HERA and the LHC

A Workshop on the implications on HERA for the LHC physics



15th International Topical Conference on Hadron Collider Physics, HCP2004



Michigan State University, June, 14-18, 2004

Uta Stösslein (DESY Hamburg)

From HERA to LHC ...





Experimental Determination of α_{c}

in all reactions which contain quark-gluon vertices









- e+e-annihilation
- total hadronic cross section hadronic decay width of Z bosons and τ leptons
 - jet rates and event shapes observables
- deep inelastic lepton-nucleon-scattering
 - scaling violations of structure functions
 - sum rules of structure functions
 - jet rates and event shapes observables
- proton-(anti-)proton collisions
 - jet rates
 - photoproduction
 - inklusive production of b-guarks
- heavy guarkonia decays

 $\alpha_{\rm S}$ least known of all couplings!

ep and pp Colliders





x = momentum fraction of quark in proton

$$\sigma = \sum_{a,b} \int dx_a \, dx_b \, f_a \, (x_a, Q^2) \, f_b \, (x_b, Q^2) \, \hat{\sigma}_{ab} \, (x_a, x_b)$$

ep collisions (HERA)

- Ideal tool to study the structure of hadrons via deep inelastic scattering
- → structure functions/parton densities
- Can use the photon as a point-like or hadronic particle through its virtuality

• Main contributions are in the area of QCD: Small-x, diffraction, saturation, high densities, jets...

Tests of new approaches/QCD

pp collisions (LHC)

- Highest energies reachable, can reach highest masses for new particles production
- Precision often limited by knowledge of quark/gluon structure of proton
- QCD effects need to be controlled to the best of our knowledge

A workshop on the implications of MERA for LNC abysics

March 2004 - January 2005

Parton density functions

Multijet final states and energy flow Heavy guarks

Diffraction Monte Carlo tools Startup Meeting March 26-27 2004 Midterm Meeting 11-13 October 2004 CERN,Geneva Final Meeting January 2005

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Hamburg

 Barters Membergi, M. Oyda Beyra (CERN), Anna (CERN), A Randon Minne, Generical (CERN), B. Bydalows (CERN), More (CERN), R. Kalance (DEN), More (CERN), R. Kalance (DEN), Scher (CERN), R. Kalance (DEN), Scher (CERN), R. Scherker (DEN), Schereige (DEN), J. Scherker (DEN), Schereige (DEN), J. Scherker (DEN), Schereige (DEN), J. Scherker (DEN), Schereige (DEN), S. Scherker (DEN), Scherker Scherker (DEN)

heralhe.workshop@cern.ch

 identify and quantify measurements to be made at HERA which have impact on LHC physics reach

 encourage and stimulate knowledge transfer between HERA and LHC communities

 examine and improve theoretical and experimental tools related to the goals

• 5 working groups with conveners from LHC and HERA experiments and theory

framework:
 startup at CERN,
 March, 26-27, 2004
 ...working group meetings...
 DESY June, 1-4, 2004
 midterm at CERN, Oct 11-13, 2004
 finish at DESY, Jan, 17-21, 2005

Workshop Goals \iff Working Groups

Varton Density Functions (PDFs)

gluons and quarks at high and low x from HERA fits un-integrated PDFs (k_t factorization) parton luminosities and precision cross section measurements at LHC

Multi-jet final states and energy flows

underlying event and minimum bias rapidity gaps and survival probabilities multi-jet topologies and multi-scale QCD parton shower/ME matching

Weavy Quark (HQ) production: charm, beauty, quarkonia

production cross sections fragmentation charm and beauty in the proton distributions

Working Groups cont'd

biffraction

diffractive PDFs and (non)factorization rapidity gaps and physics with forward proton tagging forward physics and low-x dynamics heavy ions and QCD at high parton density

Monte Carlo tools

general Monte Carlos (Pythia, Herwig, Phojet) new Monte Carlos with k_t factorization NLO calculation (MC@NLO, NLOLIB...) MC validation & tuning (JetWeb, HZTOOL)

illustrate workshop goals with my personal selected topics

Outline

- bigh precision PDF fits
- parton densities at high x
- Higgs and beauty
- >w multi-particle interactions and underlying events
- >> phenomena at low x
- **Higgs and diffraction** (talk by B.Cox)
- > concluding remarks

→many thanks to the speakers/conveners for delivering the work and material, see also at <u>www.desy.de/~heralhc</u> for further details

Structure of the Proton : PDFs from HERA



Parton Distributions at LHC

- interpretation of SM and BSM cross sections requires precision PDFs: $\delta\sigma_{th} = \delta\sigma_{pdf} + ...$
- `standard candle' processes

associate production with W/Z: $q\bar{q} \rightarrow VH$ massive vector boson fusion : $qq \rightarrow Hqq$ the gluon gluon fusion mechanism : $gg \rightarrow H$ associate production with top quarks : $gg, q\bar{q} \rightarrow t\bar{t}H$

- ➔ Higgs/new physics discovery requires to know PDFs
- measure luminosity to 1% using PDFs via Z and W production?
 - → How well do we know the current PDF uncertainties really? How well do we know α_s ?
- What we may learn more about PDFs from LHC measurements (e.g. high- E_T jets \rightarrow gluon, W⁺/W⁻ \rightarrow sea quarks)?



What HERA DIS Data taught us

DIS 1993

... and now



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Х

Gluon from HERA

→ large PDF uncertainties have potentially serious consequences for the determination of production rates at LHC



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high x and low x partons are correlated by momentum sum rule

for gluon: low-to-mid x: determined by F_2 scaling violations \rightarrow HERA II further constrained by F_L mid-to-high x: better constraints possible by \rightarrow Tevatron Run II data may help here, but by how much ?

→ include HERA jet data [H1 Collab., C. Adloff et al.,

EPJ C19 (2001) 289] 14

[M.Klein, DESY-WS-04, R.Wallny, PhDThesis01]

Correlation of α_s and $xg=ax^b(1-x)^c$



The accurate determination of α_s will profit from the longitudinal structure function $F_L(x)$ at low x and from adding jets and more precise DIS data at mid-to-high x \rightarrow all will provide additional constraints on the gluon distribution

[E.Lobodzinska, DESY-WS-04]

F_L from reduced inclusive DIS cross section



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F_L LO, NLO, NNLO and resummed - Simulation of Low E_p H1 Data



[R.Thorne, DIS04-WS, M.Klein, DESY-WS-04]

accurate F_L data at low x and Q^2 are required to test HO QCD and pin down xg(x,Q²) and α_s

such a measurement is challenging but possible at HERA via Rosenbluth separation using protons at e.g. 400, 465, 575 in add. to 920 GeV

it delivers also data at large x, medium Q2 besides measuring the W,E dependence of various cross sections, e.g. for vector mesons



Charged Current Cross Sections

unfolding of parton distributions using CC and NC cross sections ... important! but difficult to reach very large x>0.7



[M.Klein, DESY-WS-04] Access of Large x at Lower Q^2 coverage of F_2 data Q^{2}/GeV^{2} using E_{P} =460 GeV HERA Experiments: H1 1994-2000 mm ZEUS 1994-2000 Fixed Target Experiments: \sum NMC H1 simulation 460 GeV*27.5 GeV L=30 pb⁻¹ 10^{3} $\overline{}$ BCDMS 10 E665 x=0.07 F_2 SLAC 10^{2} x=0.10 x=0.14 10 x=0.18 x=0.23 1 1 x=0.25 x=0.35 10 $10^{\overline{-3}}$ 10^{-5} 10^{-2} $10^{-\bar{6}}$ x=0.45 10 Х x=0.55 extend measurements to lowest y **BCDMS data** x=0.65 with 0.1 10 100 1000 Simulation of resonance region (SOPHIA) O^2/GeV^2 v=0.3 • Low noise calorimetry (upgraded electr.)

Upgraded Forward tracking

at 280 GeV

5

[A.M.Cooper-Sarkar, C.Gwenlan, DESY-WS-04]

Add Inclusive DIS Jet Data

INCLUSIVE JET DEEP INELASTIC SCATTERING (ZEUS Coll., Phys. Lett. B547 (2002) 164)

We have chosen to use only the 6 cross sections differential in E_T^B in bins of Q^2 . This avoids correlations between cross sections with same events

30 NEW DATA POINTS -



Precision jet data from HERA !

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[A.M.Cooper-Sarkar, C.Gwenlan, DESY-WS-04]

Add Photoproduction Dijet Data

TWO-JET PHOTOPRODUCTION AT HIGH-ET (ZEUS Coll., Eur. Phys. J C23 (2002) 4)

We have chosen only the 6 cross sections at high x_{γ} (to avoid complications from uncertainty in photon structure)

38 NEW DATA POINTS —





[A.M.Cooper-Sarkar, C.Gwenlan, DESY-WS-04]

Extrapolation to LHC Energies



- Uncertainty in high-x (> 0.1) gluon can still be large, even at LHC scales
 → dominant uncertainty in production rates for many processes at LHC
- Addition of HERA jet data provides visible improvement even at LHC energies

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[simulation:M.Botje,C.Pauscaud, M.Klein,HERA-WS-97]

HERA-II, now starting



High precision at large Q², precise determination of the gluon distribution, flavor separation (via CC), u(x)/d(x) for $x \rightarrow 1,...$

\rightarrow but what will be the status of the theoretical uncertainties?

