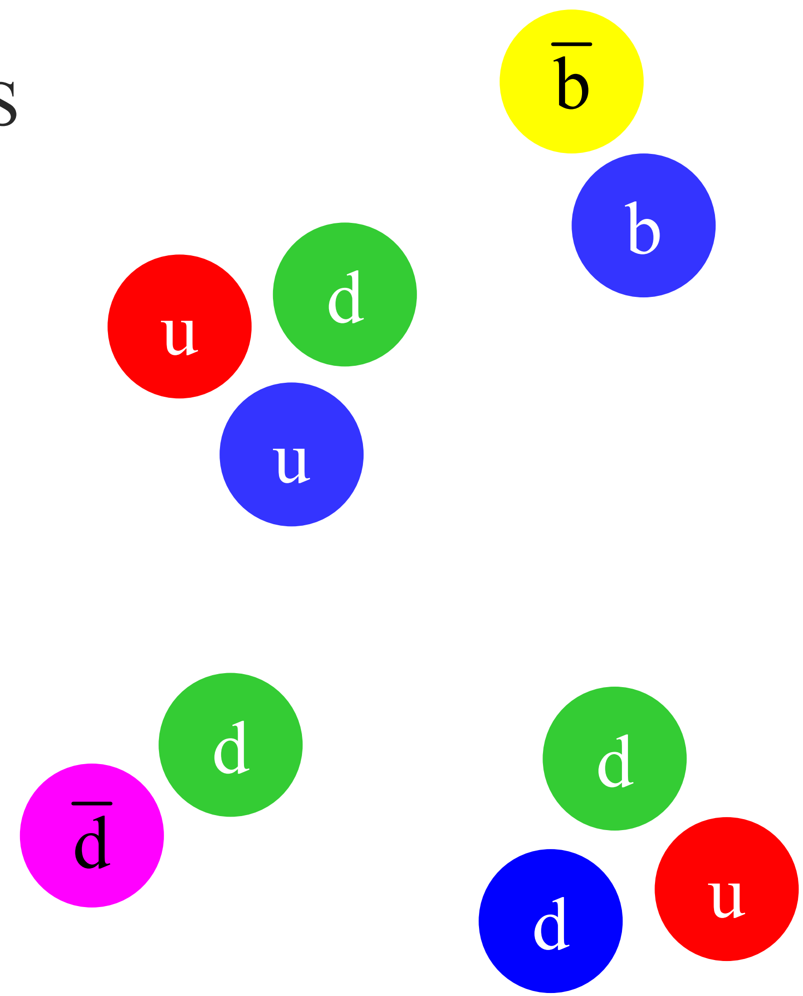
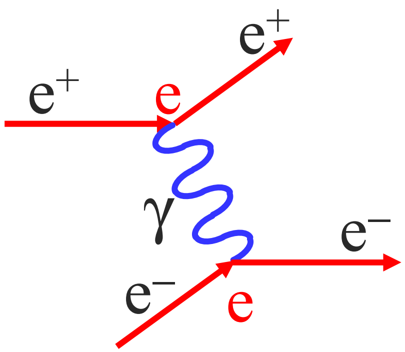
QCD and Hadron Structure

- Quark and hadron masses
- Measuring proton structure in DIS
- Proton structure and the hadronic final state
- Proton spin
- Low Q^2 and x
- Summary



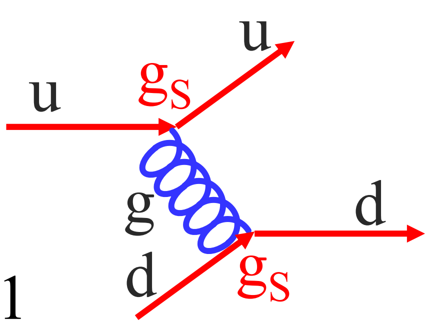
Quark and hadron masses

- Ratio of the mass of the parts to the mass of the whole:
  - Positronium
    $2 \times 511\,\mathrm{keV} / 1022\,\mathrm{keV} = 1$



$$\alpha = \frac{e^2}{4\pi}$$

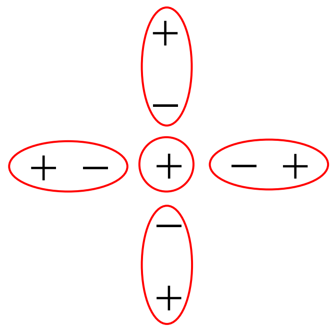
  - Bottomonium
    $2 \times 4.2\,\mathrm{GeV} / 9.5\,\mathrm{GeV} = 0.9$
  - Proton
    $\left(2 \times 3\,\mathrm{MeV} + 6\,\mathrm{MeV}\right) / 938\,\mathrm{MeV} = 0.01$
  - Pion
    $\left(3\,\mathrm{MeV} + 6\,\mathrm{MeV}\right) / 135\,\mathrm{MeV} = 0.07$

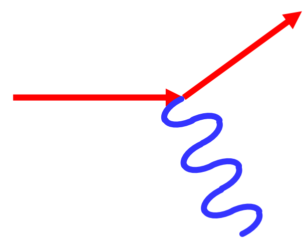


$$\alpha_S = \frac{{g_S}^2}{4\pi}$$

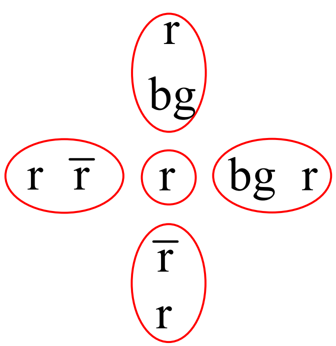
Coupling constants

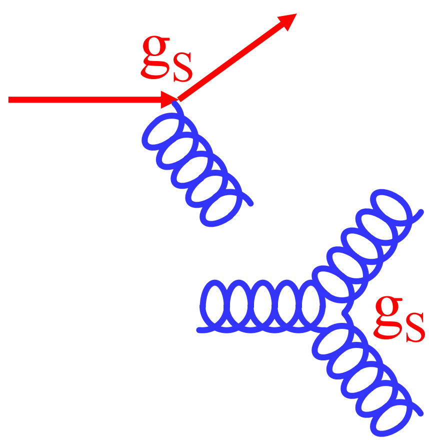
- QED, screening, polarization of vacuum leads to $\alpha \uparrow$ as $Q^2 \uparrow$

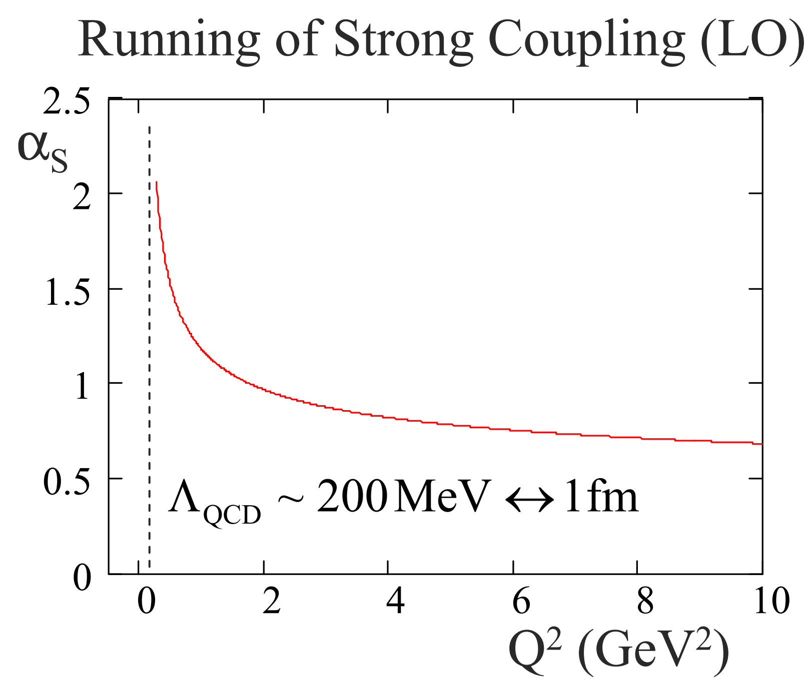




- QCD, anti-screening, polarization of vacuum leads to $\alpha_S \downarrow$ as $Q^2 \uparrow$





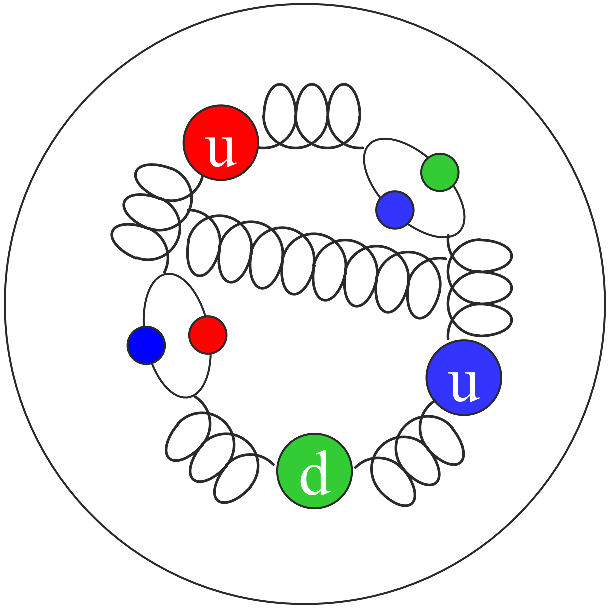


Hadron structure

- Strong force confines quarks to small space.
- Uncertainty principle dictates

$$\Delta p \sim \frac{\hbar c}{R_p} \sim \frac{200\,\text{MeV fm}}{0.8\,\text{fm}} \sim 250\,\text{MeV}$$

- Quark energies E ~ 250 MeV.
- Quarks within proton highly relativistic, suggests lots of gluon radiation…
- …and yet all quarks and gluons conspire to ensure the proton's spin is ½.



