

Multiplicities & Factorial Moments

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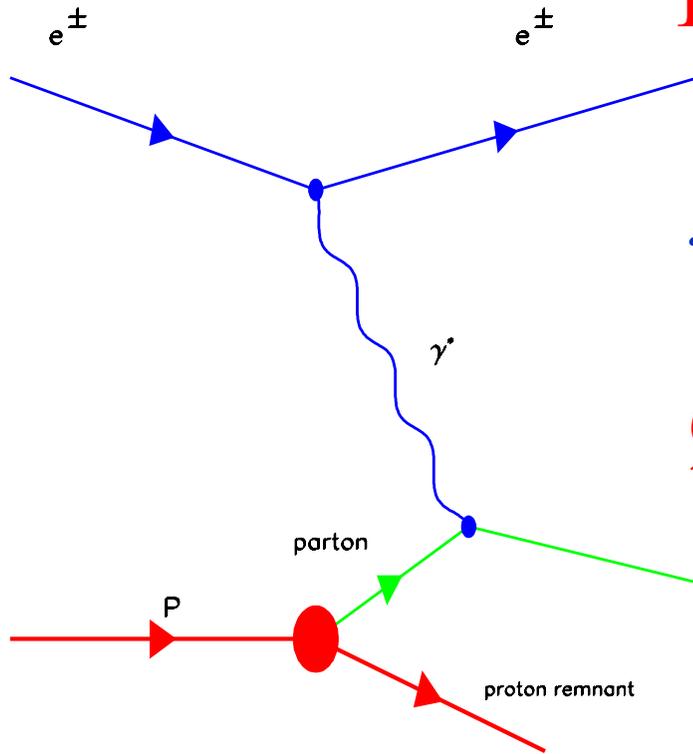


University of Bristol
(on behalf of the ZEUS Collab.)

- Introduction & Motivation
- Results
- Conclusions



Naïve Quark Parton Model (QPM)



Lorentz scalars: y, Q^2, x

$$y = 1 - \frac{E'_e}{2E_e} (1 - \cos \vartheta)$$

$$Q^2 = 2E'_e E_e (1 + \cos \vartheta)$$

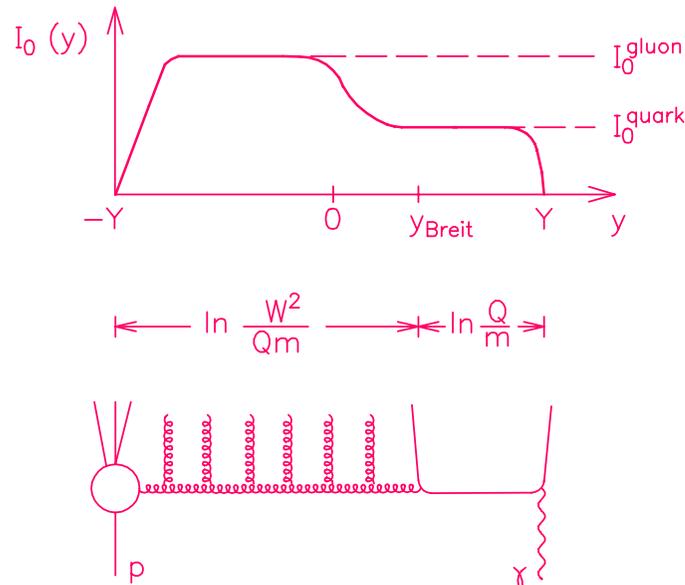
$$x = \frac{Q^2}{ys}$$

where \sqrt{s} is the eP centre of mass energy

Multiplicity Distributions

Investigation of dynamics of hadronisation process - study the $\langle n_{ch} \rangle$ as a function of invariant mass (M_{eff}) in a fixed rapidity range (in lab. frame)

M_{eff} is essentially measuring the rapidity “along” the gluon ladder



Gluon or quark initiated - measurement of colour charge

Factorial Moments

fractal dimension techniques (eg Rényi dimensions) to multiparticle production processes

investigation of cascading dynamics & “self-similarity” of branching processes

predictions exist within framework of pQCD

use of DLA and MLLA (in conjunction with LPHD)

[theoretical work: Dokshitzer, Ochs (MLLA)

Dremin, Wosiek (Factorial moments)]

Mathematical Interlude

Fractal dimension, F_D

$$M \sim l^{F_D}$$

- M is mass & l is length. $F_D = 1$ for a line, $F_D = 2$ for a square...