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[S.Moch, DESY-WS-04] [Moch,Vermaseren,Vogt hep-ph/0403192 & 0404111]

 $P_{0}^{(2)}(i) = 10 C_{0} C_{0} n_{f} \left(\frac{1}{2} + i^{2} \right) \left| \frac{11}{2} R_{1,1,1} - \frac{21}{2} R_{1} + \frac{1}{2} R_{1,1,1} - R_{-1,-1,1} - 2R_{-1,1,1} \right|$ $- \Pi_{-\frac{1}{2} \frac{1}{2}} \Big| + \frac{1}{8} (\frac{1}{2} - \pi^2) \Big| \frac{10}{3} \xi_2 + \Pi_{\frac{1}{2} \frac{1}{2}} + 9 \xi_1 + \frac{9}{4} H_{\frac{1}{2} \frac{1}{2}} - \frac{4541}{216} + \frac{271}{21} H_{\frac{1}{2}} + \frac{11}{2} \Pi_{\frac{1}{2} \frac{1}{2}} + 10 \xi_1 + \frac{1}{2} H_{\frac{1}{2} \frac{1}{2}} - \frac{1}{2} H_{\frac{1}{2} \frac{1}{2}} - \frac{1}{2} H_{\frac{1}{2} \frac{1}{2}} + \frac{1}{2} H_{\frac{1}{2} \frac{1}{2} + \frac{1}{2} H_{\frac{1}{2} \frac{1}{2}} + \frac{1}{$ $-984_{1,01} + 286_{1,1,0} + 284_{1,1,1} + (1 - a)(\frac{181}{2}86_1 + \frac{116}{2} + \frac{287}{24}84_{1,0} - \frac{71}{2}84_{-1,0} + 384_{10,0,0}$ $+\frac{11}{6}$ (1) + 5a4(1) + 11.10 + 11.12 + 201.1, (1 + 101.1)(1 + $\frac{1}{2}$ (1)(2) + $\frac{1}{6}$ (1)(2) - $\frac{9}{2}$ (1)(2)
$$\begin{split} & \frac{1}{4} \mathcal{U}_{1,1} + \mathcal{U}_{1,1,2} + \mathcal{U}_{1,1,1} + \left(1 + e_1^2 \frac{1}{12} a_1 b_{1,1}^2 + \frac{11}{4} b_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} - \frac{11}{12} b_{1,1} \\ & + \frac{1}{2} \mathcal{U}_{1,1} + \frac{1}{14} b_{1,1} + \mathcal{U}_{1,1,1} + \frac{11}{4} a_1 b_{1,1} - \frac{11}{12} a_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} \\ & + \frac{1}{2} \mathcal{U}_{1,1} + \frac{11}{14} b_{1,1} + \frac{11}{14} a_1 b_{1,1} - \frac{11}{12} a_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} + \frac{11}{12} b_{1,1} \\ & + \frac{1}{2} \mathcal{U}_{1,1} + \frac{11}{14} b_{1,1} + \frac{11}{14} a_{1,1} + \frac{11}{14} a_{1,1} + \frac{11}{14} a_{1,1} + \frac{11}{14} b_{1,1} \\ & + \frac{1}{14} \mathcal{U}_{1,1} + \frac{11}{14} a_{1,1} \\ & + \frac{1}{14} \mathcal{U}_{1,1} + \frac{11}{14} a_{1,1} \\ & + \frac{1}{14} a_{1,1} \\ & + \frac{1}{14} a_{1,1} \\ & + \frac{1}{14} a_{1,1} \\ & + \frac{1}{14} a_{1,1} \\ & + \frac{1}{14} a_{1,1} + \frac{1}$$
 $+\frac{1}{2}R_{-1}(j)-2R_{-1}j+1f_{2}^{2}-\frac{1}{2}H_{1}(j+R_{-1,-1}j+2R_{1}j,j+H_{2})-\frac{1}{2}H_{1}\Big]+2R_{-1}j+R_{1}j$ $+H_{1,0}(1) - \frac{1}{1}\frac{1}{6}t^{2} + 4H_{-1,0} + 4H_{0,1} - \frac{32}{9}H_{0,1} - \frac{29}{12}H_{0} - \frac{293}{11}H_{0} - \frac{411}{11} - \frac{97}{11}H_{0} + \frac{19}{4}H_{1} - H_{0}$ $-\frac{11}{2}60(j_2-\frac{31}{2}(j_1-\frac{3}{2}0(j_1-100(j_1)+\frac{1}{2})^2)\frac{100}{2}60(j_1-\frac{200}{2}0(j_1-\frac{100}{2})+\frac{611}{2}+\frac{611}{2}+\frac{61}{2}0(j_1-\frac{3}{2}0)$ $-u_{4}^{i} - u_{3}^{i} + u_{5} + u_{5} - u_{5,1} + \frac{1}{2} + u_{5,2} - u_{5,1} + \frac{1}{2} + u_{5,2} + \frac{1}{2} + \frac{1$ $-\frac{19}{2}\theta_0\left[+\frac{3}{9}(\frac{1}{2}-s^2)\left[\theta_{11}+\frac{3}{9}\theta_1+\frac{2}{9}\right]+(1-s)\left[\frac{1}{9}\theta_{11}-\frac{3}{9}\theta_1+s\theta_1+\frac{19}{29}\theta_1+\frac{39}{21}\right]$ $+\frac{1}{2}(1+a)\left|\frac{4}{3}M_2 - \frac{6}{3}\xi_2 + \frac{5}{3}+M_{2,1} - 2M_2 + 2H_{2,2}\frac{2H}{3}M_{2,3} + M_{2,3,3}\right|^2 + 1.6 \Box_2^2 a_{2,1}\left(\frac{4D}{3}M_2 + 2M_2 + M_2 +$ $\frac{25}{4}64(1-16(10+\frac{141}{32}+\frac{14}{32}+\frac{11}{4}\frac{1}{4}\frac$ $\frac{4}{31} (1 - \frac{31}{3} 0_{1j}) - \frac{31}{6} - \frac{31}{14} 0_{1} + \frac{32}{16} 0_{1} - \frac{34}{16} 0_{1} + \frac{34}{16} 0_{1} - \frac{34}{16} 0_{1} 0_{1} + \frac{1}{16} 0_{1} + \frac{4}{16} 0_{1} - \frac{34}{16} 0_{1}$ $\frac{12}{12}H_{1/2}\left[+\frac{4}{3}(\frac{1}{2}-2)\left[\frac{12}{12}H_{1/2}-\frac{422}{124}H_{1}-H_{0}^{2}+\frac{44}{12}+\frac{1}{3}H_{1/2}+H_{1/2}-H_{1/2}-H_{1/2}\right]\right]$ $+(1-a)\left|\frac{1}{a}\Theta_{1,1,2}+\frac{2}{1a}\Theta_{1,1}-\frac{2241}{a}\Theta_{2}-\frac{44}{1a}\Theta_{1,2}-\frac{261}{a}\Theta_{1}-\frac{3}{a}\frac{1}{a}+\frac{5}{a}\Theta_{2}-\frac{5}{a}\Theta_{1,2}+3a\Theta_{1,2}\right|$ +984[2 - 181 - 181](1 - 181](1 - 181](1 - 181](1 - 181) $-34_{0,0} + 48_{10} \frac{1}{6} - 18_{1,0,0} + 84_{1,0} - 28_{0,1} - 28_{0,1} - 18_{0,0} - 8_{1,1} - 48_{0}$

$$\begin{split} \eta_{1}^{(0)}(t) &= 16 - \zeta_{1} \cos \left(\lambda_{2} \left(\sqrt{\frac{10}{2}} \pi \right) \zeta_{1} - 6 \delta_{1,1} + 18 \zeta_{1,0} - \frac{15}{2} \delta_{1,1} + \frac{2}{4} \delta_{1,1} + \frac{2}{4} \delta_{1,1} + 3 \delta_{1,0} + \frac{16}{4} \delta_{1,0} + \frac{$$

