$

- Study of the $E_T^{\text{jet}}$-dependence of $\langle \Psi(r) \rangle$: the jets become narrower as $E_T^{\text{jet}}$ increases

- Comparison to NLO QCD calculations corrected for hadronisation effects ($< 5\%$ for $E_T^{\text{jet}} > 21$ GeV at $r = 0.5$)

- NLO QCD calculations provide a good description of the data: $\rightarrow (\text{DATA-NLO})/\text{NLO}$ smaller than $0.2\%$ for $r = 0.5$
Jet Substructure in Neutral Current DIS and extraction of $\alpha_s(M_Z)$

- The measurements of $\langle \Psi(r = 0.5) \rangle$ for $E_T^{jet}$ (Lab) > 21 GeV have been used to extract the value of $\alpha_s(M_Z)$:

  $$\alpha_s(M_Z) = 0.1176 \pm 0.0009 \text{ (stat.)} \quad +0.0009 -0.0026 \text{ (exp.)} +0.0091 -0.0072 \text{ (th.)}$$

- Small experimental uncertainties: $\Delta \alpha_s / \alpha_s = \frac{+0.8}{-2.2} \%$

- The theoretical uncertainties dominate:
  - terms beyond NLO $\Delta \alpha_s(M_Z) = \frac{+0.0089}{-0.0070}$
  - parton-to-hadron effects $\Delta \alpha_s(M_Z) = \pm 0.0018$
  - uncertainty due to those of the proton PDFs (negligible)

Improvements depend upon further Theoretical Work
Summary of $\alpha_s$ determinations

- Wealth of determinations of $\alpha_s$ at HERA from a variety of observables:
  - NLO QCD analyses of structure functions
  - Inclusive jet production in NC DIS
  - Dijet production in NC DIS
  - Tri-jet/Dijet rate in NC DIS
  - Jet substructure in NC DIS
  - Jet substructure in CC DIS
  - Inclusive jet photoproduction

- Theoretical uncertainties are dominant
  - Biggest contrib. from terms beyond NLO

- Average of HERA determinations
  $$\alpha_s(M_Z) = 0.1186 \pm 0.0011\text{(exp.)} \pm 0.0050\text{(th.)}$$

- Consistent with world average (Bethke, 2004):
  $$\alpha_s(M_Z) = 0.1182 \pm 0.0027\text{ (only NNLO results)}$$
The running of $\alpha_s$ from HERA data alone

**HERA**

$\alpha_s$ vs. $\mu$ (GeV)

- ZEUS (inclusive jet $\gamma p - \mu = E_T^{jet}$)
- ZEUS (dijet DIS - $\mu = Q$)
- ZEUS (inclusive jet DIS - $\mu = E_T^{jet}$)
- H1 (inclusive jet DIS - $\mu = E_T^{jet}$)

**Determinations of $\alpha_s(\mu)$:**
- Dijet NC DIS ($\mu = Q$)
- Inclusive jet NC DIS ($\mu = E_T^{jet}$)
- Inclusive jet $\gamma p$ ($\mu = E_T^{jet}$)

**Combination of $\alpha_s(E_T^{jet})$ determinations at similar energy scales**

Observation of the running of $\alpha_s$ from HERA jet data alone

- Consistent with the running predicted by QCD over a large range in $E_T^{jet}$
Summary and Outlook (HERA II)

- HERA I has made possible precise measurements of jet cross sections and jet substructure in neutral current deep inelastic $ep$ scattering and photoproduction
- These measurements have provided
  - tests of perturbative QCD beyond LO
  - precise determinations of $\alpha_s$
  - improved determination of the gluon density in $p$
  - tests of the partonic structure of the photon
- In many areas the measurements have reached a level of precision such that the theoretical uncertainties dominate in the accuracy of the final results
- To further improve the accuracy of the determination of the fundamental parameters of QCD and of the tests at higher orders, and to fully exploit the HERA II programme, further theoretical work will be useful