

4 **Combination and QCD analysis of beauty and charm production**
5 **cross section measurements in deep inelastic ep scattering at**
6 **HERA**

7

8 **The H1 and ZEUS Collaborations**

9

Abstract

10

11 Measurements of open beauty and charm production cross sections in deep inelastic ep
12 scattering at HERA from the H1 and ZEUS Collaborations are combined. Reduced cross
13 sections for beauty and charm production are obtained in the kinematic range of photon
14 virtuality $2.5 \leq Q^2 \leq 2000 \text{ GeV}^2$ and Bjorken scaling variable $3 \times 10^{-5} \leq x_{\text{Bj}} \leq 5 \times 10^{-2}$.
15 The combination method accounts for the correlations of the statistical and systematic un-
16 certainties among the different data sets. The combined data are compared to perturbative
17 QCD predictions and used together with the combined inclusive deep inelastic scattering
18 cross sections from HERA to determine the charm and beauty quark masses.

19

1 Introduction

Measurements of open charm and beauty production in deep-inelastic electron¹–proton scattering (DIS) at HERA provide important input for stringent tests of the theory of strong interactions, quantum chromodynamics (QCD). This note describes an extension of the previous H1 and ZEUS combination of charm measurements in DIS [1] with new charm and beauty data [2–8].

The primary aim of this analysis is to obtain a single consistent dataset which provides information on charm and beauty production in DIS in the full phase space, suitable for comparison with various theoretical predictions. The reduced charm and beauty cross-sections, $\sigma_{\text{red}}^{c\bar{c}}$ and $\sigma_{\text{red}}^{b\bar{b}}$, respectively, are combined to create one consistent set of charm and beauty cross-section measurements in the kinematic range of photon virtuality $2.5 \leq Q^2 \leq 2000 \text{ GeV}^2$ and Bjorken scaling variable $3 \times 10^{-5} \leq x_{\text{Bj}} \leq 5 \times 10^{-2}$. In this note, the combined data are compared to theoretical predictions obtained in the fixed-flavour-number scheme (FFNS) at next-to-leading order (NLO) QCD using HERAPDF2.0 and ABM11 proton distribution functions (PDFs) and approximate next-to-next-to-leading order (NNLO) using ABMP16 PDFs. The new combined data are used together with the combined inclusive deep inelastic scattering cross sections from HERA to determine the running charm and beauty quark masses in the QCD analysis at NLO.

2 Input data

The input data samples [2–14] used in the combination are listed in Tab. 1. Measurements have been obtained both from the HERA-I (1992–2000) and HERA-II (2003–2007) data-taking periods. The combination includes measurements of charm and beauty production performed using different tagging techniques: the reconstruction of particular decays of charmed mesons (datasets 2–7, 9, 10), the inclusive analysis of tracks exploiting lifetime information (datasets 1, 11) and the reconstruction of electrons and muons from heavy-flavour semi-leptonic decays (datasets 8, 12, 13).

Datasets 1–8 have been used in the previous combination [1], while datasets 9–13 are newly included. Note that dataset 9 replaces one of the datasets used in the previous combination [1] (dataset 8 from Table 1 of [1]), which is its subset. All cross sections are updated using the most recent hadron decay branching ratios [15].

3 Combination method

The quantities to be combined are the reduced charm and beauty cross sections, respectively, defined as:

$$\sigma_{\text{red}}^{Q\bar{Q}} = \frac{d^2\sigma^{Q\bar{Q}}}{dx_{\text{Bj}}dQ^2} \cdot \frac{xQ^4}{2\pi\alpha^2(1+(1-y)^2)}. \quad (1)$$

¹In this note, ‘electron’ is used to denote both electron and positron if not stated otherwise. For D mesons, charge-conjugate modes are implied.

50 Here $Q\bar{Q}$ stands for $c\bar{c}$ or $b\bar{b}$ quark-antiquark pairs, and y is the inelasticity. The cross section
 51 $d^2\sigma^{Q\bar{Q}}/dx_{Bj}dQ^2$ is given at the Born level without QED and electroweak radiative corrections,
 52 except for the running electromagnetic coupling α .

53 The combined cross sections are determined at common (x_{Bj}, Q^2) points. The grid points for
 54 $\sigma_{\text{red}}^{c\bar{c}}$ are chosen to be the same as in [1], where 52 points were used, while for $\sigma_{\text{red}}^{b\bar{b}}$ a subset of 27
 55 of these points is used. The combined reduced cross sections are provided at the centre-of-mass
 56 energy $\sqrt{s} = 318$ GeV.

All measurements to be combined are already corrected to Born level (using running α).
 The results of the H1 inclusive lifetime analysis (dataset 1) are directly taken from the origi-
 nal measurement in the form of $\sigma_{\text{red}}^{c\bar{c}}$ and $\sigma_{\text{red}}^{b\bar{b}}$.² For all other measurements the inputs to the
 combination are visible cross sections, $\sigma_{\text{vis,bin}}$, defined as the D -, μ -, e - or jet-production cross
 sections in a particular transverse momentum p_T and pseudorapidity η range, in bins of Q^2
 and y or x_{Bj} . The reduced cross sections are obtained from $\sigma_{\text{vis,bin}}$ using theoretical predictions:

$$\sigma_{\text{red}}^{Q\bar{Q}}(x_{Bj}, Q^2) = \sigma_{\text{vis,bin}} \frac{\sigma_{\text{red}}^{Q\bar{Q},\text{th}}(x_{Bj}, Q^2)}{\sigma_{\text{vis,bin}}^{\text{th}}}. \quad (2)$$

57 To calculate the predictions for the reduced cross sections $\sigma_{\text{red}}^{Q\bar{Q},\text{th}}(x_{Bj}, Q^2)$ and visible cross sec-
 58 tions $\sigma_{\text{vis,bin}}^{\text{th}}$, the theoretical calculations described in Section 4 are used. For charm production,
 59 they are consistent with those used in the previous combination [1].

60 The combination of reduced cross sections is based on the procedure described elsewhere
 61 and used in previous HERA combinations [1, 16–19], accounting for all correlations in the un-
 62 certainties. In the present analysis, the correlated and uncorrelated systematic uncertainties are
 63 predominantly of multiplicative nature, i.e. they change proportionally to the expected central
 64 values. The statistical uncertainties are mainly background dominated and thus are treated as
 65 constant. All experimental systematic uncertainties are treated as independent between H1 and
 66 ZEUS. For datasets 1, 8 and 11 statistical correlations between charm and beauty cross sections
 67 as reported in the original papers are accounted for. Where necessary the statistical correla-
 68 tion factors are corrected to take into account differences in the kinematic region of charm and
 69 beauty measurements (dataset 11) or binning schemes (dataset 1) using theoretical predictions.
 70 The consistent treatment of the correlations of statistical and systematic uncertainties, includ-
 71 ing the correlations between the charm and beauty data sets where relevant, yields a significant
 72 reduction of the overall uncertainties of the combined data.

73 4 Theoretical predictions

74 The cross-section predictions are obtained using the HVQDIS program [20] and the xFIT-
 75 TER (former HERAFITTER) framework [21] which provide NLO QCD ($O(\alpha_s^2)$) calculations
 76 in the 3-flavour FFNS for charm and beauty production in DIS. The predictions obtained with
 77 HVQDIS, which allows fully differential cross sections to be calculated, are used for phase-
 78 space corrections, while for the comparison to the combined data the predictions obtained with

²These measurements are transformed, when needed, to the common grid (x_{Bj}, Q^2) points using theoretical
 predictions, while the uncertainties on the resulting scaling factors are found to be negligible.

79 the xFITTER framework are used, which provides reduced cross-sections only, but has the ad-
 80 vantage of using the running heavy-quark mass definition as implemented in the OPENQC-
 81 DRAD program [22].

82 The following parameters are used in the calculations and are varied within the limits quoted
 83 below for estimating the uncertainties in the predictions introduced by these parameters:

- 84 • The **renormalisation and factorisation scales** are taken as $\mu_r = \mu_f = \sqrt{Q^2 + 4m_Q^2}$,
 85 where m_Q is the charm or beauty quark mass, respectively. The scales are varied si-
 86 multaneously up or down by a factor of two.
- 87 • For the extrapolation to the full phase space, the **pole masses of the c and b quarks** are
 88 set to $m_c = 1.50 \pm 0.15$ GeV, $m_b = 4.50 \pm 0.25$ GeV, respectively. For the comparison
 89 with the combined data, the **running c and b quark masses** are set to the PDG values
 90 $m_c(m_c) = 1.27 \pm 0.03$ GeV, $m_b(m_b) = 4.18 \pm 0.03$ GeV [15]. These variations also affect
 91 the values of the renormalisation and factorisation scales.
- 92 • For the **strong coupling constant** the value $\alpha_s^{n_f=3}(M_Z) = 0.105 \pm 0.002$ is chosen which
 93 corresponds to $\alpha_s^{n_f=5}(M_Z) = 0.116 \pm 0.002$.
- 94 • The **proton PDFs** are described by a series of FFNS variants of the HERAPDF1.0 set [18]
 95 at NLO determined within the xFITTER framework. These proton PDFs are the same
 96 as those used in the previous combination [1]. In the determination of these PDF sets
 97 no charm measurements were included. For all parameter settings used here, the corre-
 98 sponding PDF set is used. In the extrapolation to the full phase space, an uncorrelated
 99 uncertainty of 2% is assigned to each extrapolated data point to cover the PDF uncertain-
 100 ties. As a cross check of the extrapolation procedure, the cross sections are also evaluated
 101 with the 3-flavour NLO versions of the HERAPDF2.0 set (FF3A) [19] and the differences
 102 are found to be well within the uncertainties quoted. For the comparison to the combined
 103 data the HERAPDF2.0 FF3A set [19] is used.

104 The NLO calculations performed with the HVQDIS program yield fully differential cross
 105 sections for charm and beauty quarks. For those cross-section measurements needing extrapo-
 106 lation factors from the visible phase space in which the measurements were performed to the
 107 reduced heavy-flavour cross sections, these predictions are extended with fragmentation mod-
 108 els to provide hadron level cross sections. The fragmentation model for c quarks is based on
 109 the measurements by H1 [23] and ZEUS [24] as described in detail in [1]. The fragmentation
 110 model for b quarks uses the Peterson et al. [25] parametrisation with $\epsilon_b = 0.0035 \pm 0.0020$ [26].
 111 The fragmentation fractions of c quarks into specific charmed hadrons are taken from [27].
 112 The branching fractions of semi-leptonic decays of heavy-quarks to a muon or electron are
 113 taken from [15] with the decay spectra of leptons modelled according to [28]. When necessary
 114 for the extrapolation procedure, the parton-level jets are reconstructed using the corresponding
 115 clustering algorithms, and the cross sections are corrected for jet hadronisation effects using
 116 corrections derived in the original papers [4, 6].³

³While no such corrections are provided in [6], an uncertainty of 5% is assigned to cover the missing hadroni-
 sation effects.