 $-484_{13} + \frac{49}{2}g_{2} \Big] + g_{2}(-\phi) \Big[\frac{12}{m} R_{-1}g_{1} - \frac{3}{2} R_{-1,-1}g - \frac{3}{m} R_{-1,2} - \frac{3}{m} R_{-1,2} + \frac{3}{2} R_{-2,2}g + \frac{3}{m} R_{-1,2}g - \frac{3}{m} R_{-1,2}g - \frac{3}{2} R_{-1,2}g$ $-24_{0,1}-24_{0}-48_{-0,1}+84_{-1,-1,0}-48_{-2,0,0}+28_{0,0}\xi_{2}+88_{-2,0,0}+28_{-1,0,0}-28_{-1,0,1}$ $-mt_{-1,0}\left[+\left|\frac{1}{2}-i^{2}\right|\frac{16}{12}-i\frac{1}{2}\left[+\frac{27}{12}m_{1,0}-\frac{4}{2}m_{1,0}\right]+\left|\frac{1}{2}+i^{2}\right|\frac{7}{12}m_{1,0,0}-\frac{271}{120}m_{1}+\frac{24}{12}m_{1,0}\right]$ $-\frac{3}{2}$ (6,1) $\left[+(1-a)\right]$ (6(1), b + (16(1)), $-\frac{5}{2}$ (6(1), 1 - 260, (3 - 261) + 9000 $\frac{1}{2}$ - (16) $\frac{1}{2}$ - $\frac{26}{2}$ $+6t_{1,12} + \frac{114}{3} a_{1} a_{1}^{2} + \frac{319}{34} b_{1,1} + \frac{111}{32} a_{2}^{2} + \frac{401}{36} b_{1} - \frac{5}{3} b_{1} b_{2} + \frac{40}{36} b_{1,12} - \frac{39}{37} b_{1,2} - \frac{11}{37} b_{1,1}$ $\frac{1100}{100}H_1 - \frac{d1}{2}H_{2,1} - 20H_{2,1} - \frac{047}{2M}_{2,2} - \frac{d1}{2}H_{2,1} - \frac{112}{3}H_2 + \frac{213}{32} - \frac{1000}{48}H_0 + 2H_{-2,1,2}$ $+(1+i)\Big(H_{-1}(0) - 10H_{-1}(1+i)H_{-1}(0) + 21H_{0}(1-i)H_{-1}(1-i)H_{-1}(1-i)H_{-2}($ $-40L_{1}L_{1}U - 4001 - 400(3 - 400(3)0) + \frac{37}{2}0L_{1}U + \frac{4}{2}(1 + 400L_{1}L_{1}) - 40L_{1}U + 300(3)$ $+H_{2}\tilde{g}_{2}-3H_{1,1,2}+2H_{20,10}+H_{-2,1}-3H_{2,1,1}-\frac{3}{2}H_{1,1,1}+\frac{11}{2}H_{1,1,1}+\frac{11}{2}H_{1,1,1}+\frac{9}{2}H_{1,1,1}+\frac{9}{2}H_{1,1,1}$ $\frac{11}{4} R_{1} \frac{1}{26} + 3 R_{1} \frac{1}{26} + \frac{5}{4} R_{-1,-1,0} + \frac{3}{4} R_{-1,1} + \frac{9}{4} R_{-1,0} + \frac{13}{4} R_{-2,0} - \frac{11}{12} R_{1} \frac{1}{26} - \frac{1128}{12} R_{1} \frac{1}{26} + \frac{11}{12} R_{1} \frac{1}{12} + \frac{11}{12} + \frac{11}{12}$ $\frac{117}{12}g_{12} - \frac{39}{4}g_{2}^{-1} - \frac{340}{18}\theta_{1} - \frac{11}{4}\theta_{1}g_{1}^{-1} - \frac{7}{5}\theta_{1}g_{1}g_{1} + \frac{141}{24}\theta_{1}g_{1} + \frac{141}{26}\theta_{1}g_{1} + \frac{141}{26}\theta_{1}g_{1} + \frac{141}{12}\theta_{1}g_{1} + \frac{1$ $\frac{15}{4}62(3 + \frac{170}{9}\frac{1}{10} + \frac{10}{4}62(3 + \frac{425}{12}61 + \frac{3404}{10} + \frac{3409}{10}61 - 36\left[461(3 + 461) - 461.21\right]$ $+315_{\mu}\sqrt{(\frac{1}{2}\mu_{\mu})}(1)[11]_{2}-10]_{1}(1-10]_{1}(1-10]_{1}(1-10]_{1}) - \frac{310}{24}(0)+\frac{3}{2}(0)[1+\frac{1}{2}]_{1}(1+\frac{1}{2})$ $\frac{21}{10}R_{1} + \frac{21}{10}R_{10} - \zeta_{1} + \frac{21}{10}\zeta_{2} - \frac{100}{100} + \frac{1}{10}r_{10}\zeta - a[R_{-1,0,0} - \frac{40}{100}] - \frac{2}{10} - a^{2}] - \frac{2}{9}(1 - a[RR_{0,0,0} - \frac{1}{100}] - \frac{2}{10}] - \frac{2}{9}(1 - a[RR_{0,0,0} - \frac{1}{100}] - \frac{2}{10}] - \frac{2}{9}(1 - a[RR_{0,0,0} - \frac{1}{100}] - \frac{2}{100}] - \frac{2}{10}(1 - a[RR_{0,0,0} - \frac{1}{100}] - \frac{2}{10}] - \frac{2}{10}(1 - a[RR_{0,0,0} - \frac{1}{100}] - \frac{2}{10}(1 - a[RR_{0,0,0} - \frac{1}{10}] - \frac{2}{10}(1 - a[RR_{0,0,0} - \frac{1}{10}] - \frac{2}{10}(1$ $\frac{1}{2}dt_1 - H_{1/2} + \frac{2}{2}dt_{1/2} \Big] + \frac{1}{2}d(1 + i)H_{-1/2} + \frac{2}{2}H_0 - \frac{14}{24}H_1 + H_{0/2} + \frac{4}{2}H_{1/2} + \frac{4}{2}H_{-1/2}$ $\frac{35}{2110} + 3632^2 \sqrt{|\rho_{0}|} \phi \left[0011 + \frac{31}{2}0110 - \frac{13}{2}011 + \frac{2}{5}(t^2 - \frac{31}{20}011) + \frac{31}{10}01 - \frac{31}{2}01(t - \frac{31}{2}01) + \frac{31}{2}01(t$ $\frac{1}{12} (6\xi_1) - \frac{41}{27} (\xi_1) - \frac{21}{12} (\xi_1) - \frac{110}{4} (\xi_1 + \frac{21}{34} 6\xi_1 - \frac{2167}{27} 6\xi_1 + \frac{347}{7} - \frac{21}{3} 6\xi_{-1} (\xi_1 + 136 - 16) (\xi_1$ $\frac{325}{48}H_{11} - \frac{26}{7}H_{-11} - \frac{3}{7}H_{-\frac{1}{2}} - \frac{7}{7}H_{1\frac{1}{2}} - 3H_{1\frac{1}{2}} - \frac{31}{12}H_{\frac{1}{2}} - \frac{311}{12}H_{\frac{1}{2}} + \frac{311}{7}H_{\frac{1}{2}} + \frac{3}{7}H_{\frac{1}{2}} + \frac{314}{48}H_{\frac{1}{2}} + \frac{314}{4$ +10.11 - 383.01 - 130. 23 - 100.021 + 1900.0 - 101.021 - 10.101 - 100.110 + 200.11.1 $-281(12 - 281(12) + p_{II}) = a \left[(0.1, .12) - 281(12 - 481, 1, .13) + 83(11 + 281, .22) - 81(13) \right]$ $+\frac{127}{245}$ (6.1,6 - H - $\frac{1}{12}$ - 2 H - $\frac{1}{2}$ (6.3 $\frac{1}{2}$ - H - $\frac{1}{2}$ (6.1,2) + 2 (6.1,-1) (6 + 2 (6.1,-1)) - $\frac{5}{2}$ (6.1,0) (7) $+661_{-1}(-1,-1)(1-266_{-1}(1)+266_{-1}(1)) + |\frac{1}{2} - a^2| (\frac{2}{2}60_{1}(1+\frac{12}{2}\frac{1}{4})-266_{1}(1+\frac{3}{2}60_{1}(1+\frac{3}{2}60_{1})) - \frac{30}{2}60_{1})$ $-\frac{3}{7}\Theta_{-1,0,0}+\frac{3}{7}\Theta_{0,0}+d\xi_0+\frac{141}{16}\Theta_1-\frac{2141}{168}\Big]+\frac{3}{7}\frac{1}{16}+a^2\Big[\frac{21}{7}\pi_{-1,0}-\frac{21}{6}\Theta_0-2\Theta_{-1,0,0}\Big]$

 $-2R_{-1,2}+R_{1,0,1}+R_{-1,0,2}+\frac{32}{2}R_{1}+R_{0,1,1}\Big|+(1-\varepsilon)\Big|12R_{0,1,0,1}-2R_{1,0,1}-\frac{42}{2}R_{1,0,1}+\frac{12}{2}R_{1,0,1}\Big|\\$ $\frac{3}{2}$ (6) + $\frac{5}{2}$ (6) (2) + (1) (3) - $\frac{11}{2}$ (1) + $\frac{13}{12}$ (1) - $\frac{531}{20}$ (1² - $\frac{29}{2}$ (1) (3) - $\frac{118}{2}$ (1) + $\frac{35001}{20}$ (1) $\frac{1200}{100} + \frac{345}{6} 6L_{-}[(1 + \frac{13}{2})6][(1 + 1006]] + \frac{81}{12} 6L_{1}] + \frac{31}{2} 6L_{2}[1 - \frac{487}{34} \frac{1}{6}] + \frac{29}{4} 6L_{2}^{2}] - \frac{100}{12} 6L_{1}^{2}]$ $\frac{100}{4} \Theta_{1,1,1} - \frac{10}{13} B_{1,1}^{1} + \frac{1225}{20} \Theta_{1} - \frac{41}{8} \Theta_{0,1,2} - \frac{10}{16} B_{1,1}^{1} + (1+\epsilon) \left| \frac{10}{10} \Theta_{1,1,2} - 4 \Theta_{-1,1} \right|$ $+ 78_{-1,-1,0} - \frac{11}{2} 8_{1,1,1} - 184_{-2} \frac{1}{4} 1 - 128_{-1,1,0} + \frac{1}{2} 8_{-1,0} + \frac{10}{2} 8_{-1,1,0} \frac{1}{2} + 88_{0,1} - 328_{-2,-1,0}$ +3000+4810-R-0+3800-8000+3800+480-0-8003-3800 $-TIH(j)(1-2HL)+\frac{2h}{4}H(j,1+\frac{2h}{3}H,2\frac{2}{3}2+\frac{2H}{3}H,2(j,1+\frac{11}{3}H,2$ $+\frac{116}{116}e^{-\frac{11}{2}}e^{-\frac{11}{12}}e^{-\frac{11}{12}}e^{-\frac{1}{12}}e^$ $\frac{10}{2} M_{1,10} - \frac{211}{20} M_{1,1} - \frac{1}{2} K_{-2,1} + \frac{10}{20} K_{2} + \frac{1}{2} K_{1/2} + \frac{11}{20} M_{1,11} + \frac{12}{10} M_{1/2} + \frac{11}{20} M_{1/2} + \frac{11}{2$ $+\frac{141}{12}601 + \frac{1211}{24}60(4) + 16 C_{20}\sqrt{2}(\frac{7}{2}60(1)4 + \frac{11}{16}60 - \frac{710}{660} + \frac{341}{24}60 + \frac{7}{260}0(4 + 201)(1)$ $-\frac{3}{n}(0, 1 - \frac{3}{n})(1 - \frac{5}{n}(0, 1 + \frac{5}{n}\frac{1}{4^2} + \frac{3}{n}p_{\frac{1}{2}}(1))\left[0.1 + \frac{81}{3} - \frac{35}{3}(0, 1 - \frac{31}{3}0, 0) + 0.1, (1 + 0.0), (1 - \frac{31}{3}0, 0)\right]$ $\begin{array}{c} & & & \\ -\zeta_{2}-2R_{1,10}+\frac{2}{9}\theta_{1} \left| +\frac{27}{12}(\frac{1}{9}-a^{2}) + (1-a) \right| \frac{1}{12}R_{1} - \frac{d4d1}{d22} - 4R_{0,1,10} - \frac{24}{9}\theta_{0,1,1} + \frac{2}{9}d\theta_{1} \right| \\ \end{array}$ $+\frac{1}{9}d\theta_2 + \frac{6}{8}d\theta_1(p - \frac{7}{9}d_2) - (1 + a)\left[\frac{1673}{1218}\theta_2 + \frac{256}{12}\theta_1(p)\right] + 16 \Box ^2 a_1 \left(p_{12}(a)\right) \left[7\theta_{12} + 7\theta_2(a)\right] + \frac{1}{12}\left(\frac{1}{12}\theta_1(p)\right) + \frac{1}{12}\left(\frac{1}{$ $-38L_1[l]-781J_0^2]+104(l+604(l+604(l+841)l+641)(l+104))+104(l+104)(l+\frac{5}{2}841)l$ $+\frac{44}{\pi}611 - \frac{811}{\pi}\frac{1}{3}1 + \frac{327}{\pi}611 + \frac{311}{\pi}61,2 + \frac{81}{\pi}61,1 + \frac{37}{\pi}61,6 - 2010(\frac{1}{2}2 + \frac{5}{\pi})61,0 + \frac{3}{\pi}60,1 - \frac{19}{\pi}\frac{1}{3}2$ $+\frac{31}{2\pi}+\frac{31}{\pi}6q-\frac{11}{\pi}6q_{2}-\frac{1}{\pi}6q_{3}q_{4}+\frac{31}{\pi}6q_{1}q_{4}+\frac{11}{\pi}6q_{1}q_{4}+\frac{11}{\pi}q_{4}^{2}+96q_{1,1}-16q_{4}^{2}-76q_{4}q_{4}$ +T2H(J-2H), 2()-7H(J\$1+9H(J)()-7H(J\$1+H0,J)()+H(J)()+2H(J)()+H0,J) $+481_{110}+481_{121}\Big)+4\mu_{10}(-a)\Big[81_{110}-81_{-10}+81_{-10}-81_{-10}+\frac{1}{2}81_{-1-10}-\frac{1}{2}81_{-10}-\frac{$