Koch Curve



→ self-similar curve with dimension $F_D = \ln(4)/\ln(3)$

Generalise to multi-fractals $F_D \rightarrow D_q$ (Rényi dimension)

(cf replace our homogenous stick with an inhomogenous one)

Factorial Moments

$$F_q(\Delta\Omega) = \langle n(n-1)\dots(n-q+1) \rangle / \langle n \rangle^q, \quad q = 2, 3, \dots$$

where n is number of charged particles inside a phase-space region of size $\Delta\Omega$ and $\langle \dots \rangle$ denotes average.

For uncorrelated particle production within $\Delta\Omega$, $F_q = 1$ (Poisson stats) rise follows power law - “intermittency”

moments probe different dynamics depending on choice of $\Delta\Omega$

Kinematic Selections

Multiplicity: 1995 data (5.5 pb⁻¹)

$$8 < Q^2 < 1200 \text{ GeV}^2$$

$$70 < W < 260 \text{ GeV}$$

Particles with angular acceptance of $|\eta(\text{lab})| < 1.75$

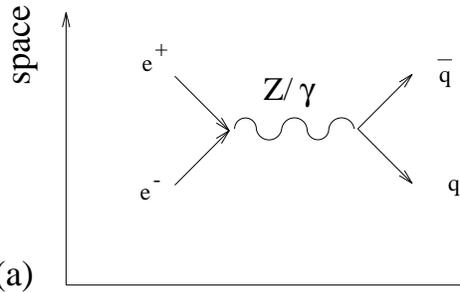
Moments: 96+97 (38.4 pb⁻¹)

$$Q^2 > 1000 \text{ GeV}^2$$

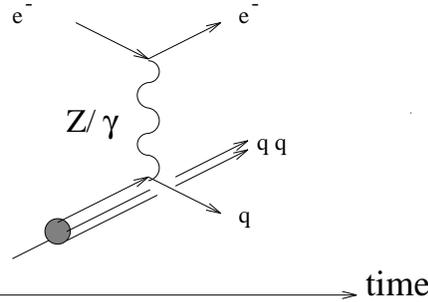
Moments measured in
current region of Breit frame

The Breit Frame 'Brickwall' frame

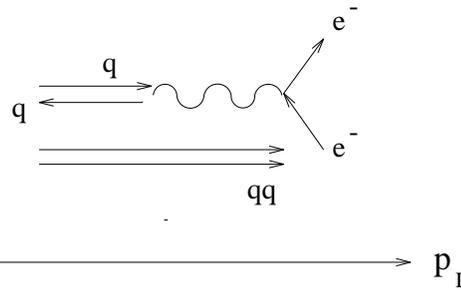
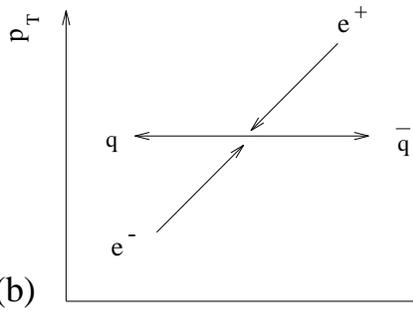
Electron-positron Annihilation



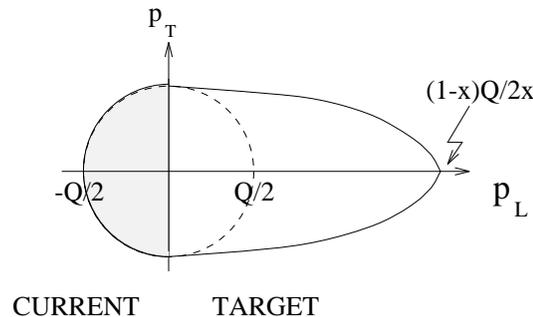
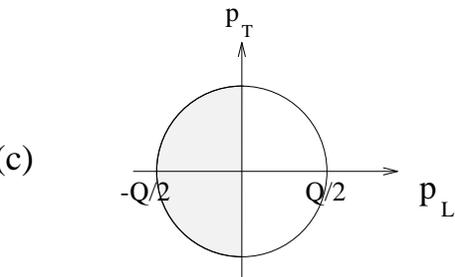
DIS in the Breit Frame