Timescales in DIS

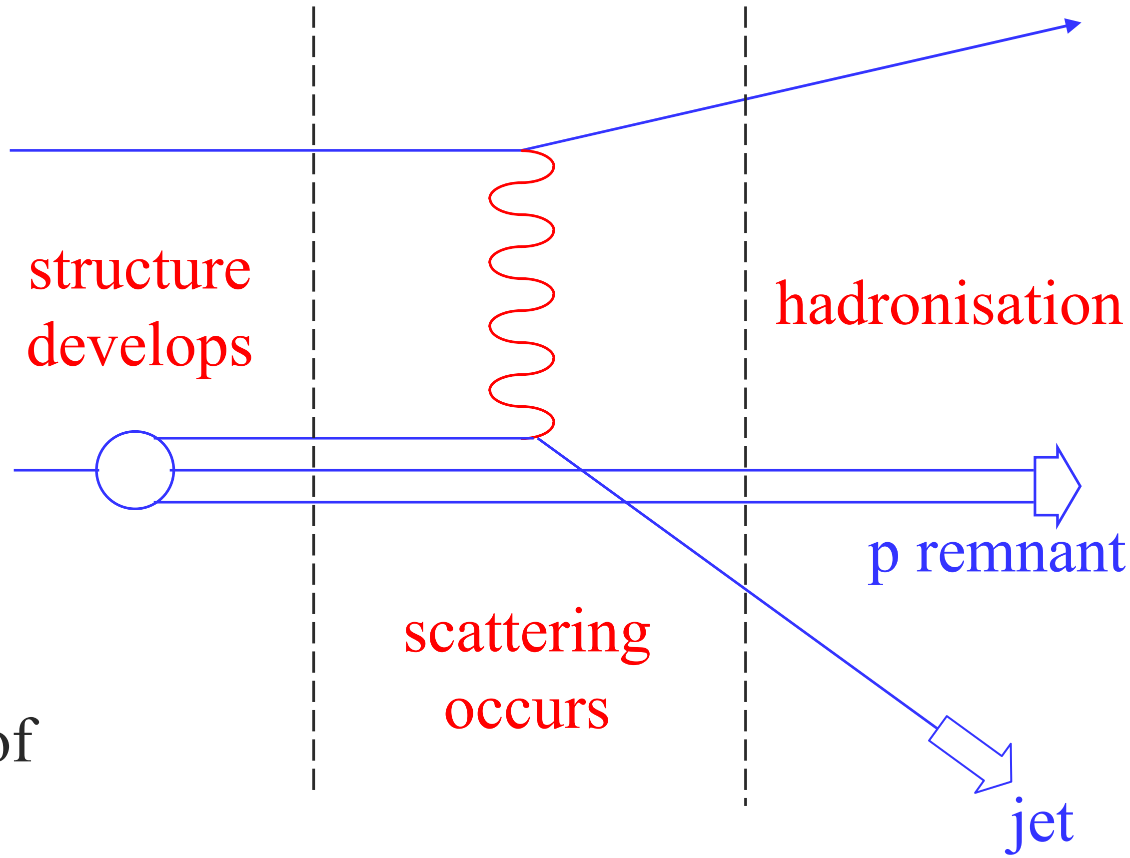
- Time scale for soft interactions

$$\Delta T_{SOFT} \approx \frac{E}{m}\frac{1}{\Lambda_{QCD}}$$

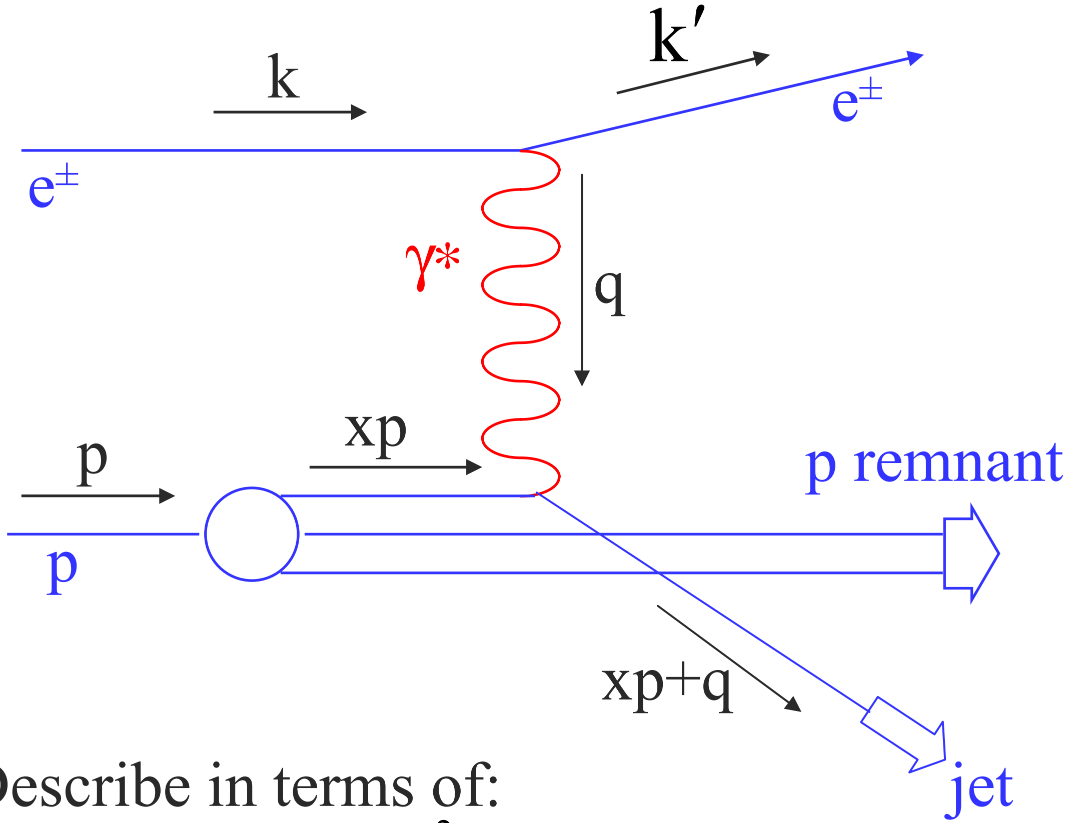
- For DIS

$$\Delta T_{DIS} \approx \frac{1}{\sqrt{Q^2}}$$

- $\Delta T_{SOFT} >> \Delta T_{DIS}$ so DIS takes snapshot of proton.



Measuring hadron structure – Deep Inelastic Scattering



- Describe in terms of:

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2p \cdot q} \text{ and } y = \frac{p \cdot q}{p \cdot k}$$

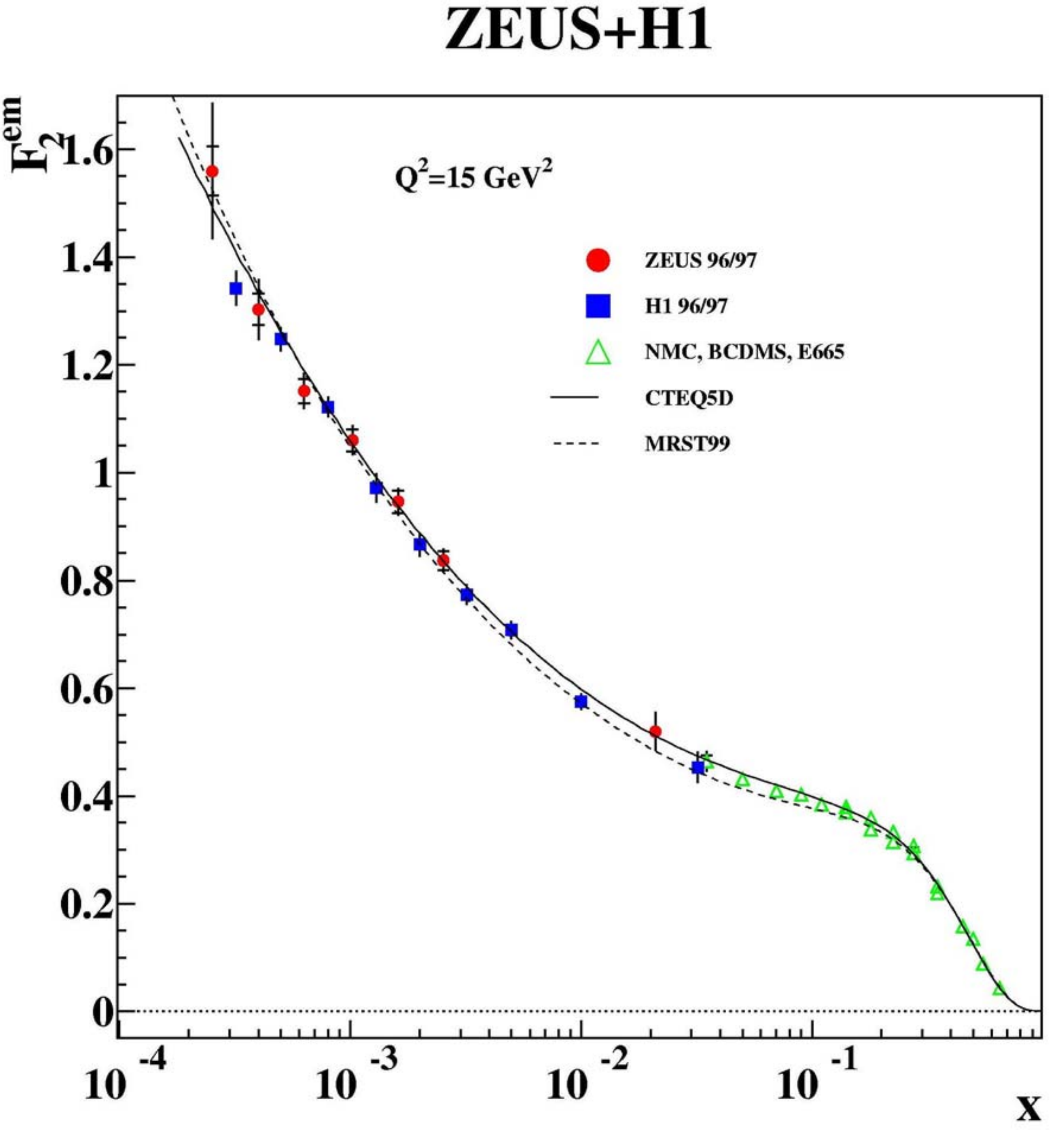
- Cross section for $Q^2 < M_Z^2$:

$$\frac{d^2\sigma_{ep\to eX}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left(y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right)$$