$$\begin{split} &- \log_{10}(q+\frac{1}{2}(t_{2},p+\frac{1}{2}(t_{1},p+\frac{1}{2}(t_{2},p+\frac{1}{2}(t_{1},$$

 $P_{\text{R}}^{(2)}(u) = 10 C_{0} C_{0} u / (\frac{3}{4}u^{2} \frac{120}{4} 01 - \frac{311}{4} + h_{0}^{2} 1 - 0...(1 - 101 + 00.) + \frac{310}{4} 00 - 00.)$ $+\frac{6}{4}\rho_{BV}\dot{\phi}$ $\left[$ $B(1+B(1+B(1+H)) + \frac{987}{120} + \frac{271}{80}60 - \frac{28}{10}B(1-3\frac{1}{4}) - \frac{3}{4}B(\frac{1}{4}) - \frac{1}{2}B(\frac{1}{4}) - \frac{4}{2}B(1+B(1)) + \frac{1}{2}B(\frac{1}{4}) + \frac{1}{2}$ $\frac{2}{8}R_{1,01} + R_{1,1,1} + \frac{2}{9}R_{0,1} + \frac{2}{9}R_{1,1} | -i \rangle \left[2R_{-1}i_{1} + \frac{2}{9}i_{2} + \frac{2}{12}R_{-1,0} - \frac{211}{92}R_{0} + \frac{1}{9}R_{-2,0}\right]$ $\frac{5}{4}R_2 + 2R_{-1,-1,0} - R_{-1,0,0} - R_{-1,0} + \frac{2}{4}(1-a)\left[R_{-1,0} + 3k_0 - R_0\right] + (1+a)\left[\frac{129}{1100}R_1\right]$ $\frac{\delta}{2}[2 + \frac{21}{3}R_{-1}] = \frac{\delta}{24}[0] = \frac{242}{34}[0] = \frac{1}{4}[0] = \frac{1}{4}[0] = \frac{1}{4}[0] = \frac{1}{22} + \frac{1}{4}[0] = \frac{1}{2}[R_{-1}] + 10[$ $\frac{1}{2} 61(1 + \frac{217}{27}661 - \frac{90}{27}66(1 - 81)) - \frac{2}{7}86\frac{1}{27} + \frac{20}{3}61.2(1 + 9\frac{1}{27} + 286) + \frac{2}{3}66(1)) + \kappa \frac{11}{7}\frac{1}{4}$ $\frac{\lambda_{22}}{\lambda_{22}} = \frac{29}{42} \hat{u}_{2} + \frac{271}{4} \hat{u}_{3} - \frac{\Lambda}{2} \hat{u}_{1,3} \Big] + 34 c_{2} q_{1}^{-2} \Big[e^{2} \Big] \frac{1}{2} + \frac{129}{42} \hat{u}_{1} - 2 \hat{u}_{2} - \frac{2}{2} \hat{u}_{1,1} - \frac{34}{2} \hat{u$ $+\frac{4400}{100}$ = 1 + $\frac{1}{2}$ = 0, 1 + $\frac{82}{2}$ = 0, 1 + 481 + $\frac{8}{2}$ = 0, 1 + 410, (1 + 200, (1 - 822) + $\frac{41}{2}$ = 0, 1 + 10, 2 $+\frac{18}{112}81(3+381(3-381(1)-\frac{3}{2}81.1\frac{1}{6})-\frac{19}{12}81\frac{1}{6}1+188\frac{1}{6}2(1+88\beta\frac{1}{6})+\frac{3}{2}81\beta(3-381)\frac{1}{6})$ $+\frac{1}{12}H_{114}+M_{0,116}-M_{0,116}-4H_{111}-H_{111}-2H_{111}+H_{116}+\mu_0|-a||H_{-14}|$ $+ H_{-1,0} \dot{g}_{0} + \frac{5}{2} H_{-1,0,0} + \frac{17}{12} \dot{g}_{1}^{-2} - 1 H_{-1,-1,0} - \frac{11}{2} H_{-1} \dot{g}_{1} - 1 H_{-1,-1,0} - \frac{5}{2} H_{-1,0,0,0} - 1 H_{-1,1}$ +384.1, 141 - 884.1, 103 - 384.1, 10 + 488.4, 1, 13 + 284.101 + (1 - 6) 8841 - 1011 $+\frac{22}{10}$ H(1) $-\frac{25}{100}$ H(1) $-\frac{4411}{344}$ H(1) $-\frac{32}{3}$ H(1)(1 - H-1)(1 - 240)(1 - 24 $\frac{2}{2}g_{0}^{2} - \frac{2}{2}\Theta_{11} + \frac{25}{2}\Theta_{22} - 4\Theta_{21} + |1 + a_{1}|\frac{49}{2}\Theta_{1} - \Theta_{-11} - \frac{21}{2}\Theta_{21} - \frac{1109}{2}\Theta_{1} - \frac{1109}{2}\Theta_{1}$ $\frac{411}{210} - \frac{111}{2}g_{2} - \frac{311}{21}H_{1,1} + \frac{1}{2}H_{1,1} - \frac{14}{9}H_{2} + \frac{3}{9}H_{1} - \frac{171}{4}H_{-1,1} - \frac{171}{4}H_{-1,1} - \frac{171}{4}H_{-1,1} + 7H_{-1,1}$ $+34t_{-1,-1,0}+\frac{3}{2}\theta_{1,0}+\frac{9}{2}\theta_{2,0,1}+i\theta_{0,0,0,0}\Big]-128t_{-1,0}-\frac{179}{29}\theta_{0}+\frac{2001}{100}\theta_{1,0}-\frac{18}{4}\theta_{0,0,0}$