Phase space for e^+e^- annihilation evolves with $Q/2 = \sqrt{s}/2$



Current hemisphere of Breit frame evolves as $Q/2$

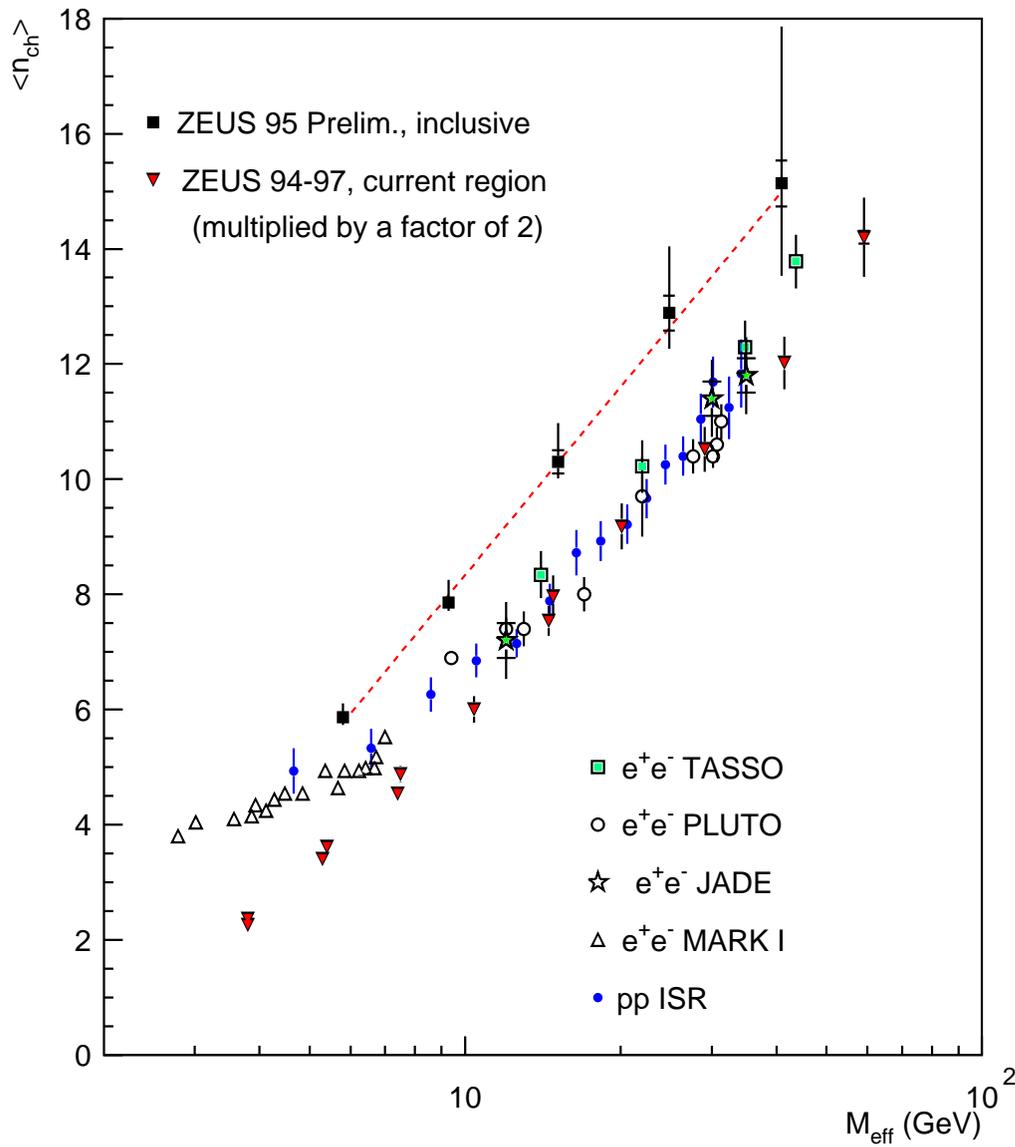


Current region $\equiv e^+e^-$ annihilation

Data grows linearly with
 $\log(M_{eff})$

$\langle n_{ch} \rangle$ higher than e^+e^-
data ($M_{eff} \equiv \sqrt{s_{ee}}$), low
energy pp data and
(Breit) current region in
DIS ($M_{eff} \equiv Q$)