- Structure functions related to quark densities:

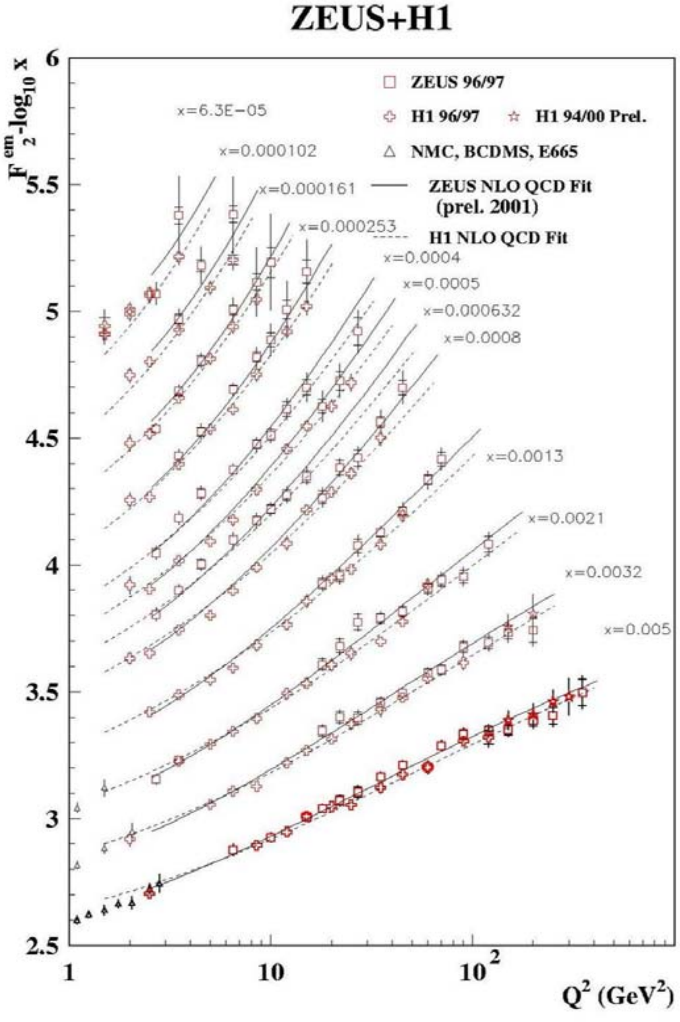
$$F_2(x,Q^2) = 2xF_1(x,Q^2)$$
$$= x\sum_q e_q^2\left(q(x,Q^2) + \bar{q}(x,Q^2)\right)$$

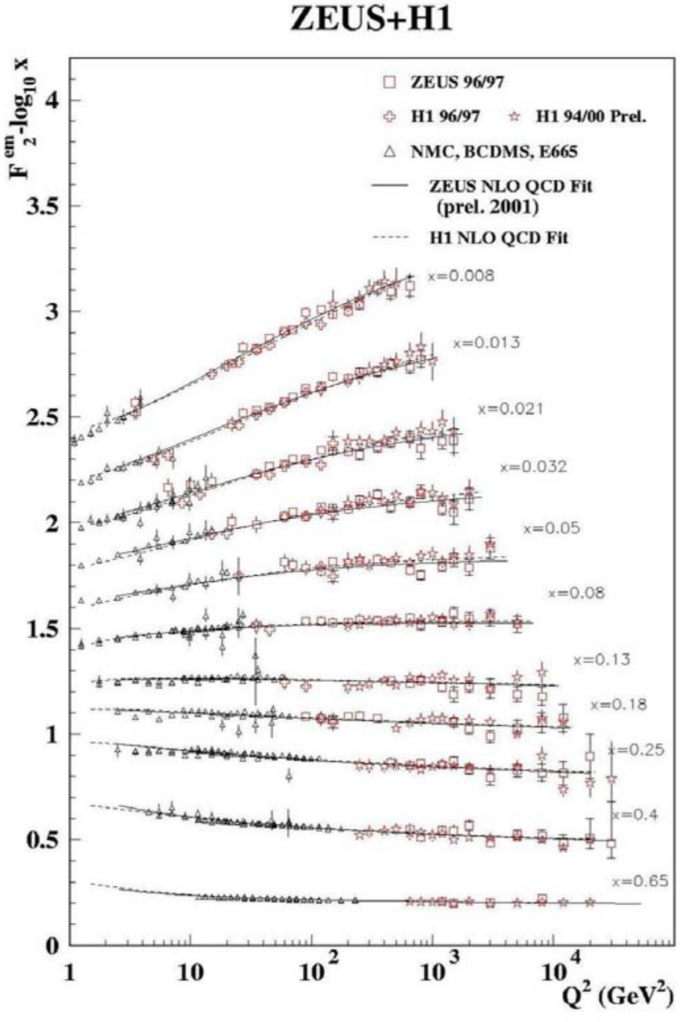
Measurements of $F_2(x,Q^2)$



- Steep rise of $F_2(x,Q^2)$ observed at low x as x decreases

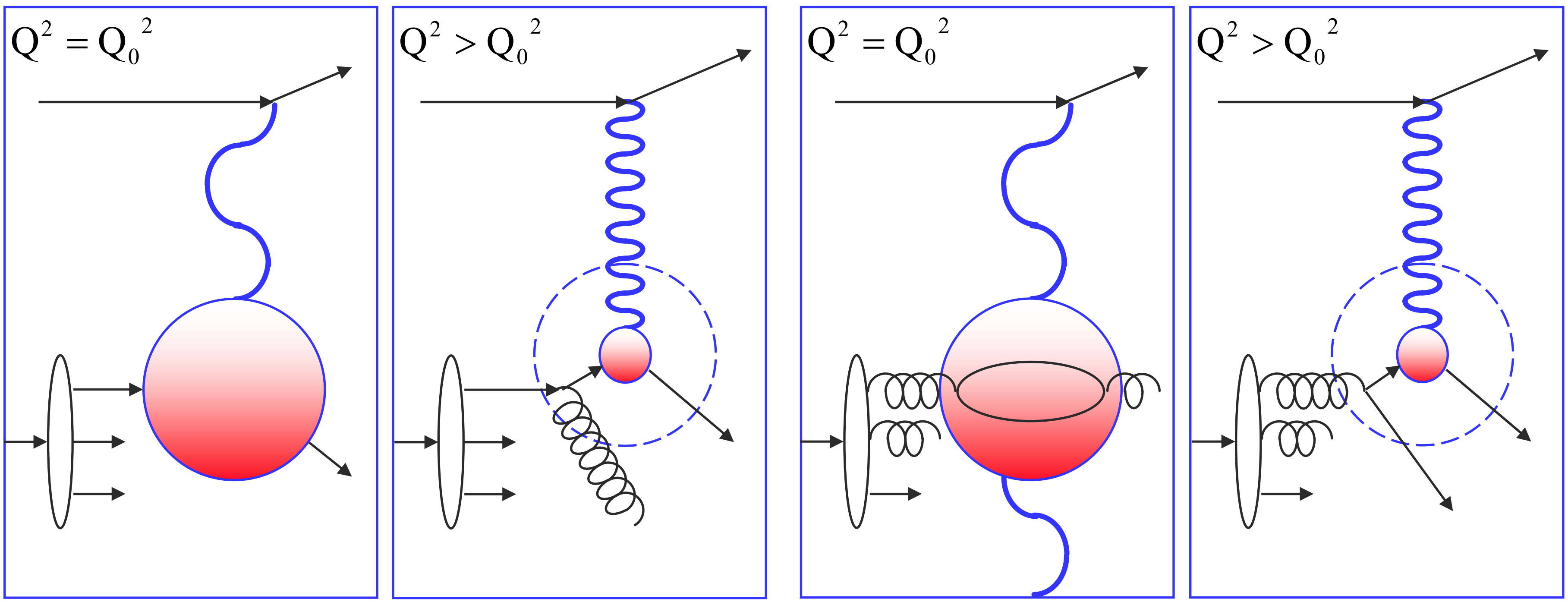
Measurements of $F_2(x,Q^2)$





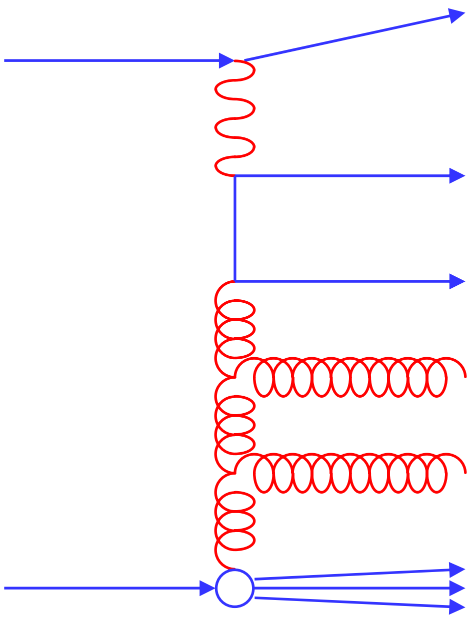
QCD and scaling violations

- If quarks dominate:
  $Q^2 \uparrow \Rightarrow F_2 \downarrow$ at a given x
- If gluons dominate
  $Q^2 \uparrow \Rightarrow F_2 \uparrow$ at a given x



QCD calculations using DGLAP equations

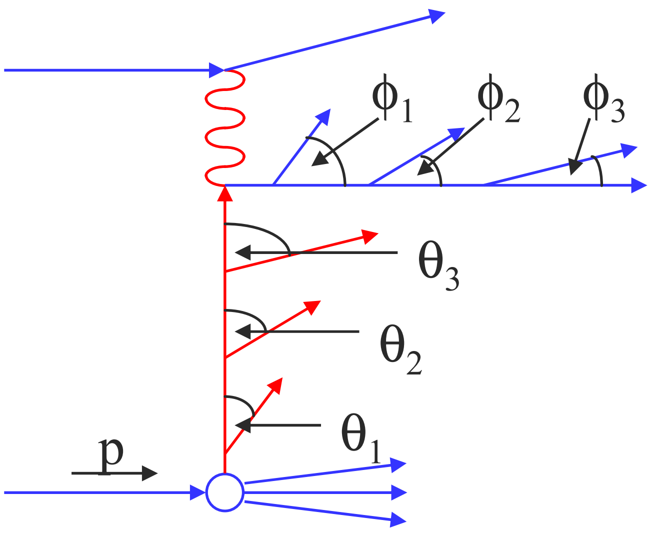
- Given parton distributions at some Q_0^2, determine how these evolve with Q^2.