 $-3\theta_{0,0} - \frac{11}{2}\theta_{0,0} - 11\theta_{-2,0} - \frac{11}{2}\theta_{0,1} + \frac{15}{2}\theta_0 - \frac{300}{44} + \frac{317}{2}\theta_0 + 3\theta_0 + \frac{319}{24}\theta_1 + \frac{31}{24}\theta_1 + \frac{31}{24}\theta_1$ $+\frac{3}{2}(0)+\frac{1}{2}(1)+\frac{27}{2}(1-1)+\frac{11}{2}(0,0)+3(1)(1-3(1))(1-3(1))+\frac{3}{2}(1-10)(1+\frac{3}{2}(0,0)+3(1-1))$ $+600,0+\frac{3}{2}60,0,1-8,122+70122+600,222+1200,2,10-600,1,0,0+o$ [0010,1-80022] $+\frac{9}{2}H_{-1,[1]} - \frac{35}{2}H_{1,[1]} + 2H_{2} + 3H_{1,[1]} + H_{-1,[2]}\Big| + 18C_{2}^{2}C_{2}e_{1}^{-1}\Big| \frac{1}{2}H_{2}\frac{1}{2} - \frac{2123}{2} - \frac{71}{12}H_{2}\frac{1}{2}$ $-iH_{2} + \frac{36}{2} - 228 \downarrow_{1} - \frac{34}{2} H_{23} - \frac{3}{2} H_{13} I_{2} - \frac{34}{2} H_{23} I_{2} + \frac{34}{2} H_{23} I_{3} + \frac{32}{2} H_{13} H_{13} + \frac{32}{2} H_{13} H_{13} + \frac{32}{2} H_{13} H_{13} +$ $+\frac{4}{\lambda}H_{-\frac{1}{2},-\frac{1}{2},-\frac{214}{8}}\frac{2}{\lambda}-\frac{X}{\lambda}H_{\frac{1}{2},+}+\frac{115}{12}H_{\frac{1}{2},+\frac{1}{2}}H_{\frac{1}{2},-\frac{1}{2}}+\frac{2}{2}H_{\frac{1}{2},-\frac{1}{2}}-\frac{2}{27}H_{\frac{1}{2},+\frac{1}{2}}+\frac{1}{2}H_{\frac{1}{2},-\frac{1}{2}}+\frac{1}{2}H_{\frac{1}{2}, +i34.1\beta+\frac{334}{21}60\left|+\rho_{30}(s)\right|\frac{2}{7}615^{2}_{4}+\frac{133155}{2160}-\frac{1}{3}62\beta+\frac{31}{4}61.4^{2}_{4}+281(1+\frac{11}{2}61)(61+\frac{11}{2}61)(1+\frac{11$ $+i001J - \frac{45}{4}H(J,J) - \frac{259}{23}Q^2 - \frac{17}{8}H(J - \frac{75}{24}H(J - \frac{11}{16}H,2)) - \frac{21}{3}Q^2 + \frac{5}{2}H(J(J) - H(J,2))$ $+\frac{1.0}{10}R_{1} - 22k_{1}g_{2}^{2} - 2k_{1}g_{2}^{2} - \frac{51}{12}R_{1,1,2} + 22k_{1,1,2} + 22k_{1,1,2} + 22k_{1,1,2} + 22k_{1,1,2} - \frac{51}{12}R_{1,2}$ $+60_{1,1,2} + 40_{1,1,1} + 60_{1,1,2} + 10_{1,2} + 10_{1,2} + p_{11} - \phi \left[\frac{13}{2}0_{-1}\phi_1^2 + 10_{-1}\phi_2^2 + 10_{-1,-1,2}\right]$ $+\frac{110}{12}R_{-1,0}+H_0\frac{1}{2}+\frac{17}{2}q^2+\frac{1}{4}H_0\frac{1}{2}+10q_{11}^2-\frac{46}{22}H_{11}-\frac{29}{2}R_{-1,-1,0}-1R_{1,0}-1R_{1,0}$ $-764.1(j) - \frac{3}{2}61.(j) + \frac{1379}{210}61 - 464.2(1 - \frac{49}{2}61.1(j) - \frac{71}{2}61.1(j)) - 118.1(11\frac{12}{2} - 104.1))$ -BU_L_L_H+12H_L_H+12H_L_H+2BL_H+2BL_H+14L_LH-2H_LH-2H_LH-2H_LH $+\frac{11}{2}64\frac{1}{2}l+\frac{1}{2}l$ $-30_{1}\frac{1}{20} + \frac{11}{20}\theta_{1,1}(1 + \frac{11}{20}\theta_{1,-1,1}(1 + \frac{211}{10})\theta_{1} - 30\theta_{0,1,0}] + |1 + a| \left[4\pi_{1,1} - \pi_{1,1,1} + \frac{29}{20}\theta_{-1,2}\right]$ $+\frac{17}{4}9\xi_{1,1}-119\xi_{2}-\frac{11}{12}9\xi_{2}+\frac{1}{2}9\xi_{3}(y-1)\xi_{2}^{2}+\frac{41}{12}9\xi_{3}(y-1)\theta_{3}\xi_{3}-\frac{11}{3}9\xi_{-1}\xi_{2}-\frac{54}{8}9\xi_{-1,-1}y$ $+\frac{27}{2}H_{0}+\frac{81}{2}H_{0}\dot{q}_{1}-\frac{26}{2}H_{11}-\frac{7}{2}H_{2}+\frac{28}{2}H_{01}-\frac{78}{2}H_{10}\dot{q}_{2}-\frac{27}{2}H_{1}\dot{q}_{1}^{2}+\frac{8111}{2100}-\frac{81}{2}H_{1}\dot{q}_{2}$ $-1218.4(1-1)8+\frac{9}{2}84.1(1)8-1384.1(1-\frac{9}{2}84.1)8+\frac{21}{2}84.1(1-\frac{1}{2}84.2)(1-2)84.0(1-\frac{2}{2}84.0(1-2)84.0\frac{1}{2})$ $-\frac{47}{44}g_{1}^{-2}+\frac{29}{4}g_{-11}-H_{-12}+3H_{-21}+2M_0g_{2}+\frac{413}{4}H_1+\frac{978}{6}H_2+\frac{1}{4}H_4-82H_2-12H_{02}$ $-\frac{41}{12}\xi_2 - \frac{1}{7}H_0\xi_2 + \frac{7}{71}H_0 + \frac{709}{12}H_1 + \frac{710}{12}\xi_2 + \frac{91}{71}H_{1/2} + \frac{91}{71}H_{1/2} - \frac{31}{71}H_{1/2} - \frac{11}{71}H_{1/2}$ $+\frac{48}{12}$ (6) $+\frac{89}{4}$ (6) -286 (1) $+\frac{21}{8}$ (6) -1.81 (6) -4.81 (6) -4.81 (1) $\Big) + 16.2 m_f^{-2} \Big(\frac{1}{n} - \frac{11}{m_e}\Big)$ $+\frac{2}{9}e^{-\frac{1}{3}}e^{-\frac{1}{3}}e^{-\frac{1}{3}}e_{+\frac{1}$

 $+\frac{1}{2} \sigma_{00} \phi \left[\left[H_{11} - H_{13} - H_{13} + H_{21} + \eta_{21}^{2} + \eta_{31}^{2} + \frac{31}{21} H_{1,1} + 2 H_{-1,1} - \frac{7}{21} H_{1} + 2 H_{13} g_{2} - \frac{1473}{21} + \frac{3}{2} H_{1,3} g_{3} \right] \right]$ $+281(\beta - \frac{3}{2}81(\beta)) + \frac{11}{10000}(-\alpha (\frac{10}{100}60 - \frac{1}{2} - 8.1\beta) + \frac{1}{2}(2 - \alpha (68000) - 81 - \frac{11001}{100})$ $-\frac{11}{2}\frac{1}{6}e^{-4H_{2,2}}-H_{2,3}-\frac{1}{2}\Theta_{1,3}-\frac{1}{2}\Theta_{2,3}+3\Theta_{0,3}-\frac{4H_{3}}{24}\Theta_{1,3}\Big|+(1+s)\Big|\Theta_{0,2}-\frac{11H_{3}}{210}\Theta_{0}$ $\frac{1}{20}b_2 - \frac{11}{20}\mathbf{R}_{-\frac{1}{2}\frac{1}{2}} - \frac{104}{12}b_2 - \frac{100}{21}b_2 + \frac{215}{12}b_2 - \frac{1}{9}b_{1,1} + \frac{1}{2}M_{1,1} - \frac{1}{9}M_1 - \frac{11}{22}M_1 + \frac{21}{22}M_{-\frac{1}{2}\frac{1}{2}}$ $\frac{101}{10} + 10_{1}\tilde{g}_{2} - 10_{1,1}\tilde{g}_{4} + \frac{231}{3}10_{10} - \frac{3}{4}10_{1,1}\tilde{g}_{1} + 1012g^{2} \left(\rho_{10}(h)\right) 101_{11}\tilde{g}_{4} + 301g^{2}g_{2} + \frac{3}{4}g_{2}$ $-2H(j,j-2H(j,j)-\frac{9}{2}H(j,1-\frac{5}{2}H(j,0)-\frac{47}{24}-\frac{47}{16}H(1-\frac{15}{3}\frac{1}{3}2)+pqg(-q)\Big[2H_{-}1,-1,0]$ $+ 6 R_{-1,-1,0} + 1 R_{-1,0,0} + \frac{1}{4} R_{-1,0} - \frac{18}{4} r_{1}^{2} - 4 R_{-1,0,0} - \frac{1}{2} R_{-1,0} + 6 R_{-1,-1,0,0} - 2 R_{-1,0,0,0}$ $-16.1(0,0) + |1-a| \\ 980(0,1+80,0) - 3081(1+180,0) + 161(1-162(2+180,0) + 180,0) \\$ $-104_{\mathbb{C}}+11_{1,1,1}+104_{\mathbb{C}[1,\frac{1}{2},1]}+104_{\mathbb{C}[1}-104_{\mathbb{C}}+\frac{2711}{14}11_{\mathbb{C}}+\frac{477}{276}2^{\mathbb{C}}+(1+a)\left[114_{\mathbb{C}}+\frac{3}{4}16_{\mathbb{C}[1}+\frac{3}{4}16_{\mathbb{C$ $+\frac{10}{14}001+1001.1(1+301.1(1-1010.1,1))-701.1(1+100.1+000.0)-10(1+201.1(1-100.0))-10(1+201.0))-10(1+201$ $+2H_{-1,0}+\frac{21}{2}H_0-2H_{0,0(1)}\Big]-2H_{-1,-1,0}-H_{-1}g_0-\frac{12}{2}f_0+\frac{9}{2}H_{0,0}+\frac{9}{22}g_0^{-1}+\frac{219}{22}H_{0,0}+\frac{11}{2}H_{0,0}+\frac{$ $+34_{-1,0,0}+344_{-2,0}-344_{-2,0}-334_{-2,-1,0}-34_{0,0}+\frac{39}{2}4_{1}+8_{0,1}-\frac{31}{2}4_{0,0}+80_{0,0}$ $+21\,\mathrm{K_{-1}}_{2}+444_{-1,0}+\frac{3}{2}\mathrm{e}[\frac{33}{2}\mathrm{g}_{2}-\frac{7}{2}\mathrm{H}_{2}\mathrm{g}_{2}+4\,\mathrm{H}_{11}-\frac{5}{2}\mathrm{H}_{11,1}+\frac{3}{2}\mathrm{H}_{11,1}-\frac{12}{2}\mathrm{H}_{11,1}+\frac{12}{2}\mathrm{H}$ $+386(3-168)+168(62-8.13-160-\frac{5}{2}66(1+\frac{1}{2}66(1)+181(6)-\frac{3}{2}66-161(1-\frac{7}{2}66\frac{1}{2}))$ $+\frac{2}{3}H(1)(-\frac{20}{3}H(1)(-\frac{203}{3}H(1)))$

$$\begin{split} g_{1}^{(2)} &= 1 = 1 + 2 \sqrt{2} m_{1} \sqrt{2} \left[\frac{2}{3} m + 10 \left(1 - \frac{2}{12} m + \frac{2}{3} m \left(1 - \frac{2}{3} m + \frac{2}{3} m + \frac{10}{3} m \right) - \frac{10}{3} \left(1 + 10 m \right) - \frac{10}{3} m \right) \\ &- m (1 + 1) + m (1 + \frac{2}{3} m + \frac{2}{3} m + \frac{2}{3} m + \frac{10}{3} m +$$