Data compatible with
picture of additional
coherent gluon radiation
due to (octet) colour
charge



Systematic Checks

Event Selection:

- checks on y_e (2%), y_{JB} (3%), $E-p_z$ (1%), vertex cut(1%)

Trk selection:

- tightening of p_T cut (1%) and η cut (2%)

Monte Carlo dependence:

- LEPTO + POMPYT (up to ~16%)

Analytic QCD Results

$\Omega \equiv p_T^{cut}$ or p^{cut} ie $p_T > p_T^{cut}$ ($|p| > p^{cut}$)

$$F_q(p_T^{cut}) \approx 1 + \frac{q(q-1)}{6} \frac{\ln(p_T^{cut}/Q_0)}{\ln(E/Q_0)}, \quad F_q(p^{cut}) \approx \text{const} > 1$$

E = jet energy, Q_0 = parton shower cut-off. DLLA approx

(Lupia, Ochs & Wosiek)

p_T : prediction of correlations (presence of gluon enhances probability of emission of another one)

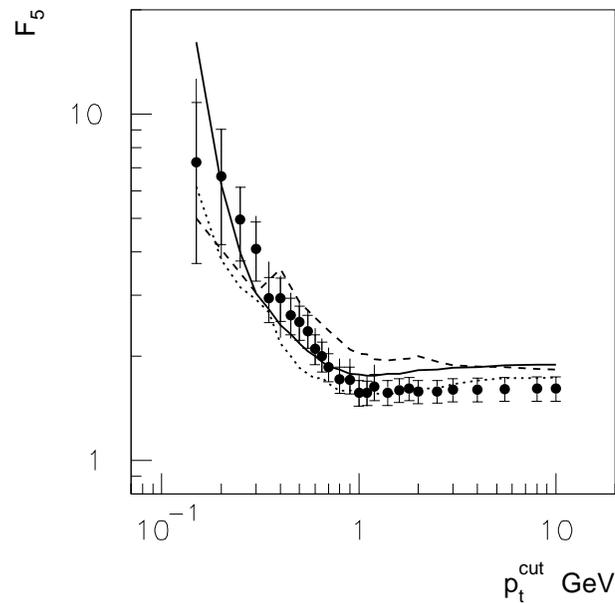
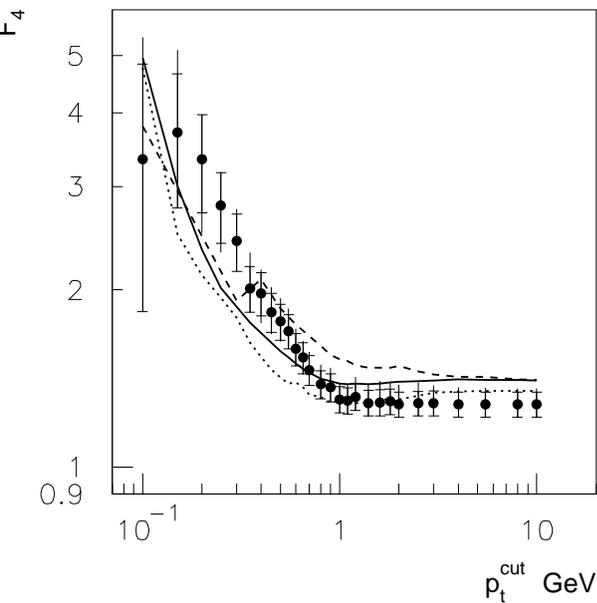
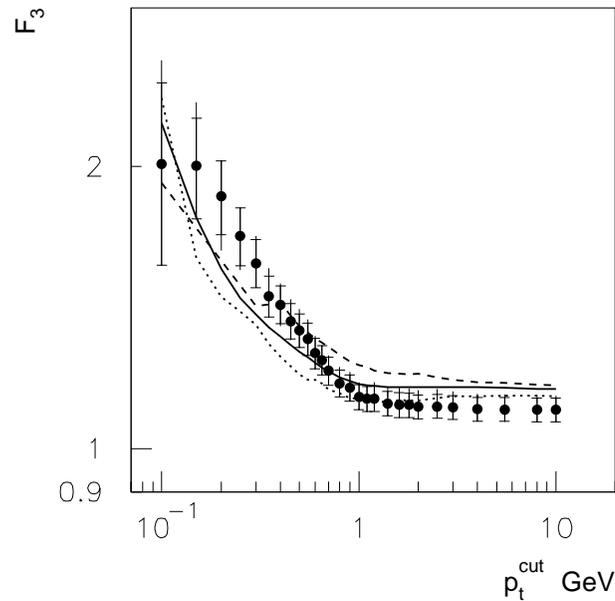
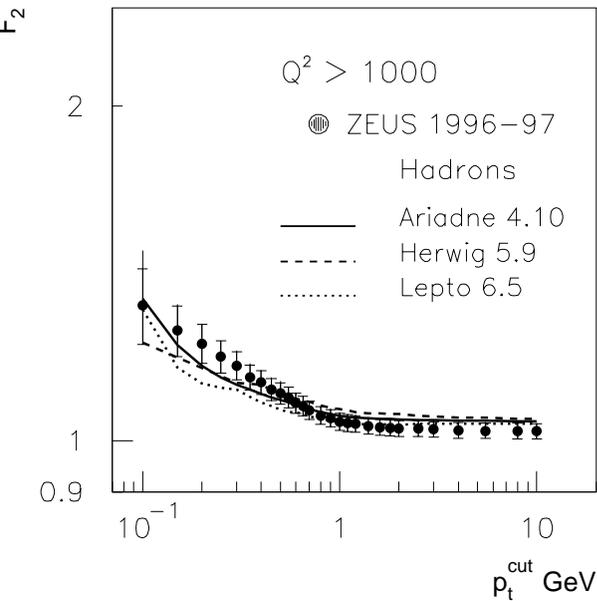
As $p_T \rightarrow Q_0$ correlations vanish due to coherence effects