- Interference effects give rise to angular ordering of partons.
- DGLAP approximation:

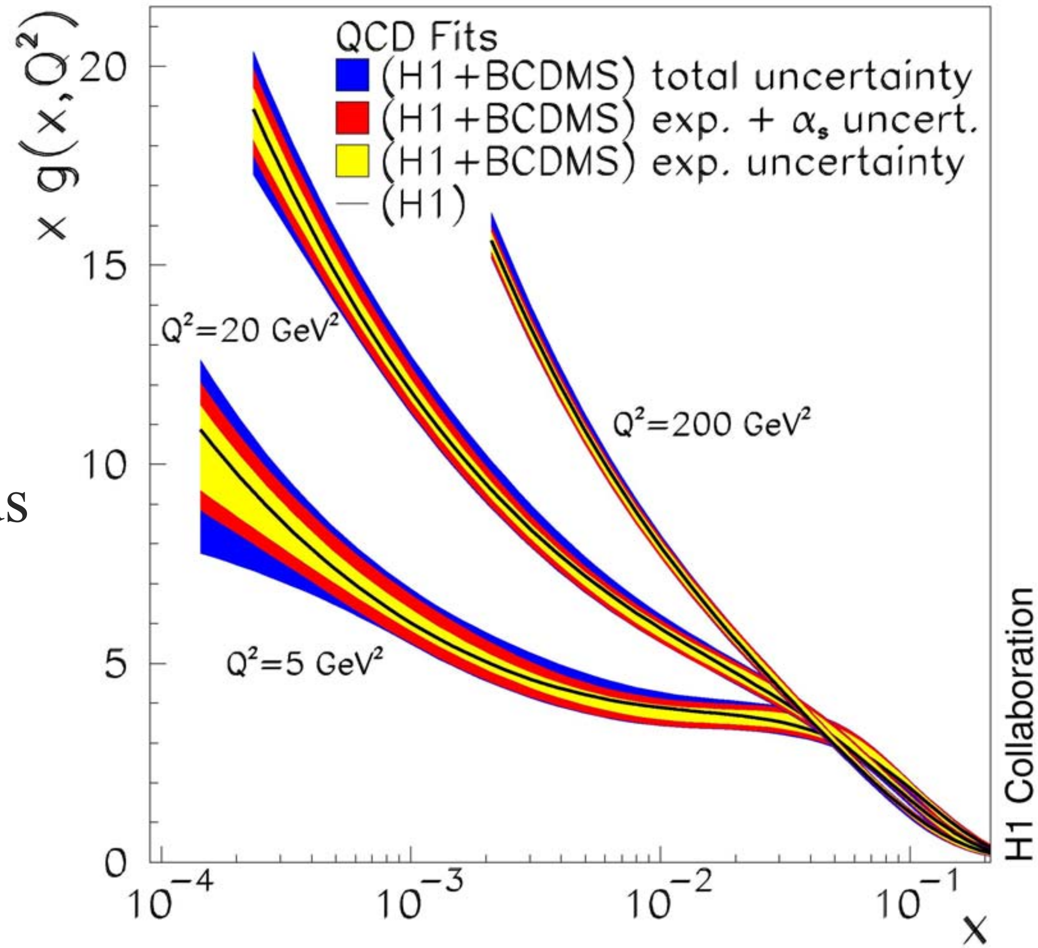
$$\theta_1 < \theta_2 < \theta_3 \cdots$$

$$\to k_{T1} < k_{T2} < k_{T3} \cdots$$



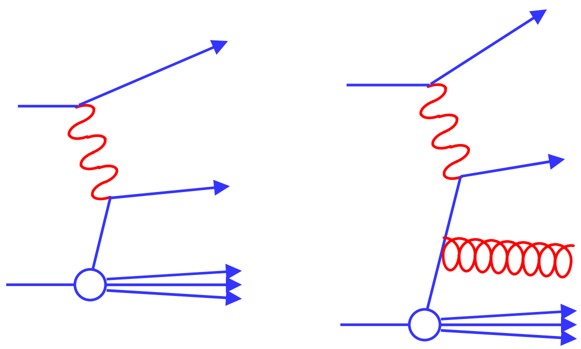
QCD and scaling violations

- NLO DGLAP fit gives simultaneously:
- $\alpha_S(M_Z{}^2) = 0.1150 \pm 0.0017(\text{exp.})$
  $^{+0.0009}_{-0.0005}(\text{mod.}) \pm 0.005(\text{scale}).$
- PDFs, e.g. gluon.
- Progress being made towards NNLO calculations.
- PDFs with complete error determinations available.



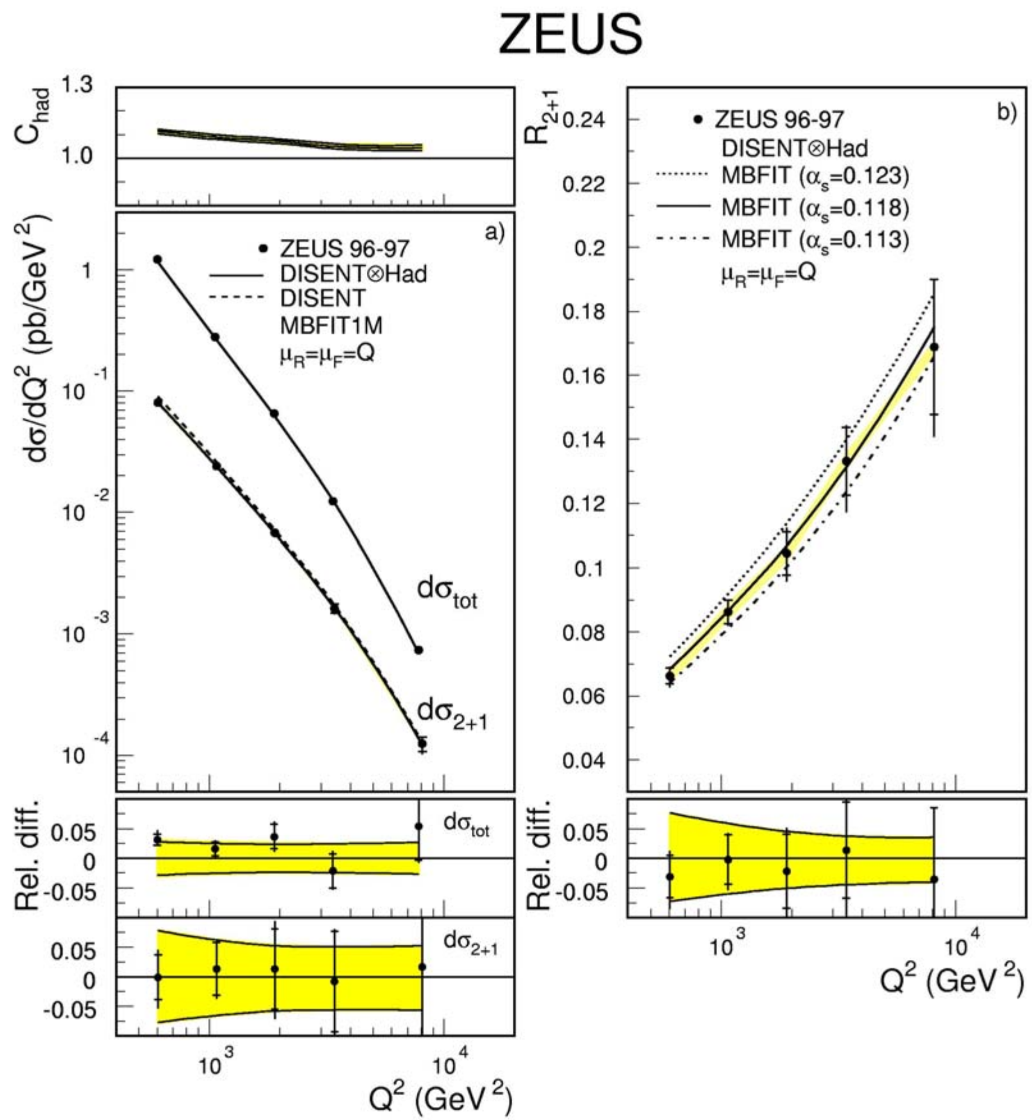
Jet production in DIS

- Described by NLO QCD.