 $+124_{1,1/1}-\frac{201}{100}+\frac{41}{2}4_{1}4_{2}-\frac{7}{2}4_{1,1}-\frac{317}{10}4_{1}-104_{1}4_{2}+344_{-2,-1,2}-104_{-2,1/2}+104_{-2}4_{2}$ $-\frac{13}{\pi}H_{-}(\beta) - \frac{10}{\pi}H_{-}(\beta) - \frac{10}{\pi}H_{-}(\beta) + \frac{1}{3}H_{-}(\beta) + \frac{1}{3}H_{-}(\beta) - \frac{10}{\pi}H_{-}(\beta) - \frac{10}{\pi}H_{ -34.10+\frac{111}{8}00(1+\frac{10}{8}02-\frac{911}{14}(1-\frac{911}{4}(1-\frac{111}{6}(1-\frac{11}{8}01)))+\frac{17}{6}0(1-\frac{11}{22}(1-\frac{11}{22}))^2+118.10)$ $-10I_{-1}[y_{1}] - \frac{241}{242}b(1-z_{1}] + 24I_{0}^{2}x_{1}^{-1} \left(\frac{27}{24}B_{0} - \frac{1}{24}B_{0} - \frac{1}{24}B_{0} + \frac{1}{24}B_{0} + \frac{1}{24}B_{0} - \frac{1}{24}B_{0} + \frac{1}{24}$ $+ (1-a) \Big[\frac{11}{12} \theta_1 - \frac{71}{124} \Big] + \frac{2}{a} (1+a) \Big[\xi_2 + \frac{13}{12} \theta_1 - \frac{1}{2} \theta_{01} - \theta_1 \Big] + \frac{2\theta}{124} \theta (1-a) \Big]$ $+ 3 \left[r_{1}^{2} n_{1} \left(a^{2} \right) \left(a + \frac{71}{2} b_{1} + \frac{11}{2} n_{1} \right) - \frac{7}{4} n_{1} + \frac{7}{4} n_{2} c_{1} + \frac{7400}{100} n_{1} - 2 n_{-2} \left(a \right) + \frac{1}{4} n_{10} \left(a \right) \left(\frac{10}{2} c_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(a \right) \left(\frac{10}{2} c_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(a \right) \left(\frac{10}{2} c_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(a \right) \left(\frac{10}{2} c_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(a \right) \left(\frac{10}{2} c_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}{100} n_{1} \right) + \frac{1}{4} n_{10} \left(\frac{10}{10} n_{1} + \frac{7400}{100} n_{1} + \frac{7400}$ $-\frac{209}{34} - 122 - 111 - 23 - \frac{1}{3}62 - \frac{11}{3}64(1 - \frac{21}{3}64(1 - 111)) - 111(1) - \frac{21}{3}21 - 102 + \frac{11}{3}999 (-4) \left[\frac{1}{32} - \frac{1}{3}62(1 - 111) + \frac{11}{3}999 (-4)\right] = \frac{1}{3} - \frac{1$ $+2 \mathbf{I}_{-1,0} + \frac{8}{10} \mathbf{g}_{0,1}^{1} - 2 \mathbf{I}_{0,0} + \frac{1}{2} \mathbf{I}_{-1}^{1} - \mathbf{a}^{2} + \left[2 \mathbf{I}_{0} - \mathbf{R} \mathbf{g}_{0,1}^{1} - \frac{71}{2} \mathbf{I}_{0} + \frac{5 \mathbf{R} \mathbf{I}_{0}}{1 \mathbf{m}} - 3 \mathbf{R} \mathbf{g}_{0,1}^{1} + \frac{2 \mathbf{R}}{2 \mathbf{A}} \mathbf{I}_{0,1} \right]$ $\frac{15}{3} \Omega_{1,1} + \Omega_{1,1,1} \bigg| + (\frac{1}{2} + r^2) \Big(\frac{111}{34} \Omega_1 - \frac{1}{8} \xi_1 + \frac{1}{2} \Omega_{-1} \xi_2 - \xi_1 + 2 \Omega_{-1,-1,1} - \frac{1}{8} \Omega_{-1,1,1} \bigg)$ $-\frac{17}{9}R_{-1[1} + \frac{2}{7}R_{-1[1]} + |1-i\rangle \left[\frac{3}{8}R_{-1[1]} + R_{-1[1]} + 2R_{[1[1]} - \frac{247}{36}g_2 - \frac{407}{216} - 3R_{-2}g_2\right]$ $-404.1_{-}10+318.2(1)-\frac{2}{\pi}814^{2}+\frac{877}{27}48.+1410+\frac{2}{7}41104+11+4\left|\frac{110}{28}48-\frac{11}{8}48.14^{2}\right|$ $+\frac{39}{20}g_{1}^{2}-\frac{7}{12}H_{1}-\frac{31}{2}H_{01}+\frac{1}{2}H_{02}g_{1}-\frac{5}{2}H_{1}g_{1}g_{1}+H_{0}g_{2}-2H_{-1,1}g_{1}+\frac{17}{2}H_{-1$ $+2R_{-\frac{1}{2}}-1R_{\frac{1}{2}\frac{1}{2}}-\frac{3}{2}R_{\frac{1}{2}\frac{1}{2}}+\frac{3}{2}R_{\frac{1}{2}\frac{1}{2}}+\frac{3}{2}R_{\frac{1}{2}}+\frac{1}{2}\frac{1}{4}\frac{1}{4}+7R_{-\frac{1}{2}\frac{1}{2}}+2R_{\frac{1}{2}}+\frac{418}{27}R_{\frac{1}{2}}+R_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}$ $+\frac{3}{2}g_{1}^{-1}+iH_{-1,j}-*\frac{(21)}{11}H_{0,j}-\frac{8}{8}H_{0,1}+\frac{3}{2}H_{0}-H_{0,j,j,j}+\frac{2}{8}H_{0,j,j}+\frac{18H}{216}H_{0}+4H_{0,0}$ $-\dot{a}(1-a)\left[\frac{21.5}{100}+\frac{3}{2}\dot{Q}+\frac{1}{12}\dot{Q}^{2}+\frac{5}{12}\dot{q}^{2}\right]+242\chi^{2}\left[a^{2}\right]2261.4(4+2100\dot{q})-\frac{1200}{12}60.6(4+310)\dot{q})-\frac{1200}{12}60.6(4+3100\dot{q})-\frac{1$ $-4000(1)(-\frac{110}{3}00)-\frac{40}{9}01(1)+\frac{30}{9}(1+\frac{800}{300}00)+p_{HP}(s)(\frac{100}{34}-\frac{80}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{300})^2+\frac{11}{9}(1-\frac{3}{9})^2+\frac{11}{9}$ $-48_{-10} + 48_{-2}\frac{1}{2} + 48_{-2-10} + \frac{11}{2}8_{-10} - 48_{-210} - 48_{-211} + \frac{1}{2}8_0 - 38_{-20} + \frac{81}{2}8_{10}$ -001041+000001-00042-001_0+1000010-000.041+000.001+000.001+000 $+\frac{11.1}{2}M_{1,2}+\frac{11}{2}M_{1,2,2}+32b_{1,2,3}+324_{1,3}+\frac{11.4}{2}M_{2}-32b_{1,2}+324_{2,3}+324_{2,3}+324_{2,3}+124_{2,3}$ $+ 60_{1,1,0} \Big| + \mu_{10} \langle -\alpha \rangle \Big(\frac{11}{3} g_1^{-1} - \frac{11}{3} M_0 g_2 - 4 M_{-2,0} + 34 M_{-2,0} - 70 M_{-2,1} - \frac{710}{9} M_{-1,0} + 2 M_0 g_2$ $\begin{array}{c} + 86 , 1 , 1 \} + 181 , \frac{1}{36} - 103 , \frac{1}{2} [1 + 301 , 1 , 2] + 341 , 1 , \frac{1}{2} [1 + 341 , \frac{1}{2} , \frac{1}{2}] + 341 , \frac{1}{2} , \frac{1}{2}] \\ + 386 , \frac{1}{36} - 103 , \frac{1}{2} [1 - 361 , \frac{1}{2}] + 341 , \frac{1}{2} - 100 \frac{1}{2} - \frac{1}{2}] + 301 , \frac{1}{3}] \\ \end{array}$ $\frac{d\mathcal{I}_{2}}{dr^{2}} + \frac{d\mathcal{I}}{dr} \mathbf{x}_{\frac{1}{2} + 1} \times \mathbf{x}_{\frac{1}{2}} \Big] + \Big\langle \frac{1}{dr} - e^{2} \Big\rangle \frac{|\frac{1}{10117} + \frac{21}{4} \mathbf{x}_{\frac{1}{2} + 1} - \frac{d\mathcal{I}_{2}}{dr} - \frac{1}{2} \mathbf{x}_{\frac{1}{2} + 1} - \frac{d\mathcal{I}_{2}}{dr} \mathbf{x}_{\frac{1}{2} - 1} - \frac{d\mathcal{I}_{2}}{dr} \mathbf{x}_{\frac{$