p : dist^{bn} of soft gluons remains non-Poisson

MC models
qualitatively
describe the data (in
detail there are
discrepancies)

data in disagreement
with theoretical
predictions

Result for
momentum look
similar to p_t



Systematic Checks

Event Selection:

- checks on y_e , y_{JB} , $E-p_z$, vertex cut

Trk selection:

- tightening of p_T cut and η cut

Typically a few % change - but can be $\sim 20\%$ at low p , p_T or high z

Analytic QCD Results

$\Omega \equiv$ polar rings of size Θ around axis centred at Θ_0 (see fig \blacktriangleright)

$$\ln \frac{F_q(z)}{F_q(0)} = z(1 - D_q)(q - 1) \ln \frac{E\Theta_0}{\Lambda}$$

E = jet energy, Θ_0 = opening half angle of jet, Λ = QCD scale

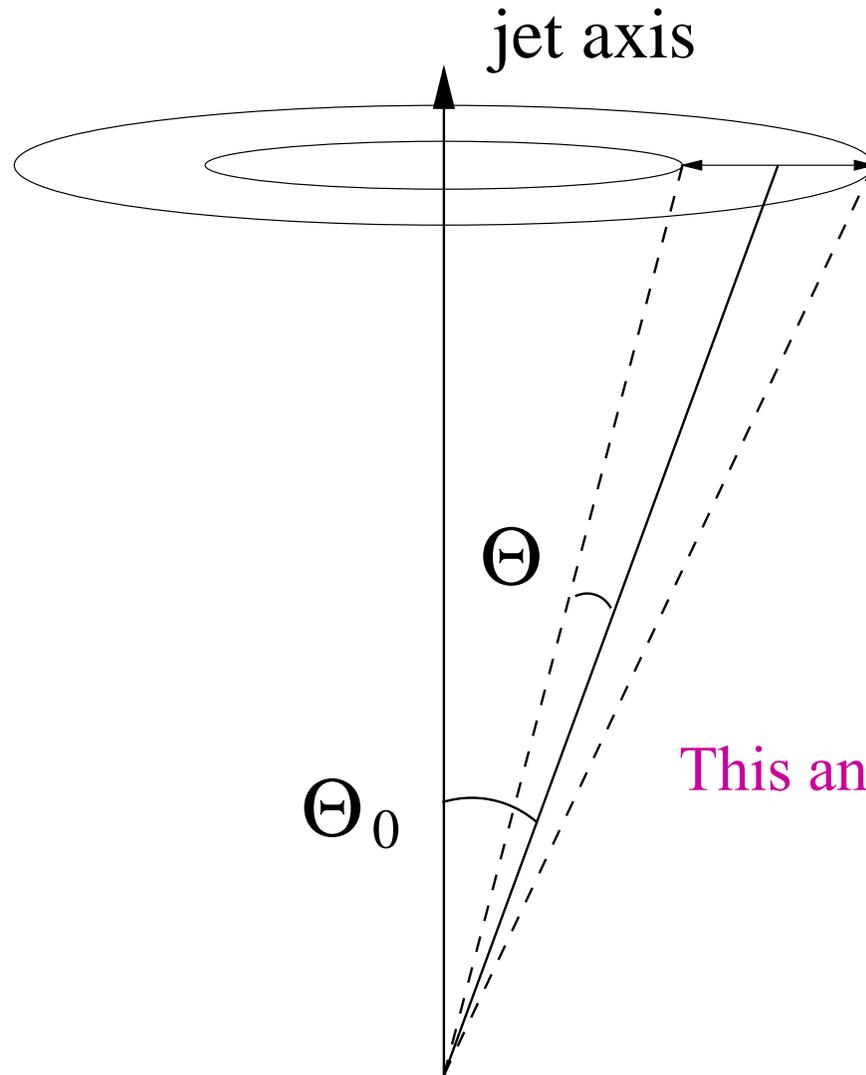
$$z = \frac{\ln(\Theta_0/\Theta)}{\ln(E\Theta_0/\Lambda)}$$