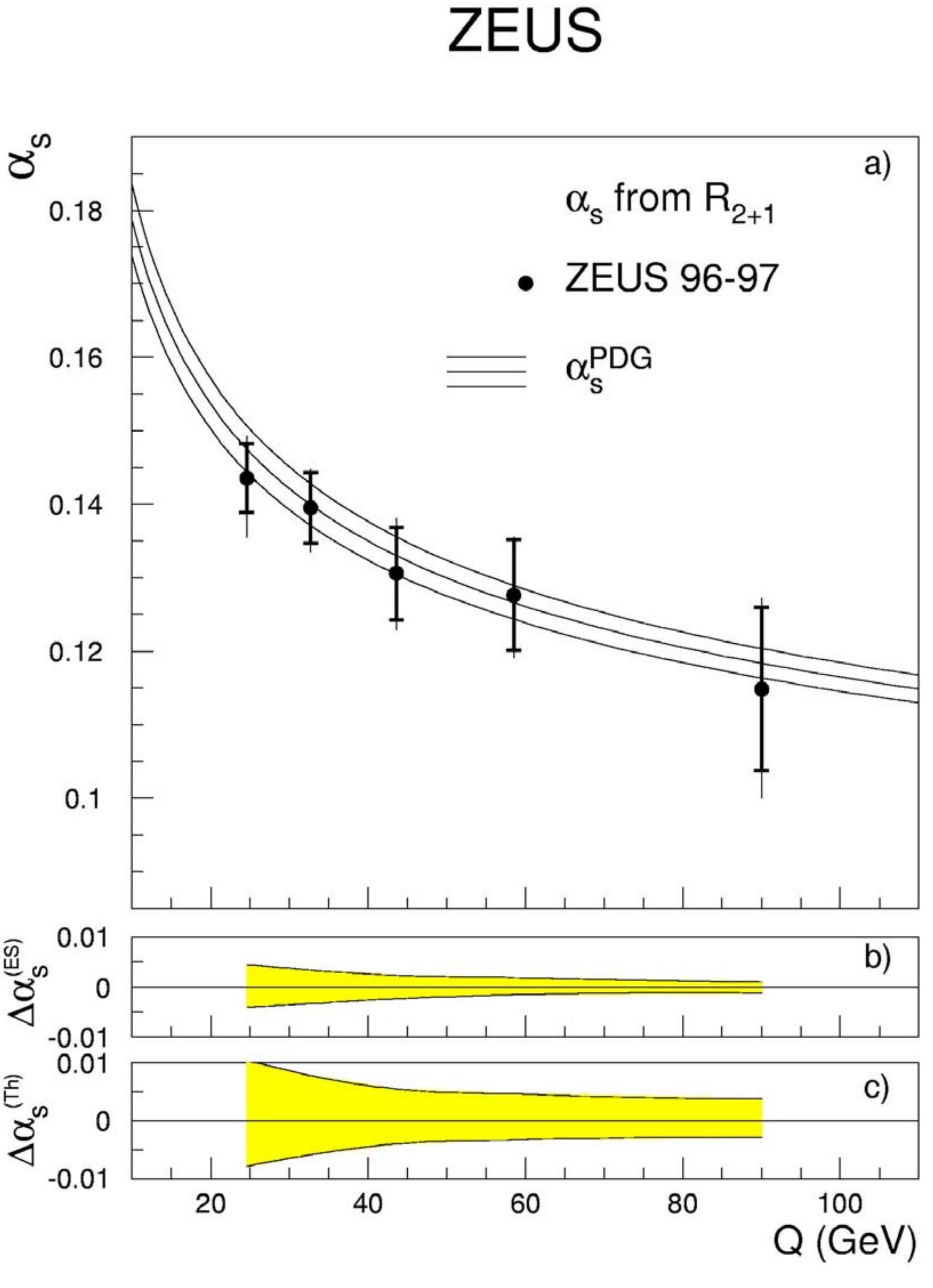


- $\alpha_S(M_Z^{\,2}) = 0.1177 \pm 0.0019(\text{stat.})\,{}^{+0.0024}_{-0.0033}(\text{exp.})\,{}^{+0.0057}_{-0.0044}(\text{theo.})$

- Again, NNLO calculations are underway and will reduce “theoretical” errors.



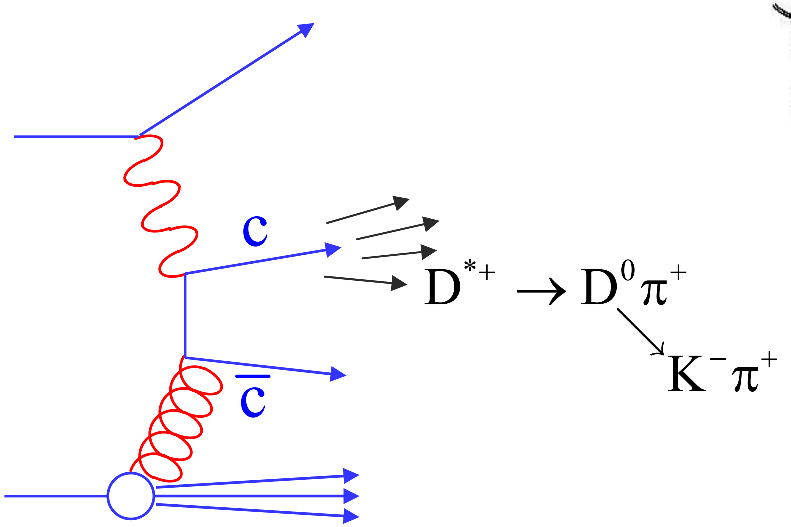
Running of strong coupling



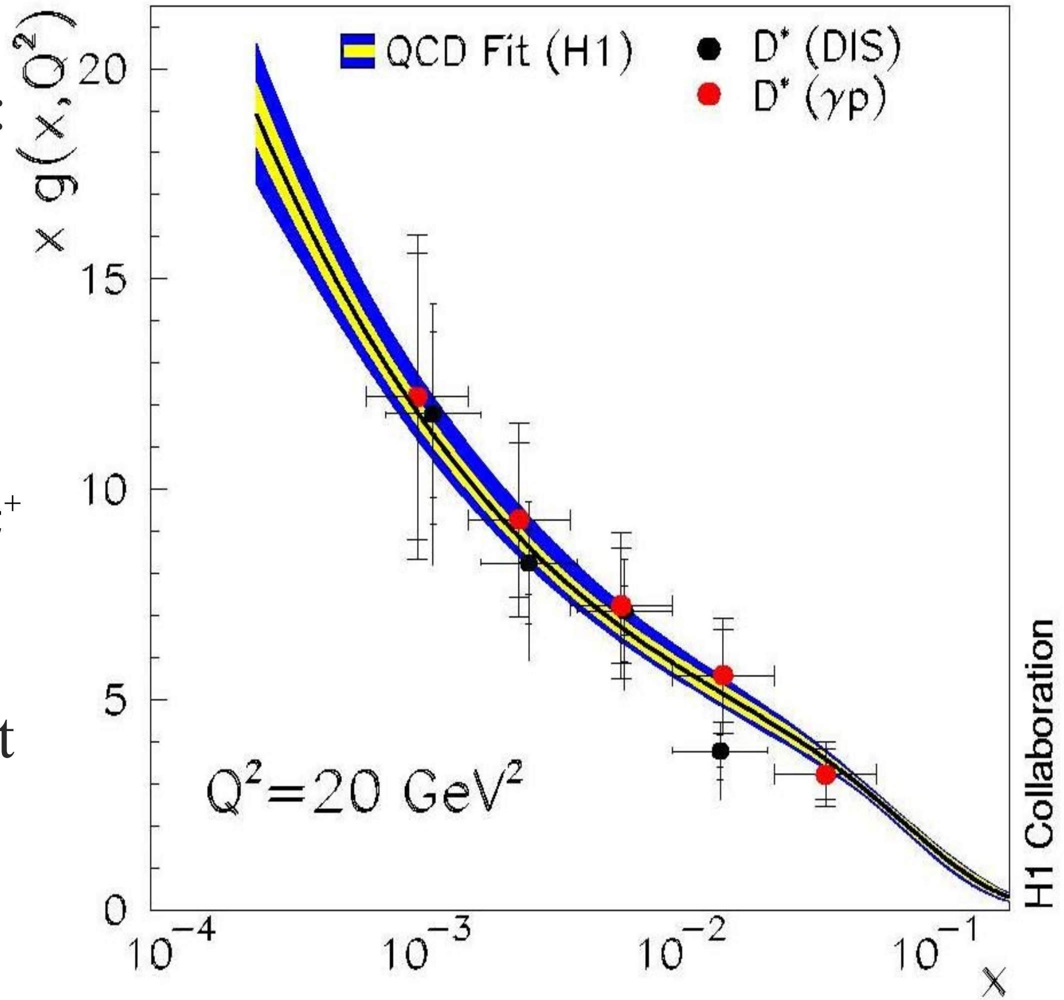
- Running of α_S observed with one experiment.

Gluon distribution from final state

- Charmed quarks produced through boson-gluon fusion:



- Process allows measurement of gluon momentum, hence $x\,g(x,Q^2)$.