117 To evaluate the extrapolation uncertainty on the extracted reduced cross sections, all the
118 above settings are varied by the corresponding uncertainties and each variation is considered as
119 a correlated uncertainty among the measurements to which it applies, except for the uncertain-
120 ties related to PDFs and hadronisation effects, which are treated as uncorrelated. Asymmetric
121 variations are symmetrised using the largest deviations.

122 5 Combined data

123 In total, 209 charm and 57 beauty data points are combined simultaneously to 52 reduced charm
124 and 27 beauty cross-section measurements, respectively. A total χ^2 of 149 for 187 degrees of
125 freedom (dof) is obtained in the combination indicating consistency of input data and conser-
126 vative estimates of the uncertainties. There are in total 167 sources of correlated uncertainties.
127 These are 71 experimental systematic sources, 16 sources due to the extrapolation procedure
128 (including the uncertainties on the fragmentation fractions and branching ratios) and 80 statisti-
129 cal charm and beauty correlations. None of these shifts exceeds 1.6 standard deviations. The
130 pull distribution of the combination is shown in Fig. 1.

131 The individual datasets as well as the results of the combination are shown in Figs. 2 and 3.
132 The combined data are significantly more precise than any of the individual input datasets. This
133 is illustrated in Figs. 4 and 5 where the measurements for $Q^2 = 32 \text{ GeV}^2$ are shown. Figs. 6
134 and 7 present a comparison of the NLO QCD predictions in the FFNS to the combined data.
135 The theoretical uncertainties on these plots present the mass, scale and PDF⁴ variations added
136 in quadrature. The predictions describe the data reasonably well within the uncertainties in the
137 whole kinematic range. Fig. 8 shows the combined reduced charm cross sections compared
138 to the results of the previous combination [1] and theoretical predictions. This comparison
139 demonstrates that the new combined charm data are also consistent with and slightly more
140 precise than the previously published results.

141 6 QCD analysis

142 The combined beauty and charm data are included in a QCD analysis at NLO, together with the
143 combined HERA inclusive DIS data [19].

144 6.1 Theoretical formalism and settings

145 The analysis is performed using an open-source QCD fit framework for PDF determination
146 xFITTER [21] (version 1.2.0). The scale evolution of partons is calculated through DGLAP
147 equations [29] at NLO, as implemented in the QCDNUM program [30] (version 17.01.11). The
148 theoretical predictions for the HERA data are obtained using the OPENQCDRAD program [22]
149 interfaced in the xFITTER framework. The number of active flavours is set to $n_f = 3$ at all
150 scales. The renormalisation and factorisation scales for heavy-flavour production are set to

⁴Only experimental uncertainties ('EIG') of HERAPDF2.0 are considered.

151 $\mu_r = \mu_f = \sqrt{Q^2 + 4m_Q^2}$, where m_Q denotes the running mass of c or b quarks. The heavy-
152 quark masses are left free in the fit if not stated otherwise. For the light-flavour contributions
153 to the inclusive DIS cross sections, the pQCD scales are set to $\mu_r = \mu_f = Q$. The massless
154 contribution to the longitudinal structure function F_L is calculated to $O(\alpha_s)$. The strong coupling
155 strength is set to $\alpha_s^{n_f=3}(M_Z) = 0.106$. The Q^2 range of the inclusive HERA data is restricted to
156 $Q^2 > Q_{\min}^2 = 3.5 \text{ GeV}^2$. No such cut is applied to the charm and beauty data since $Q^2 + 4m^2$ is
157 always $> 3.5 \text{ GeV}^2$.

158 The χ^2 definition used for the HERA DIS data follows that of Eq. (32) in Ref. [19]. It
159 includes an additional logarithmic term that is relevant when the estimated statistical and un-
160 correlated systematic uncertainties in the data are rescaled during the fit [31]. The correlated
161 systematic uncertainties are treated through nuisance parameters.

The procedure for the determination of the PDFs follows the approach of HERAPDF2.0 [19].
The parametrized PDFs are the gluon distribution $xg(x)$, the valence quark distributions $xu_v(x)$
and $xd_v(x)$, and the u - and d -type antiquark distributions $x\bar{U}(x)$ and $x\bar{D}(x)$. At the initial QCD
evolution scale $\mu_{f0}^2 = 1.9 \text{ GeV}^2$, the PDFs are parametrized as:

$$\begin{aligned}
xg(x) &= A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}},
\end{aligned} \tag{3}$$

162 assuming the relations $x\bar{U}(x) = x\bar{u}(x)$ and $x\bar{D}(x) = x\bar{d}(x) + x\bar{s}(x)$. Here, $x\bar{u}(x)$, $x\bar{d}(x)$, and $x\bar{s}(x)$
163 are the up, down, and strange antiquark distributions, respectively. The sea quark distribution
164 is defined as $x\Sigma(x) = x\bar{u}(x) + x\bar{d}(x) + x\bar{s}(x)$. The normalization parameters A_{u_v} , A_{d_v} , and A_g
165 are determined by the QCD sum rules. The B and B' parameters determine the PDFs at small
166 x , and the C parameters describe the shape of the distributions as $x \rightarrow 1$. The parameter C'_g is
167 fixed to 25 [32]. Additional constraints $B_{\bar{U}} = B_{\bar{D}}$ and $A_{\bar{U}} = A_{\bar{D}}(1 - f_s)$ are imposed to ensure
168 the same normalization for the xu and xd distributions as $x \rightarrow 0$. The strangeness fraction
169 $f_s = xs/(xd + xs)$ is fixed to $f_s = 0.4$ as in the HERAPDF2.0 analysis [19].

170 The parameters in Eq. (3) are selected by first fitting with all D and E parameters set to
171 zero, and then including them independently one at a time in the fit. The improvement in the χ^2
172 of the fit is monitored and the procedure is stopped when no further improvement is observed.
173 This leads to the same 14 free PDF parameters, as in the inclusive HERAPDF2.0 analysis [19].

174 The PDF uncertainties are estimated according to the general approach of HERAPDF2.0 [19]
175 in which the fit, model, and parametrization uncertainties are taken into account. Fit uncertain-
176 ties are determined using the tolerance criterion of $\Delta\chi^2 = 1$. Model uncertainties arise from
177 the variations of the strong coupling constant $\alpha_s^{n_f=3}(M_Z) = 0.106 \pm 0.0015$, the simultaneous
178 variation of the factorisation and renormalisation scales up and down by a factor of two, the
179 strangeness fraction $0.3 \leq f_s \leq 0.5$, and the value of $2.5 \leq Q_{\min}^2 \leq 5.0 \text{ GeV}^2$ imposed on the
180 HERA data. The parametrization uncertainty is estimated by extending the functional form
181 in Eq. (3) of all parton distributions with additional parameters D and E added one at a time.
182 An additional parametrisation uncertainty is considered by using the functional form in Eq. (3)

183 with $E_{u_v} = 0$, as the χ^2 in this variant of the fit is only 5 units worse than with the released E_{u_v}
184 parameter. Furthermore, $\mu_{f_0}^2$ is varied within $1.6 \text{ GeV}^2 < \mu_{f_0}^2 < 2.2 \text{ GeV}^2$. The parametrization
185 uncertainty is constructed as an envelope at each x value, built from the maximal differences
186 between the PDFs resulting from the central fit and all parametrization variations. This uncer-
187 tainty is valid in the x range covered by the PDF fit to the data. The total PDF uncertainty
188 is obtained by adding the fit, model, and parametrization uncertainties in quadrature. In the
189 following, the quoted uncertainties correspond to 68% CL.

190 6.2 Results

The results for the fitted heavy-quark masses extracted from the fit using the inclusive and combined beauty and charm data are:

$$\begin{aligned} m_c(m_c) &= 1290_{-41}^{+46}(\text{fit})_{-14}^{+62}(\text{mod})_{-31}^{+7}(\text{par}) \text{ MeV}, \\ m_b(m_b) &= 4049_{-109}^{+104}(\text{fit})_{-32}^{+90}(\text{mod})_{-31}^{+1}(\text{par}) \text{ MeV}. \end{aligned} \quad (4)$$

191 The model uncertainties are dominated by theoretical uncertainties arising from the scale vari-
192 ations. The fit yields $\chi^2/\text{dof} = 1435/1208$. The results obtained in the fit using the inclusive
193 data only are: $m_c(m_c) = 1798_{-134}^{+144}(\text{fit}) \text{ MeV}$, $m_b(m_b) = 8450_{-1812}^{+2282}(\text{fit}) \text{ MeV}$ (note that only
194 the fit uncertainties are quoted). In the variant of the fit using the inclusive data only and the
195 reduced parametrisation with $E_{u_v} = 0$ the central fitted values for the heavy-quark masses are:
196 $m_c(m_c) = 1450 \text{ MeV}$, $m_b(m_b) = 3995 \text{ MeV}$.

197 A cross check is performed using the Monte Carlo method [33, 34]. It is based on analysing
198 a large number of pseudo data sets called replicas. For this cross check, 500 replicas are created
199 by taking the combined data and fluctuating the values of the reduced cross sections randomly
200 within their given statistical and systematic uncertainties taking into account correlations. All
201 uncertainties are assumed to follow Gaussian distributions. The central values for the fitted pa-
202 rameters and their uncertainties are estimated using the mean and RMS values over the replicas.
203 The obtained heavy-quark masses and their fit uncertainties are in agreement with those quoted
204 in Eq. (4). We conclude that the inclusive data alone can not reliably constrain the quark masses,
205 and that the systematics from including the inclusive data in the global mass fit are covered by
206 the parametrisation uncertainties applied.

207 The predictions for the combined data are also calculated using the ABM11 PDFs [35] at
208 NLO, and ABMP16 PDFs [36] at approximate NNLO as implemented in the OPENQCDRAD
209 program interfaced in the XFITTER framework. They are compared to the combined data in
210 Figs. 9 and 10. Both calculations yield very similar description of the data to the one obtained
211 using HERAPDF2.0 FF3A, which is further illustrated by the comparisons where all results
212 are normalised to the predictions obtained using HERAPDF2.0 FF3A (see Figs. 11 and 12). In
213 order to quantify the level of agreement, the χ^2 values are calculated and reported in Table 2.