Number of predictions in DLA (Dokshitzer & Dremin; Brax, Meunier & Peschanski; Ochs & Wosiek) and in MLLA (Dokshitzer & Dremin) for the Rényi dimensions, D_q .

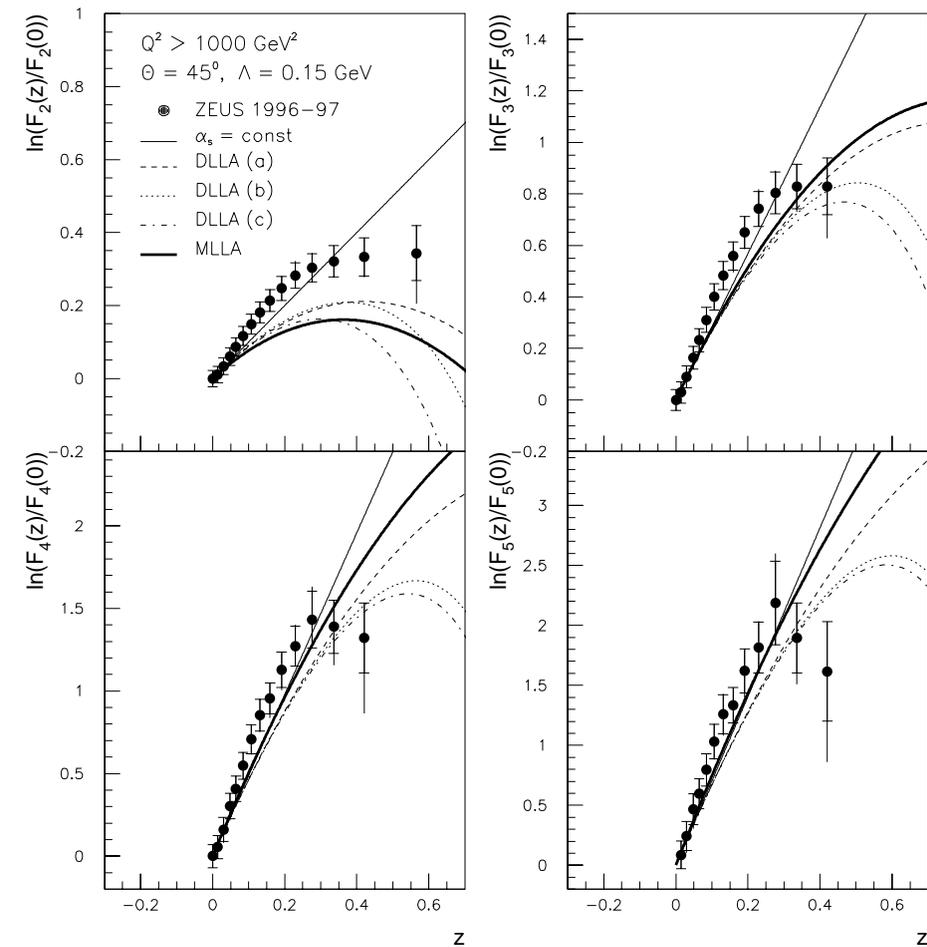
$$D_q \rightarrow 1 \quad \text{for Poisson dist}^{\text{bn}}$$