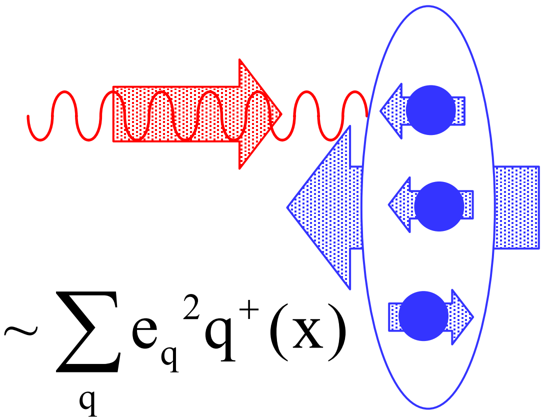
Spin structure measurements

- Use polarised target and polarised electron beam.
- Extract asymmetry

$$A = \left(\sigma_{1/2} - \sigma_{3/2}\right) / \left(\sigma_{1/2} + \sigma_{3/2}\right)$$

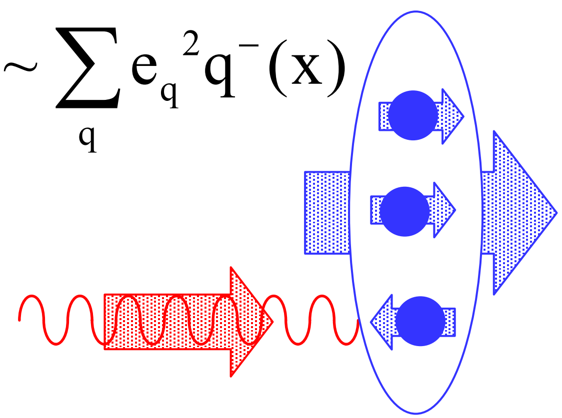
$$\approx \frac{\sum e_q^{\,2}\left(q_+(x) - q_-(x)\right)}{\sum e_q^{\,2}\left(q_+(x) + q_-(x)\right)}$$

$$\approx \frac{g_1(x)}{F_1(x)}$$



$$\sigma_{1/2} \sim \sum_q e_q^{\,2} q^+(x)$$

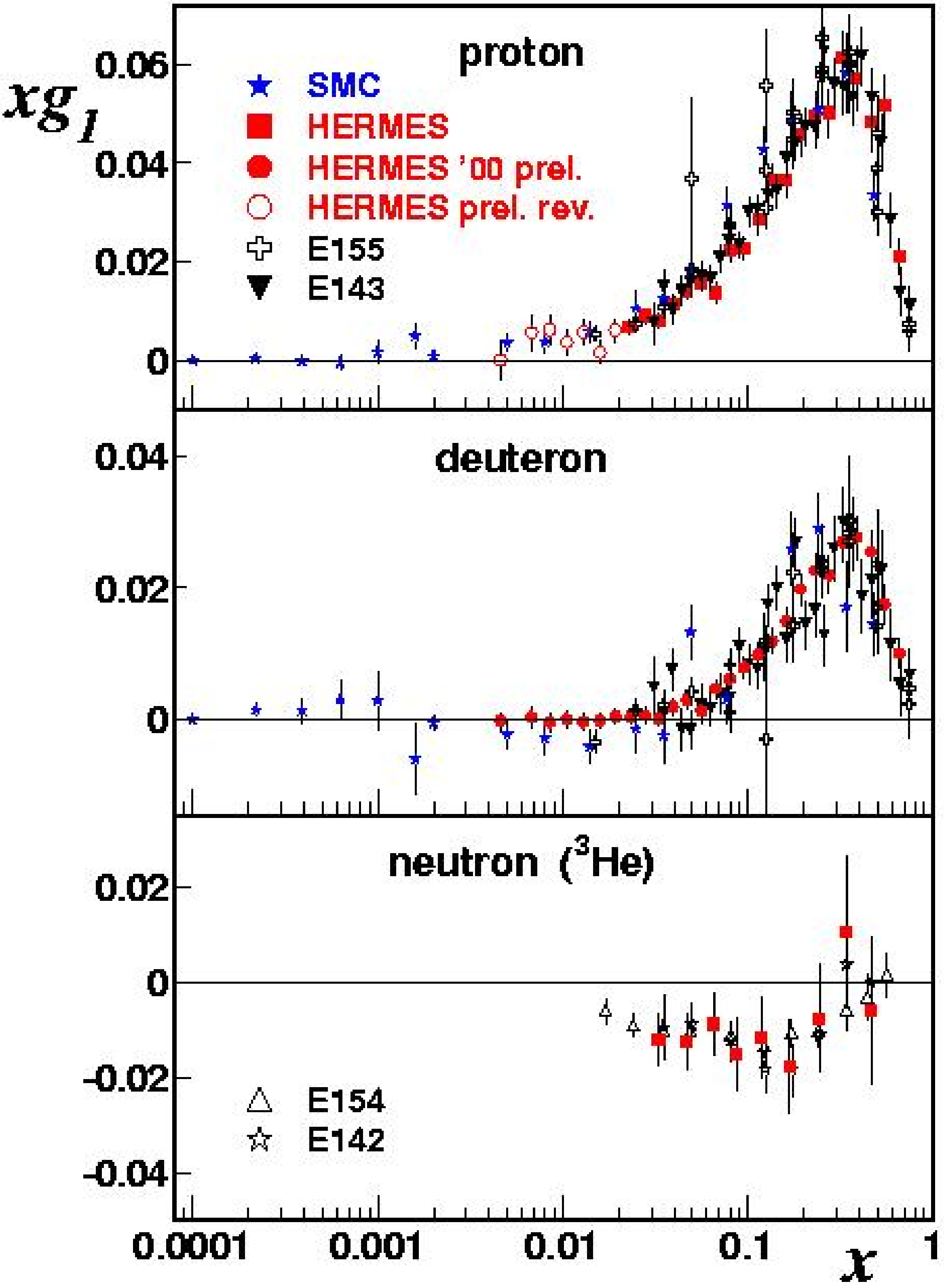
$$\sigma_{3/2} \sim \sum_q e_q^{\,2} q^-(x)$$



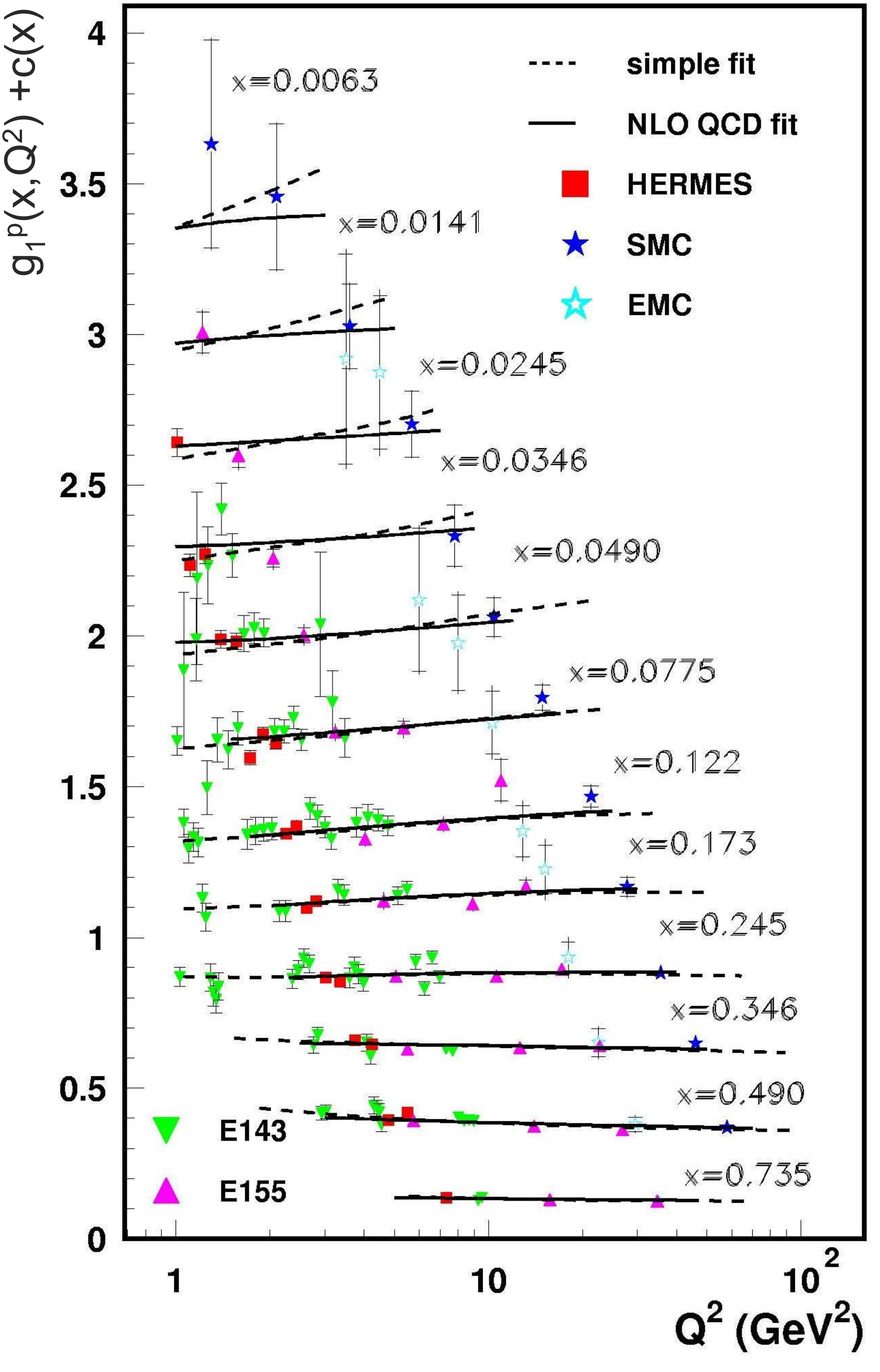
- $g_1(x) = \frac{1}{2}\sum e_q^{\,2}\left(q^+(x) - q^-(x)\right)$

$$F_1(x) = \frac{1}{2}\sum e_q^{\,2}\left(q^+(x) + q^-(x)\right) = \frac{1}{2}\sum e_q^{\,2} q(x)$$