214 7 Summary

215 Measurements of beauty and charm production cross sections in deep-inelastic ep scattering by
216 the H1 and ZEUS experiments were combined at the level of reduced cross sections, accounting

217 for their statistical and systematic correlations. The beauty cross sections have been combined
218 for the first time. The data sets were found to be consistent and the combined data have sig-
219 nificantly reduced uncertainties. The combined data were compared to NLO QCD predictions,
220 which are found to describe the data reasonably well. The running charm and beauty masses
221 were extracted in the QCD analysis using the inclusive HERA DIS and new combined charm
222 and beauty data.

223 References

- 224 [1] F. D. Aaron *et al.* [H1 and ZEUS Collaborations], “Combination and QCD Analysis of
225 Charm Production Cross Section Measurements in Deep-Inelastic ep Scattering at
226 HERA”, *Eur. Phys. J.* **C73**, (2013) 2311 [arXiv:1211.1182].
- 227 [2] H. Abramowicz *et al.* [ZEUS Collaboration], “Measurement of D^\pm Production in Deep
228 Inelastic ep Scattering with the ZEUS detector at HERA”, *JHEP* **05**, (2013) 023
229 [arXiv:1302.5058].
- 230 [3] H. Abramowicz *et al.* [ZEUS Collaboration], “Measurement of $D^{*\pm}$ Production in Deep
231 Inelastic Scattering at HERA”, *JHEP* **05**, (2013) 097 [arXiv:1303.6578]. Erratum-ibid
232 *JHEP* **02**, (2014) 106.
- 233 [4] H. Abramowicz *et al.* [ZEUS Collaboration], “Measurement of beauty and charm
234 production in deep inelastic scattering at HERA and measurement of the beauty-quark
235 mass”, *JHEP* **09**, (2014) 127 [arXiv:1405.6915].
- 236 [5] H. Abramowicz *et al.* [ZEUS Collaboration], “Measurement of beauty production in
237 deep inelastic scattering at HERA using decays into electrons”, *Eur. Phys. J.* **C71**, (2011)
238 1573 [arXiv:1101.3692].
- 239 [6] H. Abramowicz *et al.* [ZEUS Collaboration], “Measurement of beauty production in DIS
240 and F2bb extraction at ZEUS”, *Eur. Phys. J.* **C69**, (2010) 347 [arXiv:1005.3396].
- 241 [7] S. Chekanov *et al.* [ZEUS Collaboration], “Measurement of charm and beauty
242 production in deep inelastic ep scattering from decays into muons at HERA”,
243 *Eur. Phys. J.* **C65**, (2010) 65 [arXiv:0904.3487].
- 244 [8] F. D. Aaron *et al.* [H1 Collaboration], “Measurement of the Charm and Beauty Structure
245 Functions using the H1 Vertex Detector at HERA”, *Eur. Phys. J.* **C65**, (2010) 89
246 [arXiv:0907.2643].
- 247 [9] A. Aktas *et al.* [H1 Collaboration], “Production of D^{*+-} Mesons with Dijets in
248 Deep-Inelastic Scattering at HERA”, *Eur. Phys. J.* **C51**, (2007) 271 [hep-ex/0701023].
- 249 [10] F. D. Aaron *et al.* [H1 Collaboration], “Measurement of $D^{*\pm}$ Meson Production and
250 Determination of F_2^{ccbar} at low Q^2 in Deep-Inelastic” *Eur. Phys. J.* **C71**, (2011) 1769
251 [arXiv:1106.1028].

- 252 [11] F. D. Aaron *et al.* [H1 Collaboration], “Measurement of the D^{*+} - Meson Production
253 Cross Section and $F_2^{(c\bar{c})}$, at High Q^2 , in ep Scattering at HERA”, Phys. Lett.
254 **B686**, (2010) 91 [arXiv:0911.3989].
- 255 [12] J. Breitweg *et al.* [ZEUS Collaboration], “Measurement of D^{*+} - production and the
256 charm contribution to F_2 in deep inelastic scattering at HERA”, Eur. Phys. J. **C12**, (2000)
257 35 [hep-ex/9908012].
- 258 [13] S. Chekanov *et al.* [ZEUS Collaboration], “Measurement of D^{*+} - production in deep
259 inelastic e^+p scattering at HERA”, Phys. Rev. **D69**, (2004) 012004 [hep-ex/0308068].
- 260 [14] S. Chekanov *et al.* [ZEUS Collaboration], “Measurement of D^{*+} and D_0 production in
261 deep inelastic scattering using a lifetime tag at HERA”, Eur. Phys. J. **C63**, (2009) 171
262 [arXiv:0812.3775].
- 263 [15] C. Patrignani *et al.* [Particle Data Group], “2016 Review of Particle Physics”, Chin. Phys.
264 **C40**, (2016) 100001.
- 265 [16] A. Glazov, Proceedings of “13th International Workshop on Deep Inelastic Scattering”,
266 eds. W. H. Smith and S. R. Dasu, Madison, USA, 2005, AIP Conf. Proc. **792**, (2005) 237.
- 267 [17] A. Atkas *et al.* [H1 Collaboration], “Measurement of the Inclusive ep Scattering Cross
268 Section at Low Q^2 and x at HERA”, Eur. Phys. J. **C63**, (2009) 625 [arXiv:0904.0929].
- 269 [18] F. D. Aaron *et al.* [H1 and ZEUS Collaboration], “Combined Measurement and QCD
270 Analysis of the Inclusive e^+p Scattering Cross Sections at HERA”, JHEP **01**, (2010)
271 109 [arXiv:0911.0884].
- 272 [19] H. Abramowicz *et al.* [H1 and ZEUS Collaborations], “Combination of measurements of
273 inclusive deep inelastic $e^\pm p$ scattering cross sections and QCD analysis of HERA data”,
274 Eur. Phys. J. **C75**, (2015) 580 [arXiv:1506.06042].
- 275 [20] B. W. Harris and J. Smith, “Charm quark and D^{*+} - cross-sections in deeply inelastic
276 scattering at HERA”, Phys. Rev. **D57**, (1998) 2806 [hep-ph/9706334].
- 277 [21] S. Alekhin *et al.*, “HERAFitter”, Eur. Phys. J. **C75**, (2015) 304 [arXiv:1410.4412],
278 www.xfitter.org.
- 279 [22] S. Alekhin, J. Blümlein, S. Moch, “Parton Distribution Functions and Benchmark Cross
280 Sections at NNLO”, Phys. Rev. **D86** (2012) 054009, doi: 10.1103/PhysRevD.86.054009,
281 [arXiv:1202.2281];
- 282 I. Bierenbaum, J. Blümlein, S. Klein, “The Gluonic Operator Matrix Elements at
283 $O(\alpha_s^2)$ for DIS Heavy Flavor Production”, Phys. Lett. **B672** (2009) 401, doi:
284 10.1016/j.physletb.2009.01.057, [arXiv:0901.0669];
- 285 S. Alekhin, OPENQCDRAD-1.5.
286 <http://www-zeuthen.desy.de/~alekhin/OPENQCDRAD>.
- 287 [23] F. D. Aaron *et al.* [H1 Collaboration], “Study of Charm Fragmentation into $D^{*\pm}$ Mesons
288 in Deep-Inelastic Scattering at HERA”, Eur. Phys. J. **C59**, (2009) 589 [arXiv:0808.1003].

- 289 [24] S. Chekanov *et al.* [ZEUS Collaboration], “Measurement of the charm fragmentation
290 function in D^* photoproduction at HERA”. JHEP **04**, (2009) 082 [arXiv:0901.1210].
- 291 [25] C. Peterson *et al.*, “Scaling Violations in Inclusive e^+e^- Annihilation Spectra”,
292 Phys. Rev. **D27**, (1983) 105.
- 293 [26] P. Nason and C. Oleari, “A Phenomenological study of heavy quark fragmentation
294 functions in e^+e^- annihilation”, Nucl. Phys. **B565**, (2000) 245 [hep-ph/9903541].
- 295 [27] M. Lisovyi *et al.*, “Combined analysis of charm-quark fragmentation-fraction
296 measurements”, Eur. Phys. J. **C76**, (2016) 397 [arXiv:1509.01061].
- 297 [28] N. E. Adam *et al.* [Cleo Collaboration], “Absolute Branching Fraction Measurements for
298 D^+ and D^0 Inclusive Semileptonic Decays”, Phys. Rev. Lett. **97**, (2006) 251801
299 [hep-ex/0604044].
- 300 [29] Y. L. Dokshitzer, “Calculation of the Structure Functions for Deep Inelastic Scattering
301 and e^+e^- Annihilation by Perturbation Theory in Quantum Chromodynamics.,” Sov.
302 Phys. JETP **46** (1977) 641 [Zh. Eksp. Teor. Fiz. **73** (1977) 1216].
- 303 V. N. Gribov and L. N. Lipatov, “Deep inelastic ep scattering in perturbation theory”,
304 Sov. J. Nucl. Phys. **15** (1972) 438;
- 305 G. Altarelli and G. Parisi, “Asymptotic freedom in parton language”, Nucl. Phys. **B126**
306 (1977) 298, doi:10.1016/0550-3213(77)90384-4;
- 307 G. Curci, W. Furmanski, and R. Petronzio, “Evolution of parton densities beyond leading
308 order: The non-singlet case”, Nucl. Phys. **B175** (1980) 27,
309 doi:10.1016/0550-3213(80)90003-6;
- 310 W. Furmanski and R. Petronzio, “Singlet parton densities beyond leading order”, Phys.
311 Lett. **B97** (1980) 437, doi:10.1016/0370-2693(80)90636-X;
- 312 S. Moch, J. A. M. Vermaseren, and A. Vogt, “The three-loop splitting functions in QCD:
313 the non-singlet case”, Nucl. Phys. **B688** (2004) 101,
314 doi:10.1016/j.nuclphysb.2004.03.030, [arXiv:hep-ph/0403192];
- 315 A. Vogt, S. Moch, and J. A. M. Vermaseren, “The three-loop splitting functions in QCD:
316 the singlet case”, Nucl. Phys. **B691** (2004) 129, doi:10.1016/j.nuclphysb.2004.04.024,
317 [arXiv:hep-ph/0404111].
- 318 [30] M. Botje, “QCDNUM: Fast QCD Evolution and Convolution”, Comput. Phys. Commun.
319 **182** (2011) 490, doi:10.1016/j.cpc.2010.10.020, [arXiv:1005.1481].
- 320 [31] F. D. Aaron *et al.* [H1 Collaboration], “Inclusive Deep Inelastic Scattering at High Q^2
321 with Longitudinally Polarised Lepton Beams at HERA, JHEP **09**, (2012) 061
322 doi:10.1007/JHEP09(2012)061 [arXiv:1206.7007 [hep-ex]].
- 323 [32] W. J. Stirling, R. S. Thorne and G. Watt, “Parton distributions for the LHC”, Eur. Phys. J
324 **C63** (2009) 189, doi: 10.1140/epjc/s10052-009-1072-5, [arXiv:0901.0002].
- 325 [33] W. T. Giele and S. Keller, “Implications of hadron collider observables on parton
326 distribution function uncertainties,” Phys. Rev. D **58** (1998) 094023
327 doi:10.1103/PhysRevD.58.094023 [hep-ph/9803393].