 $-\frac{411}{229}H_1 - \frac{31}{2}H_2\xi_2 + \frac{31}{2}H_{1/2}[+ 11]\frac{1}{2} + s^2 [\frac{31}{129}H_1 - \frac{1}{2}H_1 - \frac{319}{129}\xi_2 - \frac{1}{2}H_{2/2} - \frac{1}{2}H_{-1}\xi_2]$ $+86.1, -10-\frac{429}{100}8.10+\frac{3}{2}86.100+86.10\right]+(1-c)\frac{(11}{100}81+\frac{27}{3}81,0-\frac{21}{3}81,0-88.10)$ $\begin{array}{c} \frac{100}{100} (1) - \frac{10}{10} 0(1)(1 - \frac{10}{100} + 2 - \frac{110}{100} + 0 - 10 + \frac{1}{100} + 0 - 10 + \frac{1}{100} + 2 - \frac{1}{100} + 2 - \frac{1}{100} + \frac$ TEN \$1 - \$1002 - 38 J. J. 8 + 180. 149 + TEN 12 + 1803.0 - \$102 + 1800 $\frac{116}{2}\theta_{1} - \frac{21}{3}\theta_{-} \frac{1}{5}\theta_{-} \frac{1}{5}\theta_{0|1} - \frac{40}{5}\theta_{0|1} + 21\theta_{-} \frac{41}{3}\theta_{0|2} - 22\theta_{1} - 22\theta_{2|1} + \frac{21}{3}\theta_{2|2}$ $+\frac{601}{10}M_0 + 2M_0^2 + 3M_0^2 + 27M_0^2 - 4M_0M_0^2 - 14M_0M_0 + 2MM_0M_0 + 3(1 - a)$ $-\frac{1}{2}\frac{1}{2}\left[1 + \frac{1}{2}\frac{1}{2}\left[1 + \frac{1}{24}\frac{1}{2}\right]^2 + \frac{4\beta^2}{4}\left[1 - h_{0}^{2}\right]^2\right] + 16\beta_{2}m_{1}^{-2}\left[\frac{1}{2}\frac{1}{4}\left[1 + 12 - \frac{1}{2}\right] + 16(1 - 0)\right] + \frac{1}{4}60$
$$\begin{split} & \frac{1}{2} \hat{G}_{2} - \frac{11}{12} \frac{1}{64} - \frac{1}{3} \hat{G}_{00} + 2 + \frac{1}{7} \hat{G}_{1}^{-1} - e^{2} \hat{G}_{1}^{-1} - 2 \hat$$
 $-\Omega_{2,1} - 2\Omega_{2,0} \left[+ \frac{31}{244} \dot{a} (3 - a_{1}^{2}) + 2M_{2}^{2} F_{2} \left[\frac{4}{3} a^{2} \left[\frac{2M}{3a} + \frac{27}{6} B_{1} + \frac{7}{2} B_{0,0} - B_{0,0} - \frac{1}{6} \frac{2}{4} + \frac{9}{4} B_{0,0} \right] \right]$ $-16(1+\frac{1}{2}60)(1+\frac{35}{12}61+16-261.1(1-\frac{5}{2}(1)+\frac{4}{12}-r^2)\left|\frac{61}{110}61-\frac{31}{2}-\frac{1}{2}61(1+\frac{1}{2}61)(6)\right|$ $-H_{1}\underline{\hat{g}}_{2}-H_{1,1}+H_{1,1,0}+H_{1,1,1}+\underline{\hat{g}}_{2}\Big|+\frac{4}{3}\frac{1}{\sqrt{2}}+e^{2}\Big(H_{-}\underline{\hat{g}}_{2}+2H_{-1,1,0}-H_{-1,0,0}\Big)+\frac{215}{12}H_{0,0}\Big)$ $+\frac{21}{3}\theta_{1}-\frac{191}{3}+3\theta_{12}+\frac{211}{12}\zeta_{2}-12\theta_{13}+\theta_{13}-\frac{81}{32}\theta_{1}+\frac{11}{2}\theta_{2}+3\theta_{12}+2\zeta_{2}^{2}-\theta_{3}\zeta_{2}$ $+ \Omega_{\mathbb{C}} + 4 R_{\mathbb{C}} \xi_{\mathbb{C}}^{1} + 1 \Omega_{-1,\mathbb{D}}^{1} - 4 d R_{\mathbb{C}} g_{\mathbb{C}}^{1} + |1 - a| \Big(\frac{2 \pi}{\pi} R_{\mathbb{C}} - \frac{5}{6} \Omega_{1,\mathbb{D}}^{1} - 4 \xi_{\mathbb{C}}^{1} + R_{\mathbb{C}} \xi_{\mathbb{C}}^{1} - 1 \Omega_{-1,-1,\mathbb{D}}^{1} \\$ $-i84.3\frac{1}{6}1+i84.3(d-i84\frac{1}{2})+\frac{2}{3}84(d)-\frac{3}{32}84(1+83)(d+84)(d+84)(d)+(1+1)\left|\frac{5}{4}82+\frac{39}{8}\right|$ $-\frac{67}{4}60(j-1001)-\frac{341}{21}00(-1101)-\frac{5}{2}01(j)-2a_{2}^{2}t+\frac{1}{2}a_{1}^{2}t^{2}+503a_{1}^{2}1-100t-110.12t$ $-mt_{-1,-1,0}+\frac{m}{2}\pi_{-1,0}+mt_{-1,0,0}+mt_{0,0}t_{1}^{2}-2m_{0,0,0}-mt_{0,0,0}+mt_{0,1,0}+2m_{0,1,0}$ $+22\xi_{111} + 8\xi_{11} - 28\xi_{1} + \frac{1}{22}\dot{a}(1-a)$.



[S.Moch, DESY-WS-04] [Moch,Vermaseren,Vogt hep-ph/0403192 & 0404111]

- NNLO analysis of deep-inelastic structure functions $F_2, F_3 \longrightarrow high \ precision$
 - stability under scale variations at NNLO
 - match experimental accuracy in final HERA data
 - NNLO parton distributions for LHC precision analyses

Methods

- → offer *easy-to-use* parameterization
- Mellin moments and nested sums powerful technology
 - apply innovative and efficient method to solve multi-loop integrals
 - formalism with wide range of applications

Upshot

- Phenomenology for deep-inelastic scattering and hard hadronic interactions
 - reach new level of precision

$$\begin{split} & +\frac{11}{10} (q+100, Q_1+100, Q_1+1$$

$$\begin{split} g_{1}^{(0)} &= 1 - 1 - 2 \sqrt{\alpha_{1}} \sqrt{c^{2}} \frac{1}{2} \log t + 10 + (t - \frac{11}{12} t) + \frac{2}{3} \log 1 - \frac{12}{3} \log \frac{1}{3} + \frac{11}{32} \log - \frac{14}{12} \log \frac{1}{3} + 10 + 10 \\ &- 8 \log \left(\frac{1}{32} + \frac{1}{32} +$$

$$\begin{split} &-4611_{-1}(1)+411_{-1}(0)-\frac{2}{3}\pi \frac{11}{3}^2+\frac{11}{3}\pi \frac{11}{3}+411(1+\frac{2}{3}\pi \frac{11}{3}(1)+\frac{11}{3}+11_{-1}(1+\frac{11}{3})+\frac{11}{3}+1_{-1}(1+\frac{11}{3})+\frac{11}{3}+1_{-1}(1+\frac{11}{3})+\frac{11}{3}+1_{-1}(1+\frac{11}{3})+1_{-1}(1+\frac{3$$

$$\begin{split} & L(J) - 3111 + 810, J + \frac{125}{9} (0 - 101) \Big| \\ & L(J) - \frac{3}{2} (0 - \frac{1}{9} (0 - \frac{1}{9}$$

 $\frac{31}{4} \Theta_{0} \frac{1}{4} - \frac{1}{2} \Theta_{0} \frac{1}{1} - \frac{1100}{34} \frac{1}{4}$ $(1) - 100 + 100 + 100 + \frac{1}{2} \Theta_{0} \frac{1}{4}$

201_110 - 200_110 + ¹

 $(1) + \frac{1}{3} (10) \frac{1}{2} + 10 \frac{1}{3} (10) \frac{1}{2} + 10 \frac{1}{3} \frac{1}{3} + 10 \frac{1}{3} +$

$$\begin{split} & = \frac{1}{2} \Phi_{1} - \frac{110}{2} (z - \frac{1}{2} U_{1,2}) - \frac{1}{2} U_{1,2} (z - \frac{1}{2} U_{1,2}) - \frac{1}{2} U_{1,2} (z - \frac{1}{2} U_{1,2}) - \frac{1}{2} U_{1,2} (z - U_{1,2}) \\ & = 0 + U_{1,2} (z - U_{1,2}) + (-\frac{1}{2} U_{1,2}) - \frac{1}{2} U_{1,2} (z - U_{1,2}) \\ & = U_{1,2} (z - U_{1,2}) + U_{1,2} (z - \frac{1}{2} U_{1,2}) - \frac{1}{2} U_{1,2} (z - U_{1,2}) + \frac{1$$

$$\begin{split} & = 0 \begin{pmatrix} -111 \\ -101(1+101) + \frac{11}{101}(1+10-101) + \frac{1}{101}(1+10) + \frac{1}{101}(1+101) + \frac{1}{101}(1+10$$