Measurements of $g_1(x,Q^2)$

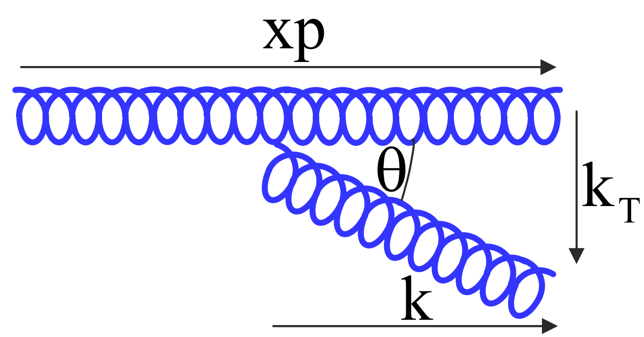


Scaling violations, $g_1(x,Q^2)$

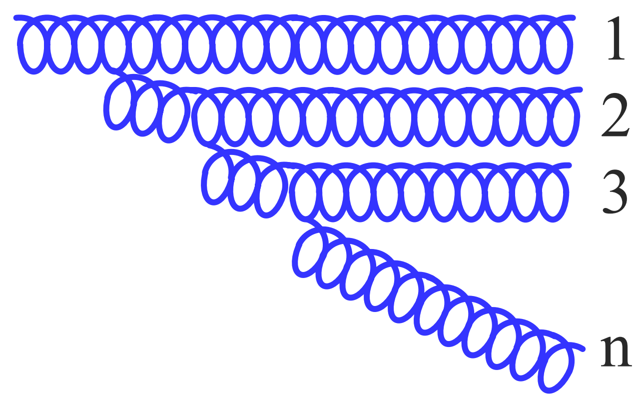


QCD radiation inside proton

- Probability of g emission:
  $$dP = \frac{\alpha_S(k_T^2)}{\pi} C_A \frac{dk_T^2}{k_T^2} \frac{dk}{k}$$
- Consider small k_T fluctuations $dk_T^2 \sim k_T^2$ and use $dk/k = dy$.
- Then have: $dP \sim \frac{\alpha_s(k_T^2)}{\pi} C_A dy$
- For n^{th} gluon $dP_n \sim \frac{\alpha_S(k_T^2)}{\pi} C_{eff} dy$
- If treat g charges as random:
  $$dP_n \sim \frac{\alpha_S(k_T^2)}{\pi} n C_A dy$$

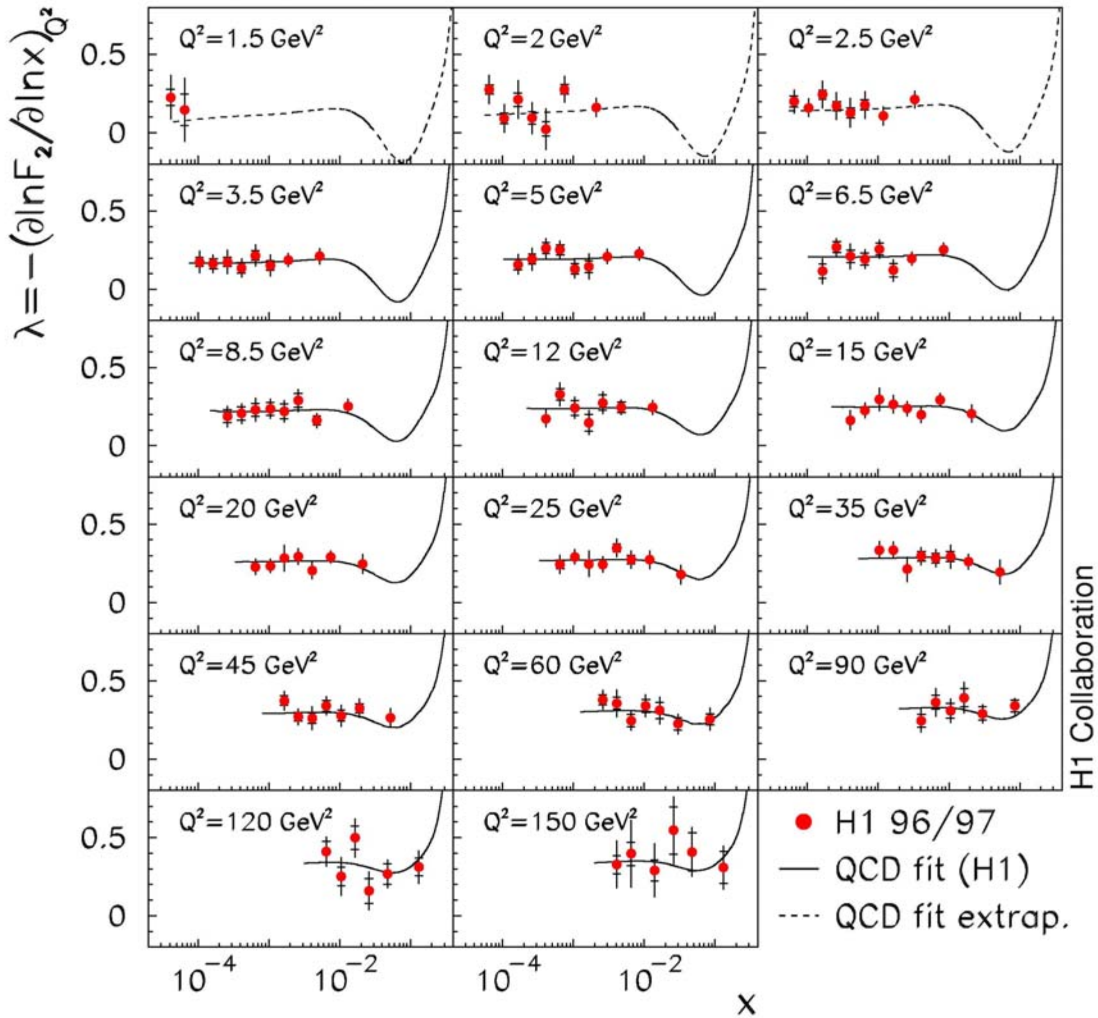


$$y \sim -\ln \tan\frac{\theta}{2} \sim \ln\frac{2k}{k_T}$$



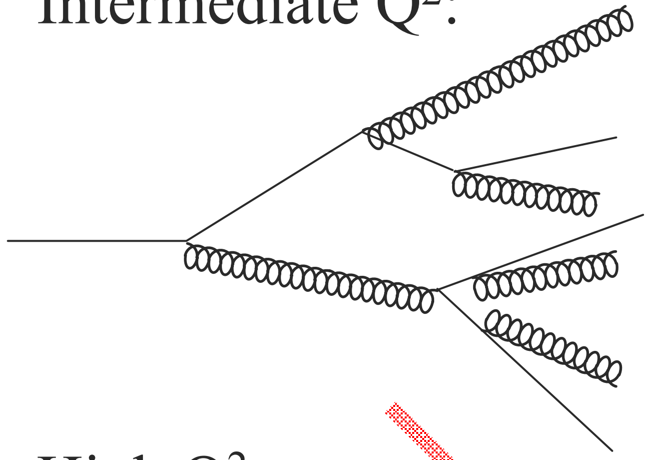
Variation of $F_2(x,Q^2)$ with x

- BFKL result:
  $$x\,q(x,Q^2) \sim x^{-\delta}$$
  $$\delta = 0.3...0.5$$
- Causes $F_2 \uparrow$ as $x \downarrow$
- Does data show
  $$F_2(x,Q^2) \sim x^{-\lambda}\ ?$$
- Study derivative
  $$\frac{\partial \ln F_2}{\partial \ln x} = -\lambda$$
- Data consistent with
  $$F_2 \sim x^{-\lambda} \text{ for } x < 0.01$$