- 328 [34] W. T. Giele, S. A. Keller and D. A. Kosower, “Parton distribution function uncertainties,”
329 hep-ph/0104052.
- 330 [35] S. Alekhin, J. Blümlein and S. Moch, “Parton Distribution Functions and Benchmark
331 Cross Sections at NNLO,” Phys. Rev. D **86** (2012) 054009
332 doi:10.1103/PhysRevD.86.054009 [arXiv:1202.2281 [hep-ph]].
- 333 [36] S. Alekhin, J. Blümlein, S. Moch and R. Placakyte, “Parton Distribution Functions, α_s
334 and Heavy-Quark Masses for LHC Run II,” arXiv:1701.05838 [hep-ph].

Data set	Tagging	Q^2 range [GeV ²]	N_c	\mathcal{L} [pb ⁻¹]	\sqrt{s} [GeV]	N_b
1 H1 VTX [8]	VTX	5 – 2000	29	245	318	12
2 H1 D^{*+} HERA-I [9]	D^{*+}	2 – 100	17	47	318	
3 H1 D^{*+} HERA-II (medium Q^2) [10]	D^{*+}	5 – 100	25	348	318	
4 H1 D^{*+} HERA-II (high Q^2) [11]	D^{*+}	100 – 1000	6	351	318	
5 ZEUS D^{*+} 96-97 [12]	D^{*+}	1 – 200	21	37	300	
6 ZEUS D^{*+} 98-00 [13]	D^{*+}	1.5 – 1000	31	82	318	
7 ZEUS D^0 2005 [14]	D^0	5 – 1000	9	134	318	
8 ZEUS μ 2005 [7]	μ	20 – 10000	8	126	318	8
9 ZEUS D^+ HERA-II [2]	D^+	5 – 1000	14	354	318	
10 ZEUS D^{*+} HERA-II [3]	D^{*+}	5 – 1000	31	363	318	
11 ZEUS VTX HERA-II [4]	VTX	5 – 1000	18	354	318	17
12 ZEUS e HERA-II [5]	e	10 – 1000		363	318	9
13 ZEUS μ + jet HERA-I [6]	μ	2 – 3000		114	318	11

Table 1: Data sets used in the combination. For each data set the Q^2 range, integrated luminosity (\mathcal{L}), centre-of-mass energy (\sqrt{s}) and the numbers of charm (N_c) and beauty (N_b) measurements are given.

Dataset	PDF	χ^2	χ^2 with PDF unc.
HERA 2012 c [1] (dof = 52)	HERAPDF20_NLO_FF3A_EIG	59	59
	abm11_3n_nlo	62	62
	ABMP16_3_nnlo	64	63
New combined c (dof = 52)	HERAPDF20_NLO_FF3A_EIG	86	85
	abm11_3n_nlo	92	91
	ABMP16_3_nnlo	101	99
ZEUS VTX b [4] (dof = 17)	HERAPDF20_NLO_FF3A_EIG	14	14
	abm11_3n_nlo	13	13
	ABMP16_3_nnlo	14	14
New combined b (dof = 27)	HERAPDF20_NLO_FF3A_EIG	33	33
	abm11_3n_nlo	34	34
	ABMP16_3_nnlo	39	39

Table 2: The χ^2 values and dof of the charm and beauty data with respect to the NLO and approximate NNLO calculations using various PDFs. The χ^2 values that include PDF uncertainties are shown separately.

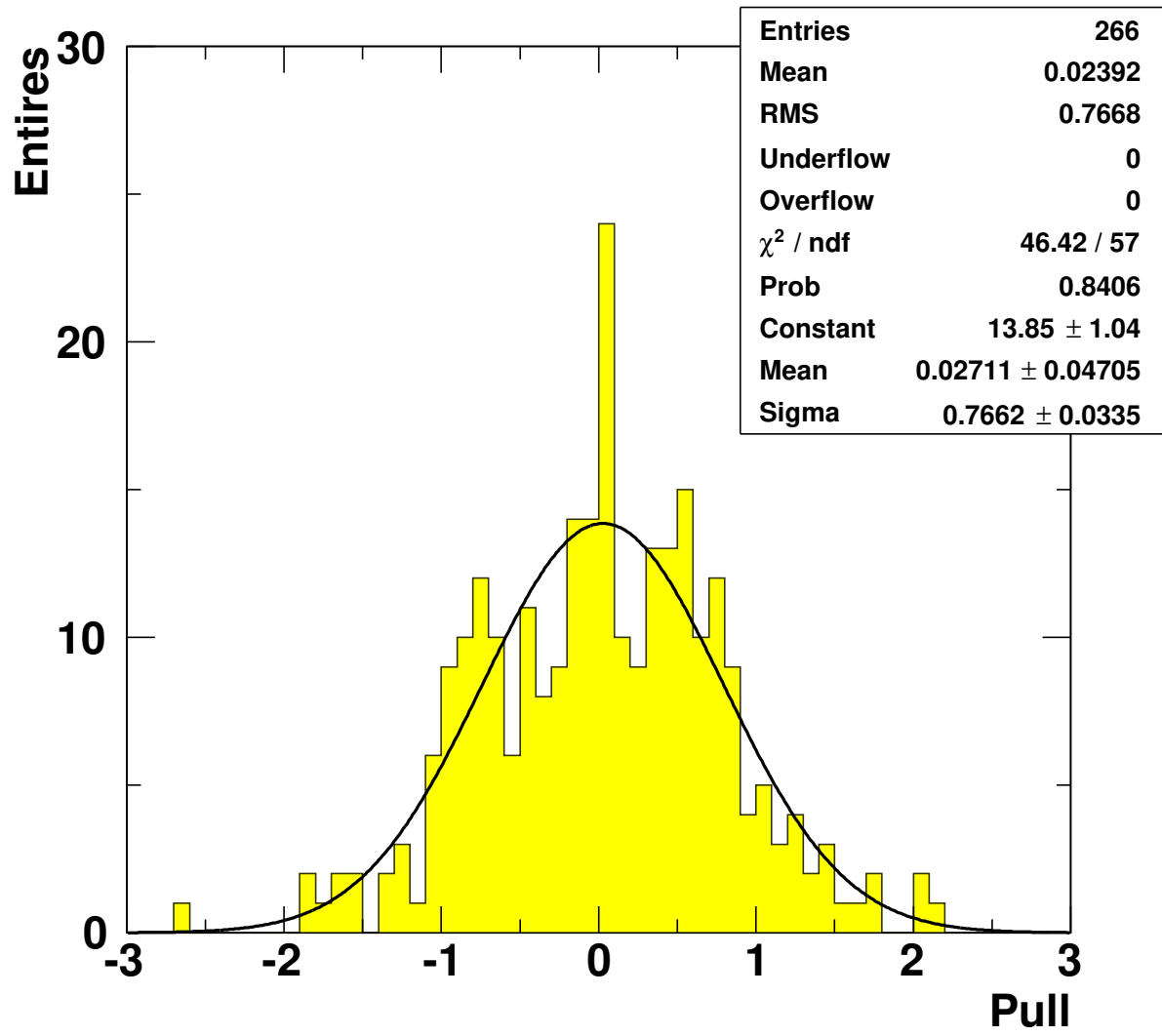


Figure 1: The pull distribution for the combination of the charm and beauty reduced cross sections.

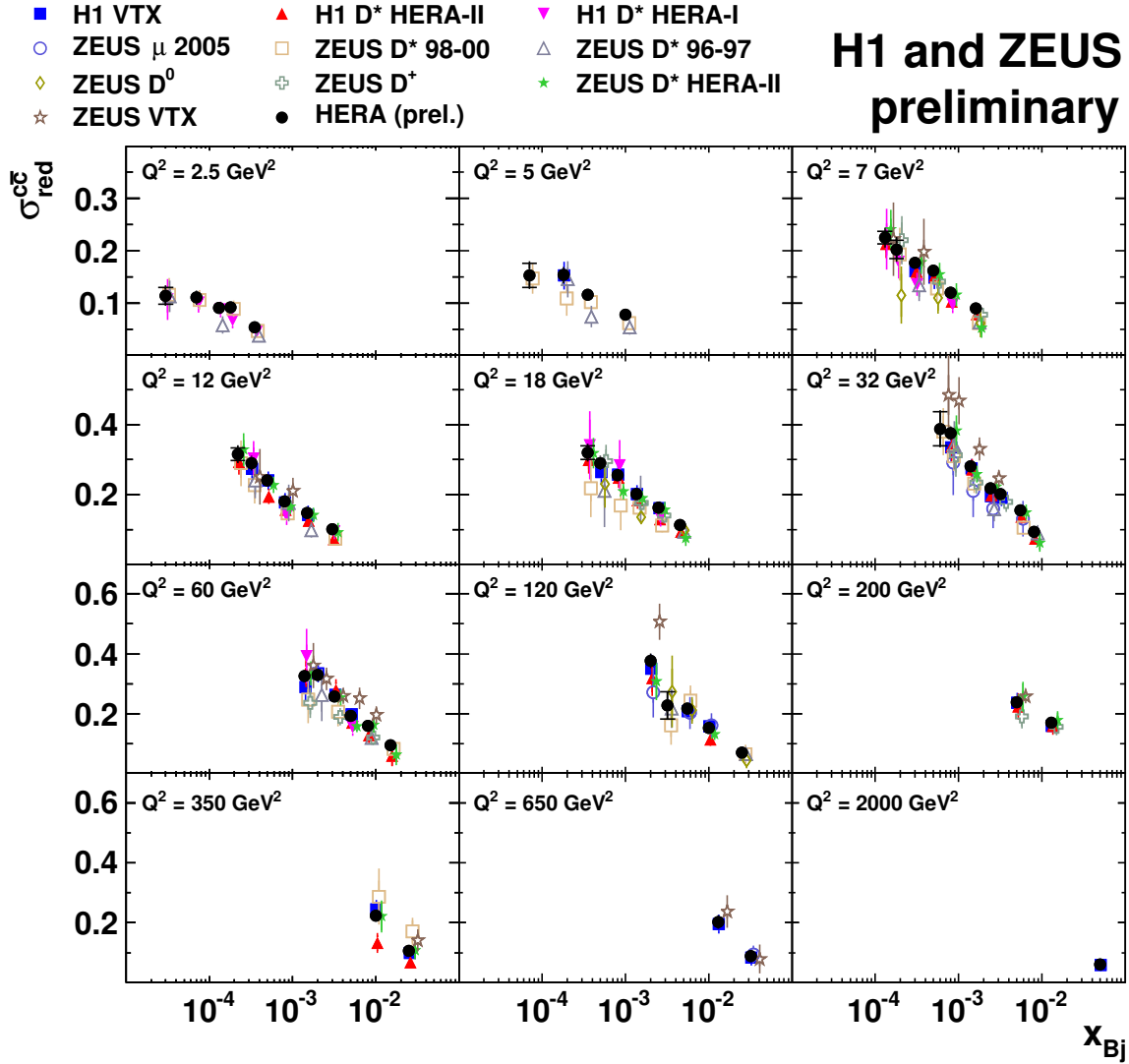


Figure 2: Combined measurements of the reduced charm production cross sections, $\sigma_{\text{red}}^{c\bar{c}}$, (full circles) as a function of x_{Bj} for different values of Q^2 . The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in x_{Bj} .