[S.Moch, DESY-WS-04] [Moch,Vermaseren,Vogt hep-ph/0403192 & 0404111]



$$\begin{split} & + \frac{112}{128} \mathbf{x}_{1,1} + 4b_{1,2} + \frac{11}{12} b_{1,1}^{-1} + \frac{11}{12} b_{1,1}^{-1} + \frac{11}{12} b_{1,1}^{-1} - \frac{$$

$$\begin{split} & \frac{1}{12} \frac{1}{16} H + 100...(1+201...(1+20$$

$$\begin{split} g_{1}^{(0)} \psi_{1} &= 1 \exp_{1} \psi_{1} \left[\frac{1}{2} \log + 10 \left(1 + \frac{11}{12} + \frac{1}{2} \log - 1 \right) - \frac{1}{2} \log \frac{1}{12} + \frac{11}{12} + 1 \right) \\ &- \exp_{1} \left(1 + 2 \exp_{1} \left(\frac{1}{2} + \frac{1}{12} + \frac{1}{12} + 1 \right) + \exp_{1} \left(\frac{1}{2} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + 1 \right) - \frac{11}{12} \exp_{1} \left(\frac{1}{12} + 1 \right) \\ &- \exp_{1} \left(1 + 2 \exp_{1} \left(\frac{1}{12} + \frac{1}{1$$

$$\begin{split} &-46.1_{-1}(1)+13.2(1)-\frac{2}{2}31\frac{1}{2}+\frac{177}{10}48+4013+\frac{2}{6}4110[\frac{1}{2}+1]+4[\frac{11}{100}48-\frac{11}{2}81,3\frac{1}{2}1\\ &+\frac{172}{2}9\frac{1}{2}-\frac{1}{2}91-\frac{13}{2}80g+\frac{1}{2}96g\frac{1}{2}+\frac{1}{2}96g\frac{1}{2}+\frac{1}{2}91g\frac{1}{2}+1\frac{1}{2}91g\frac{1}{2}+\frac{1}{2}91$$

$$\begin{split} & \frac{1}{2} (1) + \frac{1}{2} (1$$



[S.Moch, DESY-WS-04] [Moch,Vermaseren,Vogt hep-ph/0403192 & 0404111]





 $\alpha_s(M_z) = 0.1179 \pm 0.0031$





•only results in completed NNLO with total error <0.008 used

• overall error adjusted to $\chi^2/dof = 1$

→ <u>theory</u>: more NNLO calculations!

→<u>exp:</u> more high precision collider data

→consistent definitions and treatment of systematic uncertainties

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PDF : Goals and Roadmap

- study potential inclusion of HERA jet data in a rigorous manner in global QCD fits
- need improved understanding of error propagation and of consistency of experiments in order to assess relative size of exp. and theoretical uncertainties at LHC
- need to identify a list of 'reference' processes at HERA & LHC
- need to understand dependencies on rapidity and $p_T \rightarrow acceptance$ cut effects
- need to understand interplay of PDF uncertainties with MC
- need tools to access impact of resummation or non-DGLAP effects

[A.Djouadi, S.Ferrag, hep-ph/0310209]

Higgs and Heavy Quarks



Charm and Bottom

at x=0.001→ dominated by sea quarks → but valence quark PDFs although small also not well known → xG₃ from HERA II

→dramatic increase
 of sea PDFs towards
 high Q² :c,b dominant
 →ubar=dbar??
 ... ed at HERA!

→how to improve knowledge on HQ
PDFs?
→how well does evolution work?

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Charm in DIS



Fragmentation at HERA

[L.Gladilin, DIS03-WS] ZEUS



- e.g. measure charm fragmentation function in hadronic events.
- needed for beauty jet rates; minimise extrapolation uncertainties.
- should be more precise after ugrade (CST, MVD).
- should also be done for beauty.
- also measured fragmentation fractions.



Beauty Production at HERA



→ fair agreement with NLO QCD

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Charm and Beauty measurements using Si-Vertex-Detector : impact parameter analysis



HERA II \rightarrow 'b machine': much higher statistics anticipated \rightarrow F₂^{cc}, F₂^{bb}, b-fragmentation functions...

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A 'Map' of the Parton Dynamics



DGLAP at Low x?



→ but splitting functions have to be convoluted with the gluon distribution → exact NNLO trustable until $x \sim 10^{-4} - 10^{-5}$...

→ Gluon at low x needs experimental constraints

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[L.Lönnblad,StartUp-WS]

Forward Activity

DGLAP based initial-state parton showers limits emissions to be at lower scales than the hard scattering. How severe is this restriction?

For forward jets at HERA it is clearly a severe restriction.

For small-x and moderate scales it is clearly a severe restriction.



k_{\perp} -factorized (BFKL/CCFM) generators

@TEVATRON x ~m_w/S~0.01-0.1 but @LHC x an order of mag. lower



Event Generator HERA \rightarrow LHC

Who needs data when we have Pythia, Herwig and Ariadne?

 $\mathsf{LEP} \rightarrow \mathsf{HERA} :$

At HERA we have a hadron in the initial state

There are initial-state parton showers, but they are not quite up to the task at small \boldsymbol{x}

Not even in the current region of the Breit frame, where things should look like half an e^+e^- event. At small x the target region is very much larger and hard emissions there affect the current region (energy-momentum conservation).

$\mathsf{HERA} \rightarrow \mathsf{LHC}:$

All small-x problems at HERA are there at the LHC

In addition we have multiple scatterings and underlying events (also in photoproduction at HERA)

HERA has a lot to tell us about where to trust the current event generators at LHC

Forward activity

The second commandment of event generation:

Thou shalt never omit any part of phase space

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Study of Azimuthal Correlations beteen two hardest jets



Underlying Event and resolved yp



- Primary hard parton parton interaction
 - Underlying event
 - multiple soft to hard parton interactions (MI)
 - initial/final state radiation
 - fragmentation
 - beam remnants



• HERWIG

- soft underlying event: parametrized results of soft hadron hadron interactions are added in a fraction of the events
- JIMMY: "add on" to generate MI
- PYTHIA with MI (LO + unitarization)
- PHOJET includes multiple soft and hard parton interactions + unitarization scheme

Plans

[G.Grindhammer, DESY-WS-04]

Models

 Many distributions in resolved γp scattering are better described by QCD models which include MI

- * There is evidence that the effects seen are due to MI
- ☆ These effects were studied mainly in the early years of HERA with limited statistics - we should revisit
- ☆ Compare CDF-tunes of underlying event with HERA data during the workshop

Which measurements should still be done at

HERA?

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[P.Landshoff, DESY-WS-04]

Regge Theory and Total Cross Section

1984: <u>soft pomeron</u>

Fit high-energy $\sigma(pp), \sigma(p\bar{p})$ with simple power s^{ϵ_1} $\epsilon_1 \approx 0.08$

1998: <u>hard pomeron</u>



Use hardpoint, softpoint and reggeon exchange Reggeon = ρ, ω, f_2, a_2

Approximate with a single term $\epsilon_R \approx 0.5$ $\sigma(pp), \sigma(p\bar{p}), \sigma(\gamma p):$ $\sigma = X_0 s^{\epsilon_0} + X_1 s^{\epsilon_1} + Y s^{\epsilon_R}$ $F_2(x, Q^2):$

$$x^{-\epsilon_0} f_0(Q^2) + x^{-\epsilon_1} f_1(Q^2) + x^{-\epsilon_R} f_R(Q^2)$$

NEW 2004 fit



Differential Cross Section

$$\frac{d\sigma^{pp}}{dt} \sim \frac{d\sigma^{\bar{p}p}}{dt} \sim \frac{(3\beta_{I\!\!P}F_1(t))^4}{4\pi} \left(\frac{s}{s_0}\right)^{2\alpha_{I\!\!P}(t)-2}$$

$$\alpha_{I\!\!P}(t) = 1.08 + \alpha' t \qquad s_0 = 1/\alpha'$$

 $F_1(t) =$ Dirac form factor

Old fit:

$$\alpha(t) = 1 + \epsilon + \alpha' t \qquad \epsilon = 0.08 \qquad \alpha' = 0.25$$

Replace with two trajectories:

$$\epsilon_0 = 0.45$$
 $\alpha'_0 \approx 0.1$

$$\epsilon_1 = 0.067 \qquad \alpha_1' \approx 0.3$$

F₂^C is purely hard Pomeron exchange hard Pomeron is flavor blind

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LHC Energies and Unitarity



[P.Landshoff, DESY-WS-04]

Diffractive Higgs pp →pHp

A good way to discover the Higgs: the background is relatively small - Albrow

Big disagreement about size of cross section

Bialas + PVL (1991):





[C.Royon, DIS04-WS]

"Exclusive" production at the LHC

- Survival probability: estimated to be ~ 0.03
- Exclusive $b\bar{b}$ cross section (for jets with $p_T > 25$ GeV): 70.1 pb * 0.03 = 2.1 pb
- Exclusive Higgs production (in fb) after applying the gap survival probability

M_{Higgs}	σ (fb)
120	3.9
125	3.5
130	3.1
135	2.5
140	2.0

Peter Landshoff: $pp \rightarrow p$ Higgs p anywhere between 3 and 300 fb

LHC should teach us a lot

Areas of Impact & Concluding Remarks

Precision measurement of QCD inputs

 α_{S} : from jet rates, jet substructure, event shapes, global fits... Parton distributions from structure functions, jets and charm Fragmentation parameters: strange, charm, beauty, leading particles

Testing ground for non- or semi-perturbative models

Underlying events; minijets, multiparton interactions, saturation Soft underlying events, rescattering, forward neutrons & protons Diffractive structure functions, gaps between jets, survival probability

Testing ground for calculational techniques

Very forward jets, low x Multijets, matrix element/parton showers Evaluation of theoretical uncertainties Beauty & charm production cross sections and dynamics DIS/photoproduction transition; multiscale QCD "Intrinsic" transverse momentum, k_T factorization

Gain a *quantitative* understanding of hadronic production mechanisms at high energies.

Join the effort ! 48



Many thanks to the organizers for the invitation and the fruitful workshop



HCP 2004 Uta Stösslein HERA and the LHC

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Impact of Low x Constraint: $\overline{d} - \overline{u}$



H1 + BCDMS p & d



- shape of $\bar{d} \bar{u}$ in global fits reproduced by fit to H1 + BCDMS p & dwhen $x(\bar{d} - \bar{u}) \xrightarrow{x \to 0} 0$ is imposed
- uncertainty is much wider when this constraint is not applied
- test of symmetry of light sea quarks at low xrequires NEW data (p&d)