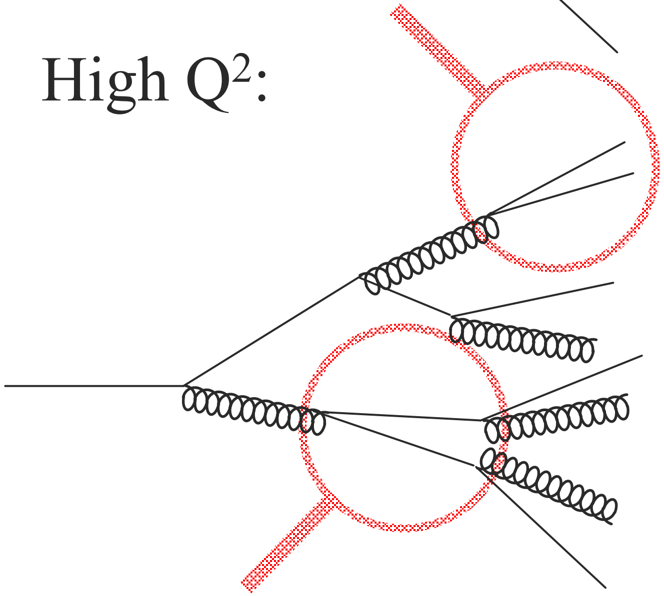


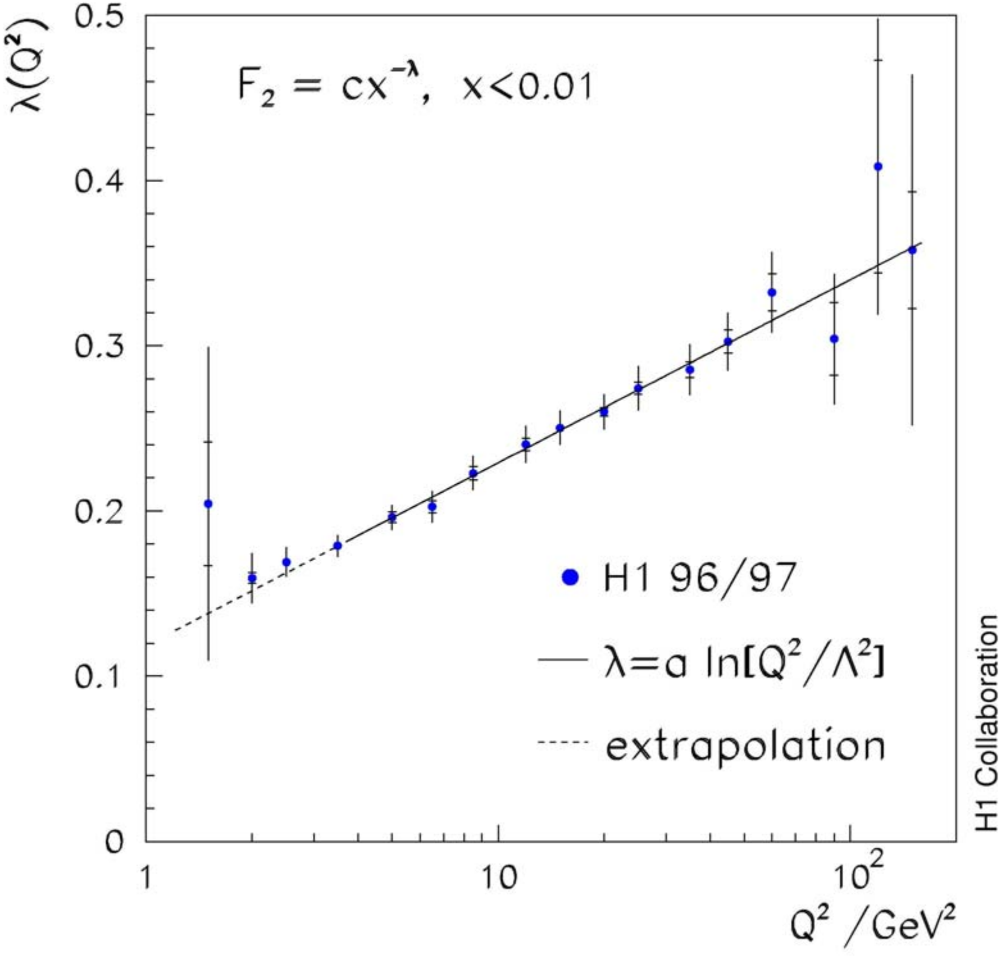
Change of F_2 with x and Q^2

- Intermediate Q^2:



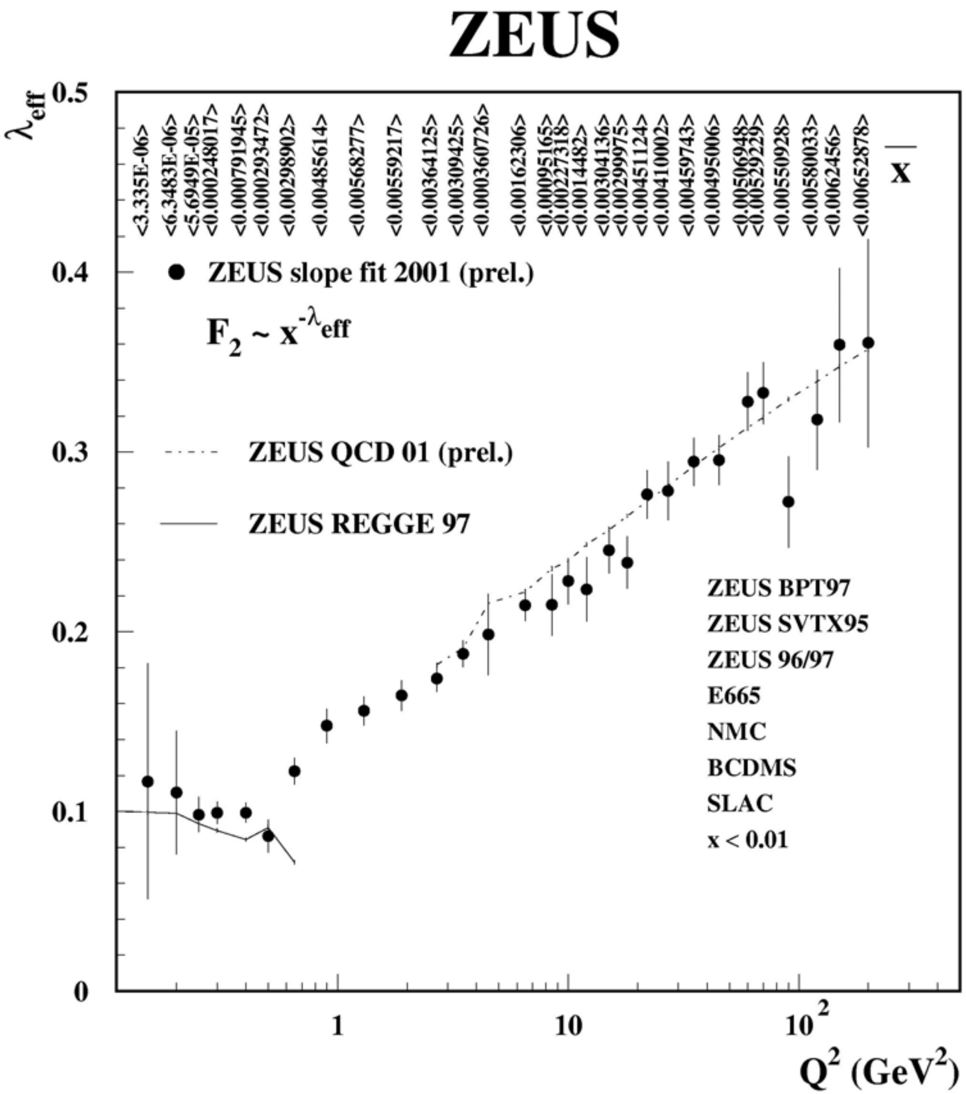
- High Q^2:





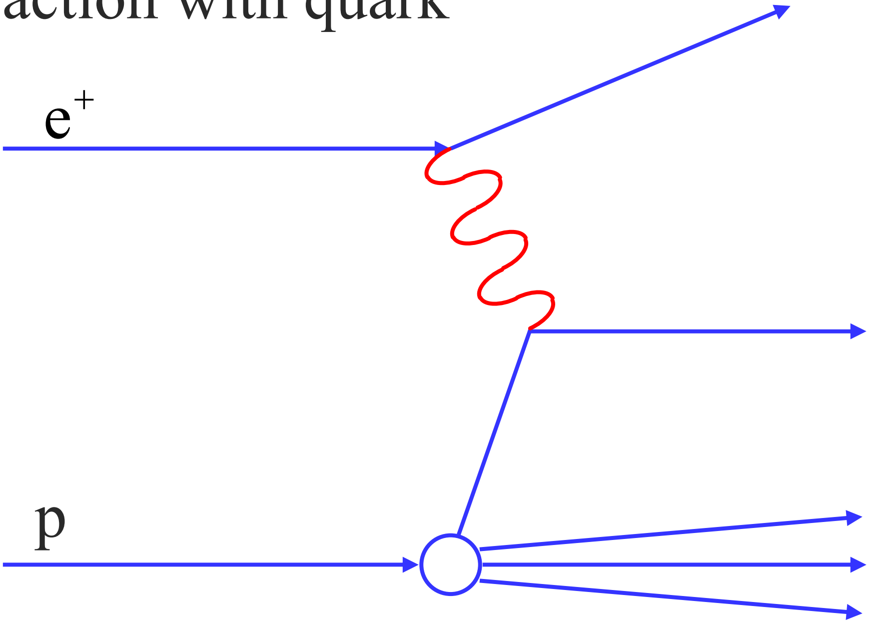
Behaviour at low Q^2

- What happens at the "ankle" at $Q^2 \sim 0.6$ GeV^2?
- Amount of radiation decreases and becomes "resolution independent".
- Corresponds to length $r \sim 0.2 \ldots 0.3$ fm.
- C.f. proton radius $r_p = 0.8$ fm.
- "Non-partonic" sub-structure within proton?
- Other evidence for this?

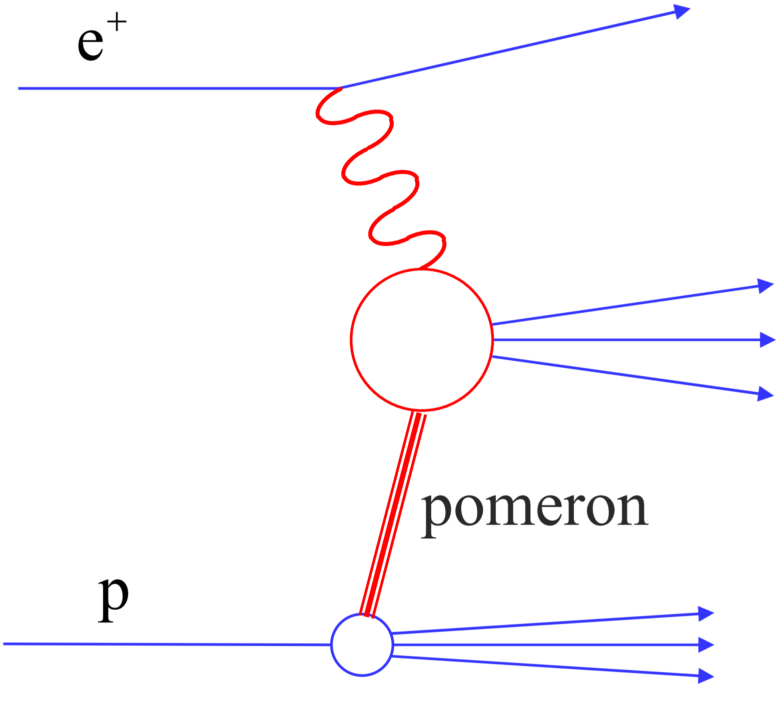


Rapidity gap events

- Interaction with quark



- Interaction with colourless component of proton



Masses of baryons from hyper-fine splitting

- Using $m_u = m_d = 363\,\text{MeV},\; m_s = 538\,\text{MeV}:$

Baryon (mass in MeV)	Composition (q = u, d)	Predicted mass (MeV)
N(939)	qqq	939
Λ(1116)	qqs	1114
Σ(1193)	qqs	1179
Ξ(1318)	qss	1327
Δ(1232)	qqq	1239
Σ(1384)	qqs	1381
Ξ(1533)	qss	1529
Ω(1672)	sss	1682

Summary

- HERA has uncovered a wealth of structure at low x in the proton.
- Continuing theoretical and experimental improvements in study of perturbative QCD, in regions of applicability obtain good description of:
  - Unpolarised and polarised structure functions.
  - Hadronic final state.
- Perhaps starting to see features in data in low Q^2 region that will help develop an understanding of confinement.