H1 and ZEUS preliminary

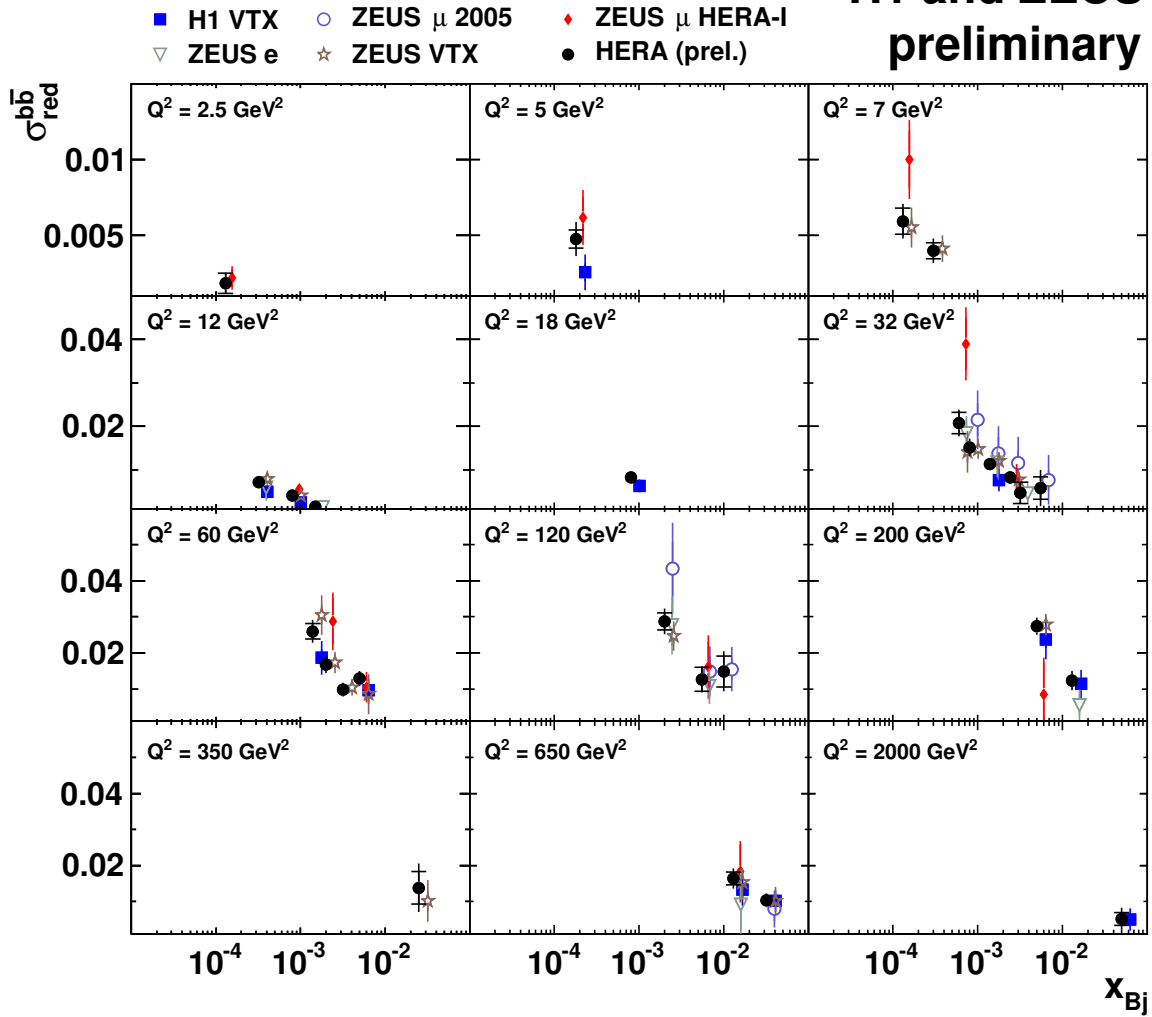


Figure 3: Combined measurements of the reduced beauty production cross sections, $\sigma_{\text{red}}^{b\bar{b}}$, (full circles) as a function of x_{Bj} for different values of Q^2 . The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in x_{Bj} .

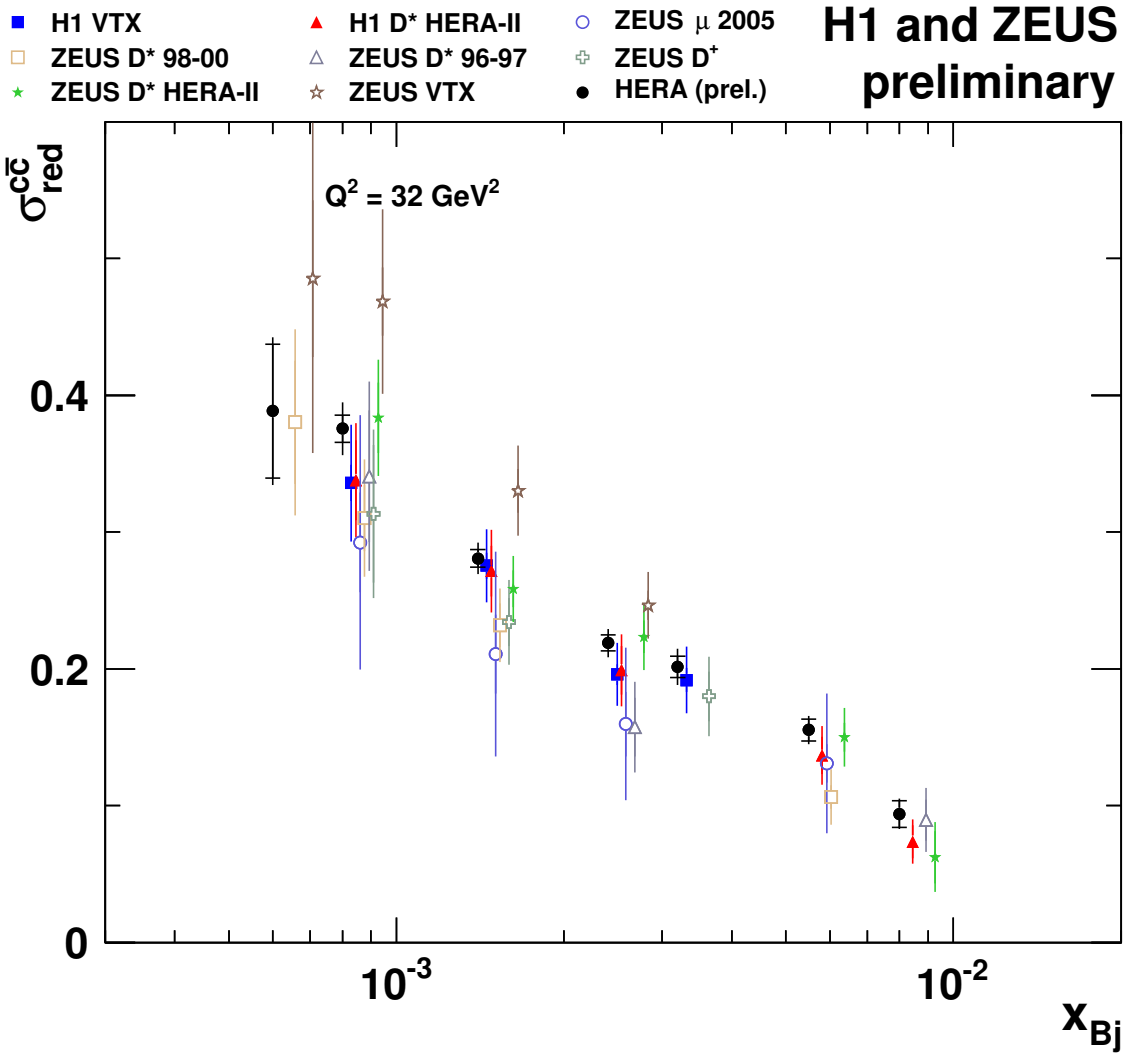


Figure 4: Combined measurements of the reduced charm production cross sections, $\sigma_{\text{red}}^{c\bar{c}}$, (full circles) as a function of x_{Bj} for $Q^2 = 32 \text{ GeV}^2$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in x_{Bj} .