QCD predictions have $D_q = D_q(q, z)$

Defⁿ of angular variable

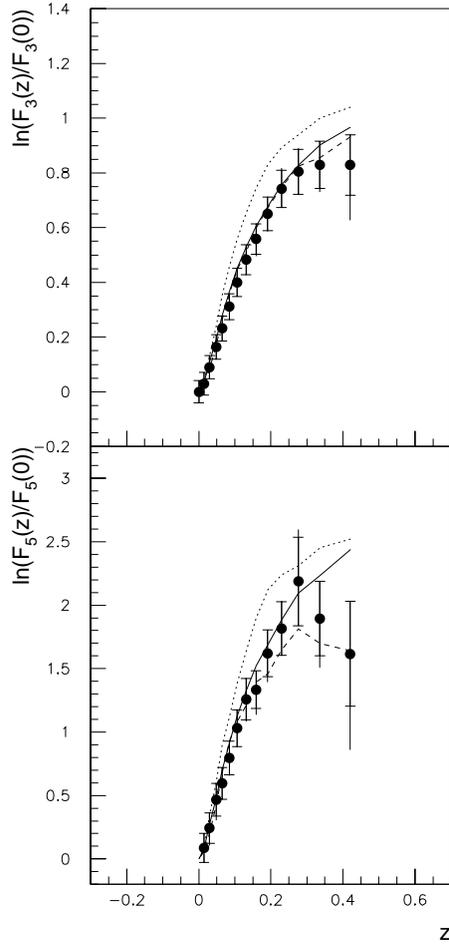
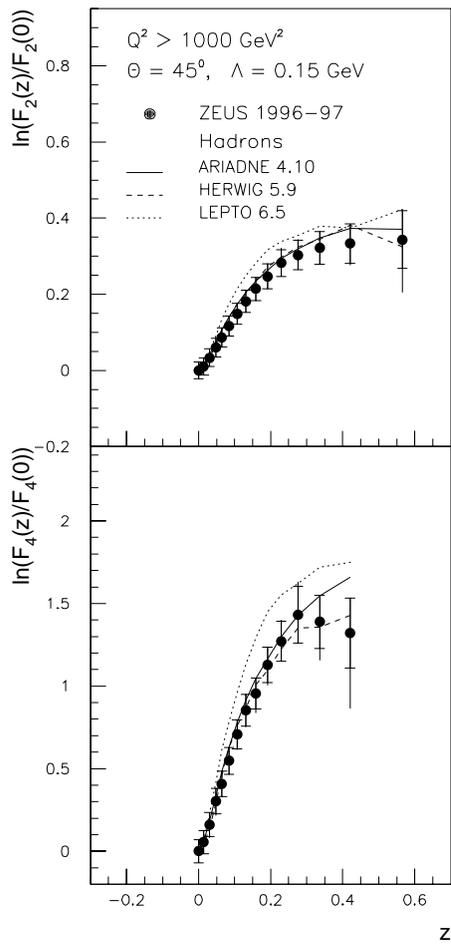


This analysis uses $\Theta_0=45^\circ$



For $\Theta_0 = 45^\circ$ & $\Lambda = 150 \text{ MeV}$,
 general trend of data & theory
 agree ie moments \uparrow as $z \uparrow$

Particular at lower order moments
 data & theory don't agree in
 detail.



ARIADNE & HERWIG
reproduce the data

LEPTO overshoots the data

Non-negligible hadronisation
effects

Conclusions

- $\langle n_{\text{ch}} \rangle$ higher than e^+e^- , low energy pp data & (Breit) DIS current region - colour charge effect ?
- Multiplicity factorial moments exhibit strong rise in restricted intervals as p_T , p & Θ decreases
- Analytic calculations (for partons) do not show the same increase for the factorial moments
- MC models (generally) reproduce the data
- Substantial contribution from hadronisation - LPHD hypothesis non-applicable