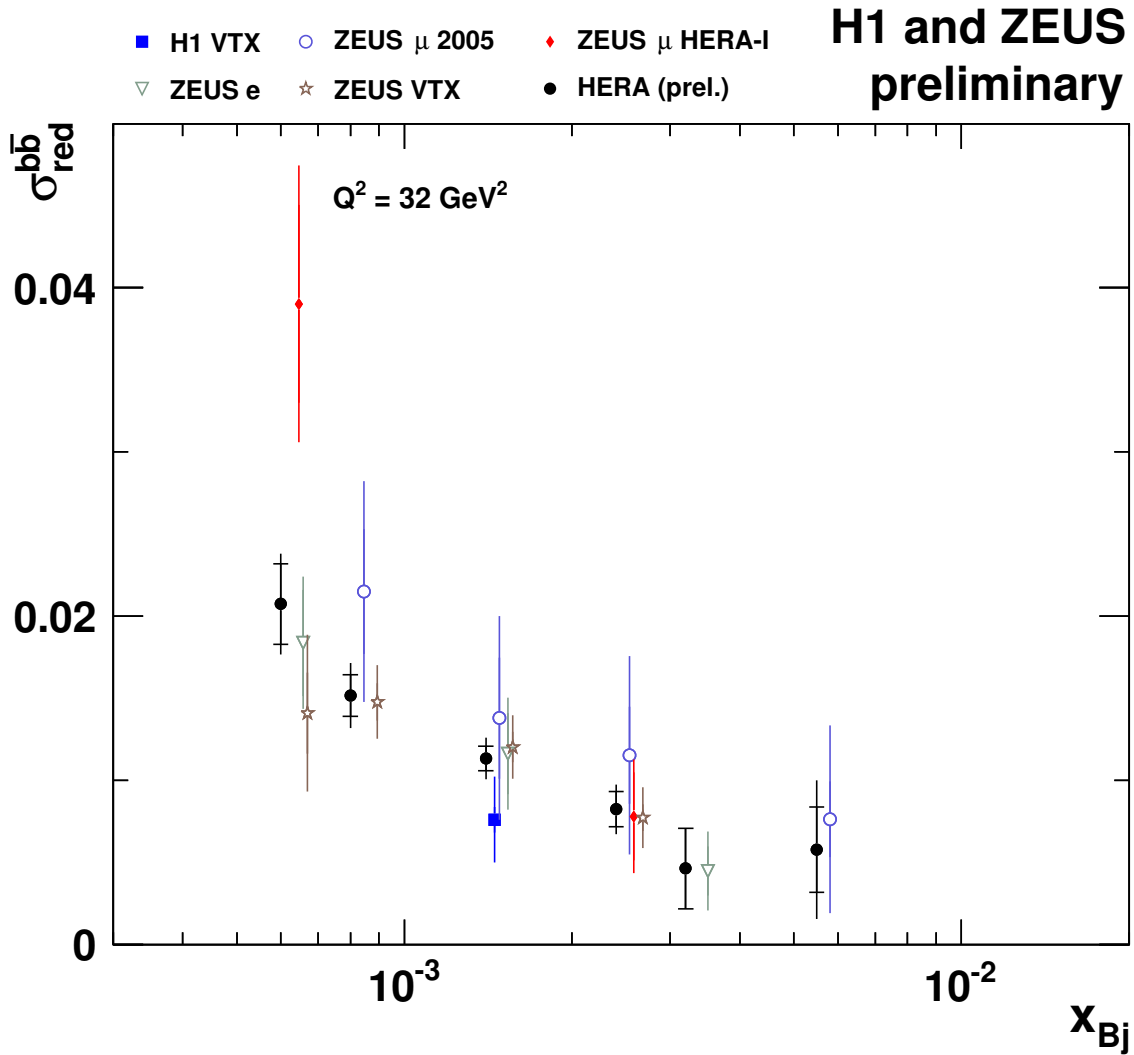


Figure 5: Combined measurements of the reduced beauty production cross sections, $\sigma_{\text{red}}^{c\bar{c}}$, (full circles) as a function of x_{Bj} for $Q^2 = 32 \text{ GeV}^2$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in x_{Bj} .

H1 and ZEUS preliminary

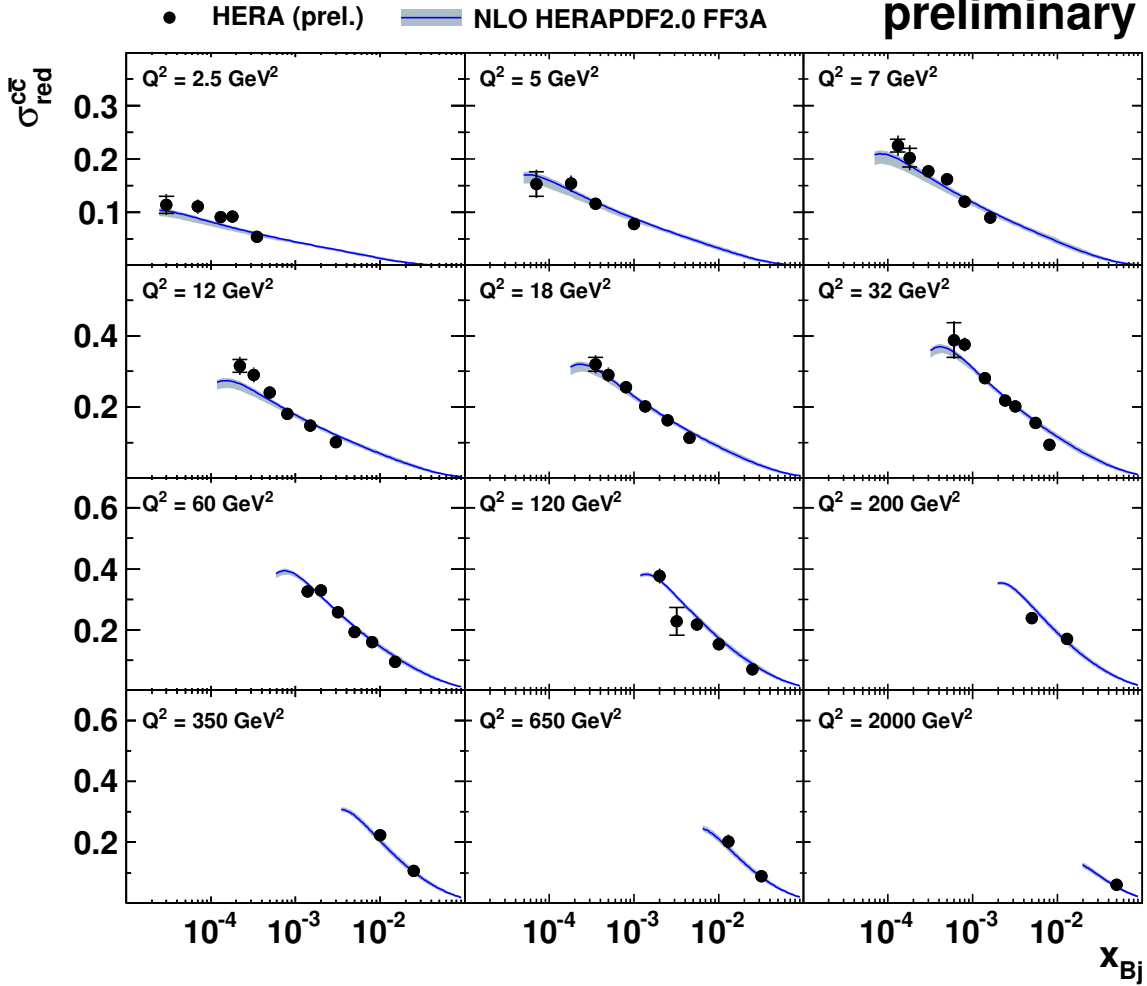


Figure 6: Combined reduced charm cross sections $\sigma_{\text{red}}^{c\bar{c}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO QCD theoretical predictions obtained using HERAPDF2.0 FF3A with their uncertainties (solid line with band).

H1 and ZEUS preliminary

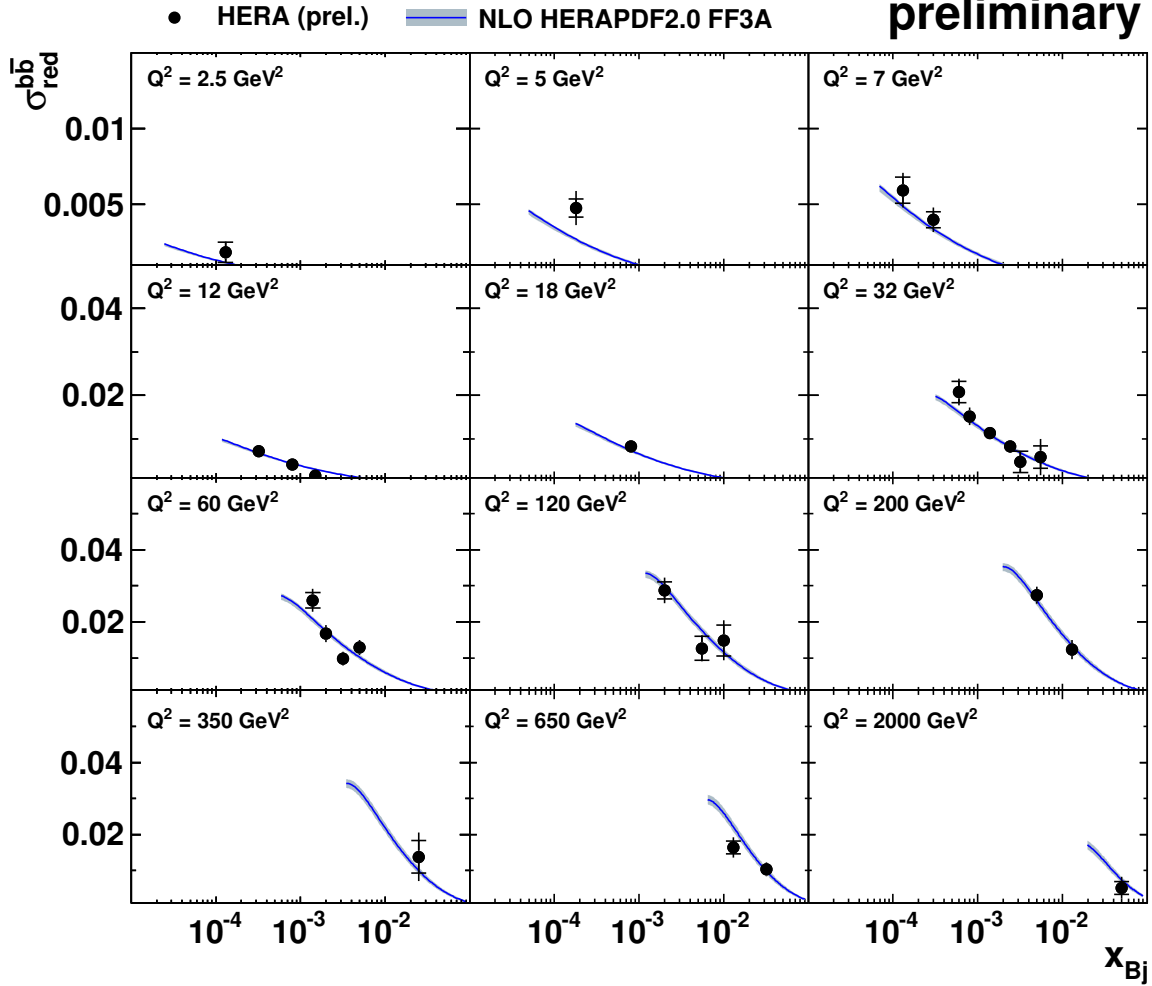


Figure 7: Combined reduced beauty cross sections $\sigma_{\text{red}}^{b\bar{b}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO QCD theoretical predictions obtained using HERAPDF2.0 FF3A with their uncertainties (solid line with band).

H1 and ZEUS preliminary

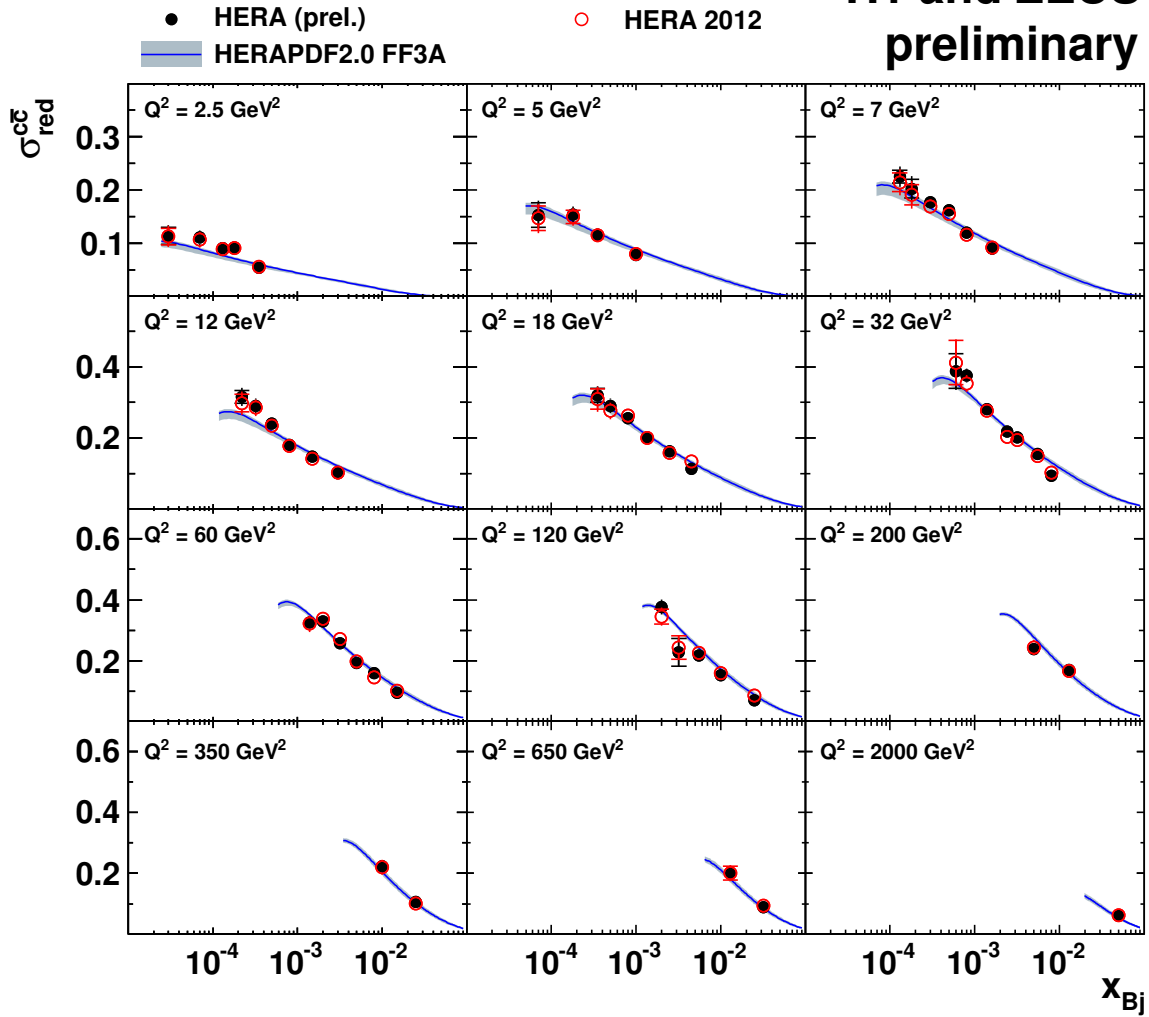


Figure 8: Combined reduced cross sections $\sigma_{\text{red}}^{c\bar{c}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the results of the previous combination, denoted as ‘HERA 2012’ (open circles), and the NLO QCD theoretical predictions obtained using HERAPDF2.0 FF3A with their uncertainties (solid line with band).

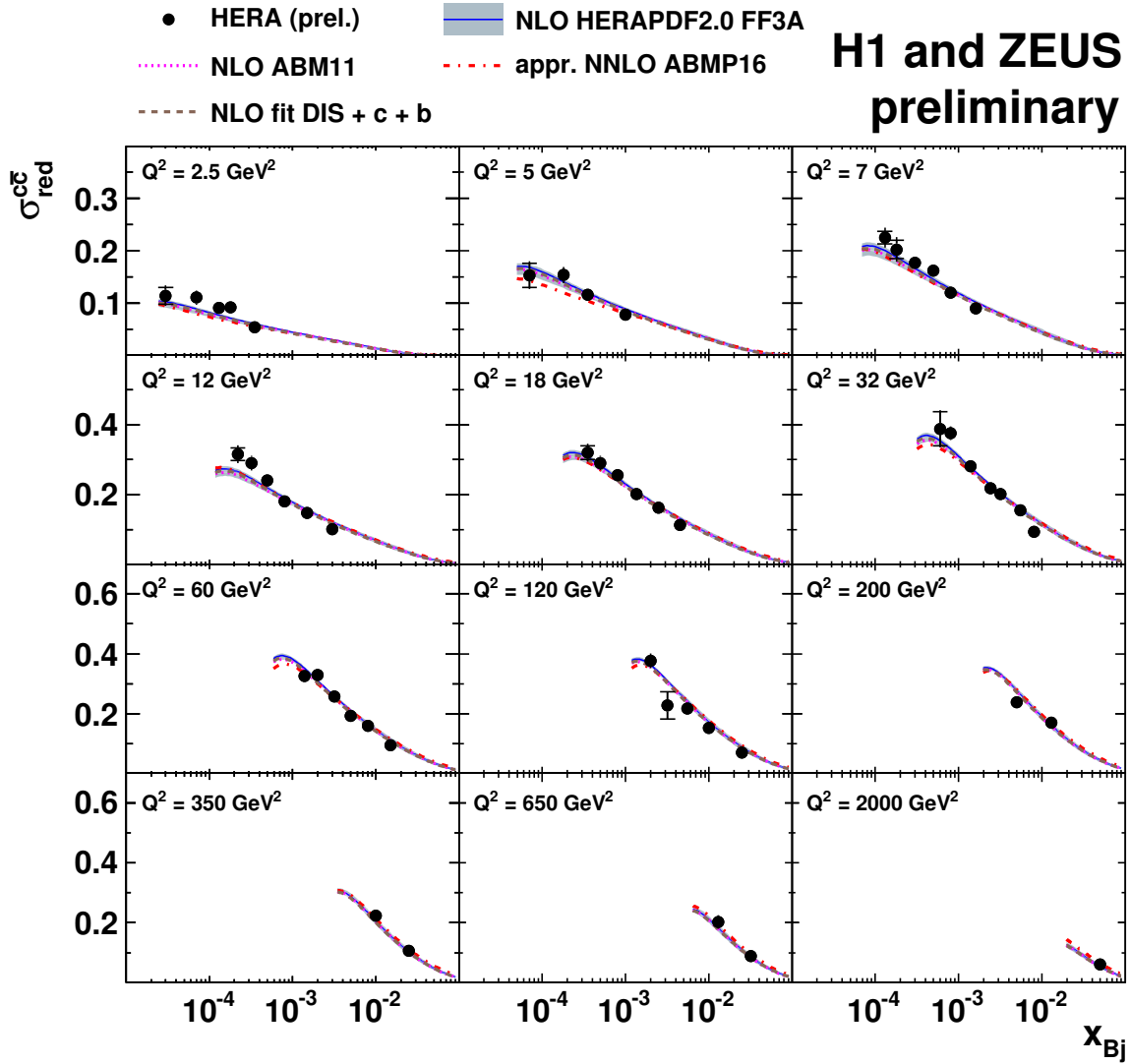


Figure 9: Combined reduced charm cross sections $\sigma_{\text{red}}^{c\bar{c}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO and approximate NNLO QCD theoretical predictions obtained using various PDFs.

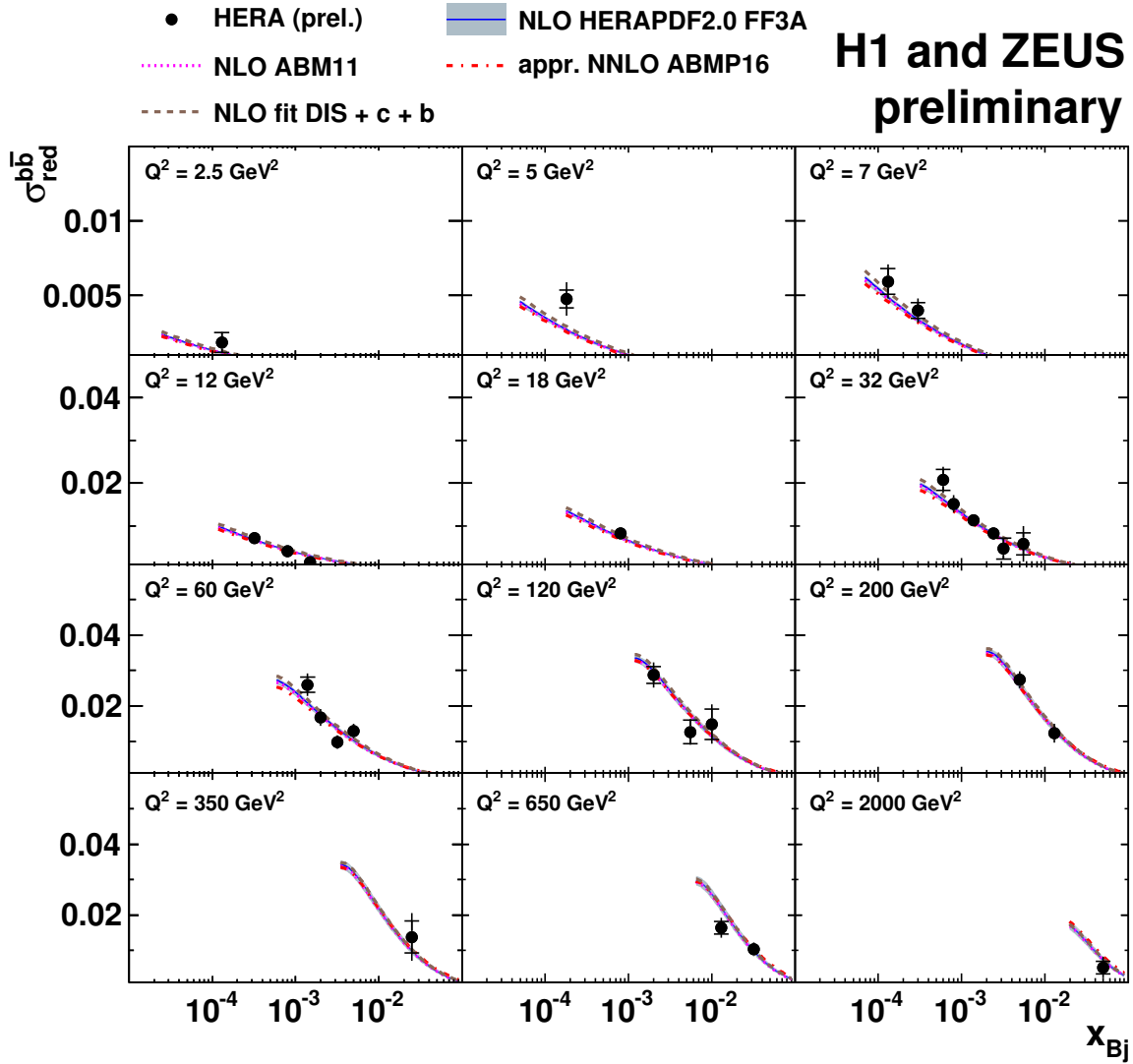


Figure 10: Combined reduced beauty cross sections $\sigma_{\text{red}}^{b\bar{b}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO and approximate NNLO QCD theoretical predictions obtained using various PDFs.

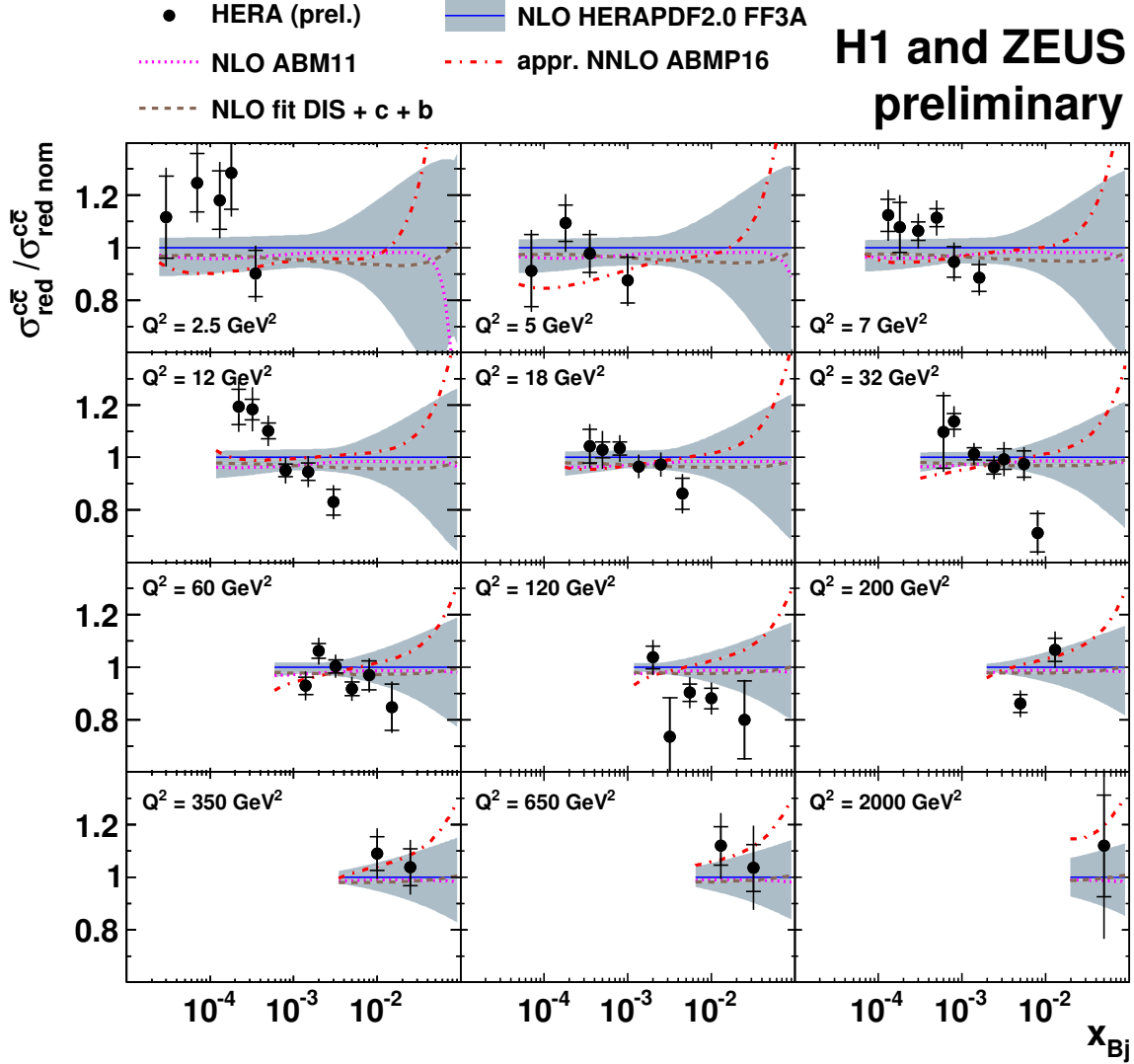


Figure 11: Combined reduced charm cross sections $\sigma_{\text{red}}^{c\bar{c}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO and approximate NNLO QCD theoretical predictions obtained using various PDFs, normalised to the predictions obtained using HERAPDF2.0 FF3A.

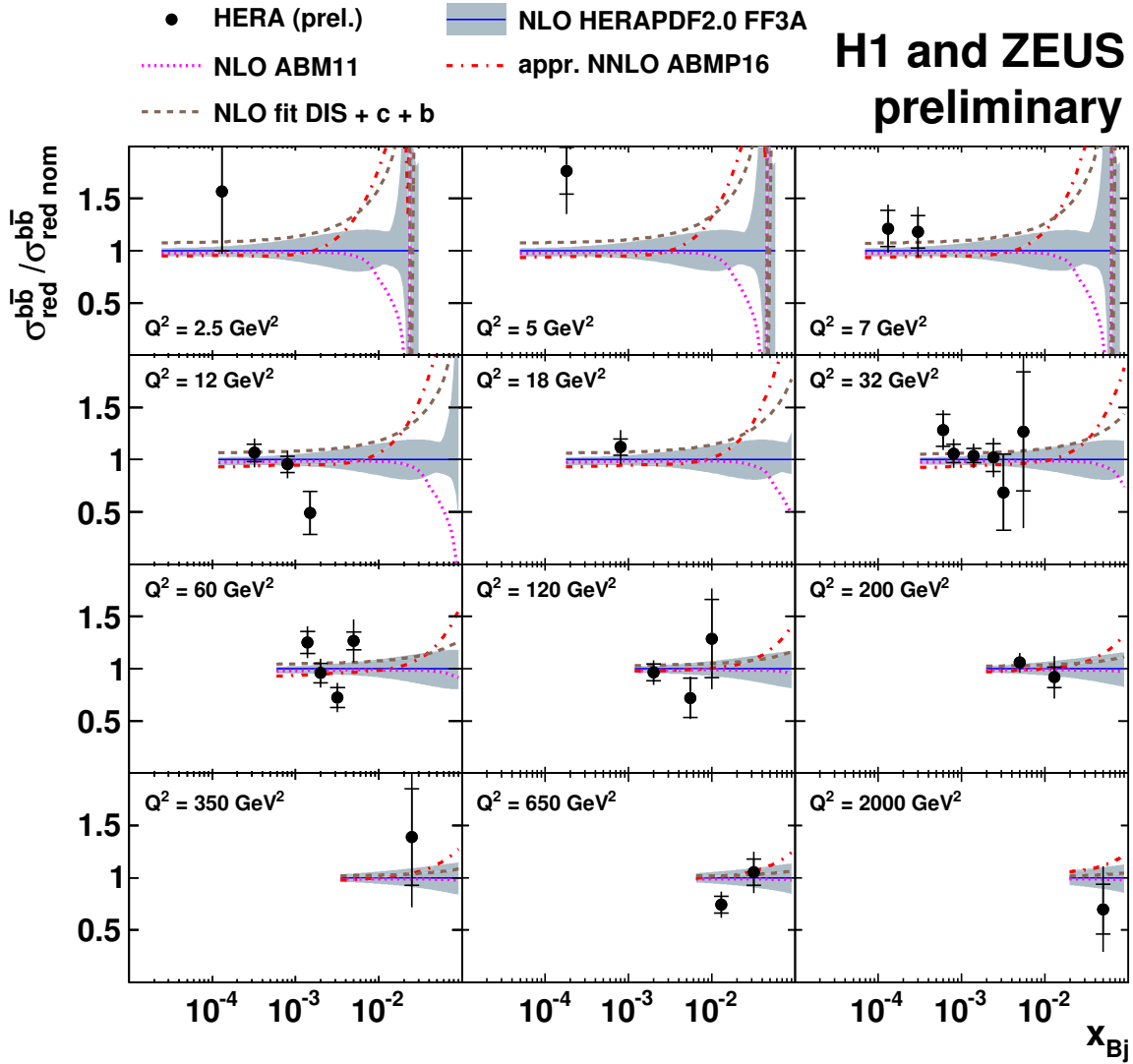


Figure 12: Combined reduced beauty cross sections $\sigma_{\text{red}}^{b\bar{b}}$ (full circles) as a function of x_{Bj} for given values of Q^2 , compared to the NLO and approximate NNLO QCD theoretical predictions obtained using various PDFs, normalised to the predictions obtained using HERAPDF2.0 FF3A.